LAND SURVEYING.

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ITS THEORY AND PRACTICE.

BY

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Illustrated with Numerous Diagrams and Examples.

LAND SURVEYING:

ITS THEORY AND PRACTICE.

CHAPTER I.

GENERAL VIEW OF THE SUBJECT.

*Purposes for which Surveys may be made—Different
   kinds of Surveys—Plan for Surveyors' Institution
   Examinations.*

It is as true of land surveying as of most other subjects,
that to know exactly what we have to do and for what we
are doing it, is a long way towards the satisfactory
accomplishment of the work. From those standpoints,
therefore, I propose to commence by taking a brief view
of the subject, with the object of giving the student a
general idea of the kind of work on which he is embarking,
before proceeding to deal in detail with the correct
methods of procedure in land surveying for one or other
of the various purposes and under the different circum-
stances and conditions met with in practice.

PURPOSES FOR WHICH A LAND SURVEY MAY BE MADE.

(**1**).—A land survey may be made for one or all of the
following purposes :—

(1) To enable the preparation of an accurate and comprehensive plan of the land to be surveyed, showing such details as shall be sufficient to convey to the mind all the essential elements connected with it, and drawn to a scale, so that all its parts may be in true proportion and from which the acreage may be correctly computed;

(2) To ascertain without preparing a plan either (*a*) the actual surface area of the land, such as would be required for finding the quantities of growing crops for a valuation thereof, or (*b*) the area of the land on a horizontal plane, such as would be obtained by scaling from an ordinary map; and

(3) To ascertain the slope of the land or the irregularities of the surface thereof; either (*a*) on defined lines, known as levelling; or (*b*) over a considerable area, known as contouring; and, in the former case, to show the result in a drawing representing a section through the land at the points where the irregularities are so ascertained, properly defined as " sections," and in the latter case to show the result on a plan by what are known as " contour lines."

DIFFERENT KINDS OF SURVEYS.

(**2**).—A land survey may be

(1) A chain survey, *i.e.*, a survey performed by the use of the chain without the aid of instruments for measuring the angles which the survey lines bear to each other;

(2) A trigonometrical survey, or a survey depending chiefly on trigonometrical calculations based on

angular measurements ascertained by the use of the theodolite or other similar instrument ;

(3) A survey which is partly a chain survey and partly depends upon trigonometrical observations and calculations ;

(4) A traverse.

SURVEY REQUIRED FOR THE SURVEYORS' INSTITUTION EXAMINATIONS.

(**3**).—I may state here that candidates for the Students' Proficiency and Associateship Examinations of the Surveyors' Institution are required to prepare plans and sections from actual survey of about 20 acres of land, and to forward finished drawings to the secretary not less than a fortnight before the respective examinations. The survey required is a chain survey, complete in itself, the accuracy of which has further to be shown by giving, on the plan, the angles which the principal lines bear to each other from readings taken with a theodolite on the land.

A longitudinal and cross section through the surface of the land has also to be prepared, necessitating the taking of levels in the field, generally at every chain distance.

The candidate is likewise subjected to examination in chaining, levelling, and theodolite work in the field, and sometimes in the use of other instruments, and the indoor work consists in plotting a plan from field notes, laying down on a plan a system of triangulation for a survey, reducing levels, plotting a section, and answering two papers of questions in writing.

It is not necessary to say more here on this point, the excellent publications of the Institution affording all necessary information.

I have now dealt in a general way with the purposes for which a land survey may be required, and it

only remains, before entering upon a description of
the correct methods of procedure under all the different
circumstances which are likely to occur in practice, to
consider what it is we shall have to do in making surveys
for the various purposes detailed.

(4).—In the case first mentioned, namely, in making a
survey for the purpose of preparing an accurate plan, we
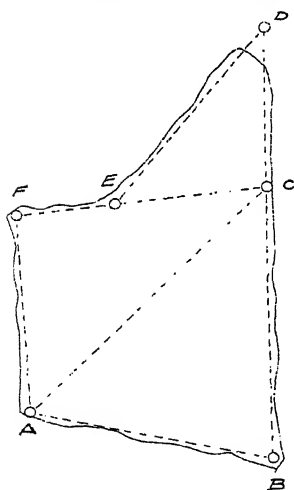shall require to ascertain the measurements on a hori-
zontal plane of lines compre-
hending the boundaries of
the land as *AB*, *BD*, *DE*,
EF, *FA*, in Illustration No.
1, and such other additional
lines as may be necessary to
enable us (*a*) to plot the
figure accurately on paper
(*b*) to show all the hedges,
ditches, fences, buildings,
etc., on the land, and (*c*) to
prove the accuracy of the
work when plotted.

I have said the measure-
ments for a plan must be
such as would be obtained on
a horizontal plane, and would

ILLUSTRATION No. 1.

lay stress on that point (leaving for the future an
explanation of the methods of arriving at those measure-
ments), and here perhaps, an illustration may be of
service.

Let Illustration No. 2 represent a section through
hilly ground, the measurements for a plan would be the

distance *AB*, which would be considerably less than the distance on the surface over the hills, the correct

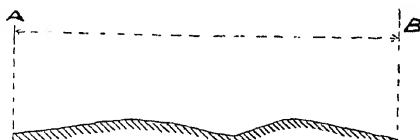
ILLUSTRATION No. 2.

measurements for computing the quantity of growing crops thereon.

A SURVEY FOR THE PURPOSE OF ASCERTAINING THE ACREAGE OF LANDS.

(**5**).—In the second case, a survey to ascertain the actual surface area of the land, we should merely have to divide the land up into imaginary triangles, and to take the measurement of the base of each triangle and the perpendicular, viz., a straight line from the apex of the triangle to the base, and at right angles thereto, these measurements being taken not on a horizontal plane, but on the surface of the land with a slack chain.

Suppose, for instance, we had a four-sided field, as shown in Illustration No. 3, we should have to divide it into two triangles, taking the measurement of the base *AB*, and the perpen-
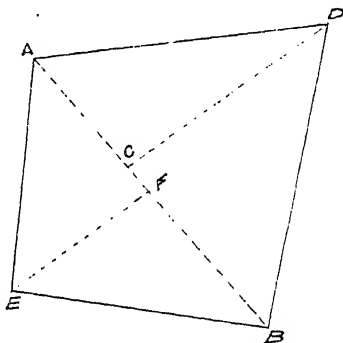

ILLUSTRATION No. 3.

dicular *EF* and *DC*, those measurements being sufficient to enable us to compute the contents or actual surface area.

Or again, in a survey to ascertain the area of the land on a horizontal plane, such as would be obtained by scaling from an ordinary plan or map, we should require to divide the land up in a similar manner, viz., into two triangles, but the measurements in this case must be those on a horizontal plane as shown by AB in Illustration No. 2.

(a) LEVELLING.

(**6**).—In the third case, levelling, or a survey to enable us to represent in a drawing a section through the surface of the land, we should require to ascertain the relative distances, vertically, from a truly level line above the highest or below the lowest point in the land along the line of section we are required to give, at which the inclination of the surface of the land changes, a, b c, d, e, f, g, in Illustration No. 4.
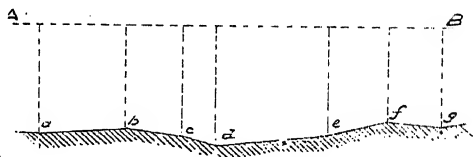

ILLUSTRATION No. 4.

Suppose it were possible for us to strain a cord or wire perfectly level above the surface of the ground, as indicated by the line AB, and then to measure with a rod, held perfectly perpendicular to that line, the distance therefrom to the several points. It is obvious we should then be in a position to plot a section, and this is practically what we arrive at, although in a different way, by the process of levelling to be described in a future chapter, the present allusion to the subject, as in the other cases referred to, being only intended to give the beginner an idea of what has to be done.

(b) CONTOURING.

(7).—In contouring, instead of, as in levelling, finding
the difference in level value between a number of points
on a given line or lines, we ascertain a number of points of
equal level value and marking these points on a plan, draw
a line through them called a contour line. By repeating
this operation at regular altitudes and showing same on
plan by a number of lines a good general idea of the rise or
fall from one point to another is given. The contour lines,
in the case of a hill in the form of a right cone, would be
concentric circles, the smaller circles representing the top
of the hill.

From these observations on the general scope of land
surveying and levelling, in theory and practice, we may
now be able to proceed to a fuller and more exhaustive
consideration of the whole subject.

CHAPTER II.

Schedule of Instruments to be Described — Parts Common to Various Instruments—The Telescope— Adjustments—Collimation—Paralax—The Magnetic Compass—Vernier—Tripod—Parallel Plates and Screws—Clamp and Tangent Screws—Vernier Reader.

Having already taken, as it were, a general survey of the subject, in order that the reader may follow the various methods of surveying for the numerous purposes, and under the varying conditions and circumstances met with in practice, with something like an insight into the matter under consideration, I ought now to deal with the methods of land surveying from a scientific and practical standpoint. Perhaps, therefore, of next importance to a knowledge of what we have to do and the purpose for which we are doing it, is a knowledge of the implements and instruments which we shall need and can procure for the work. I shall consequently be following the natural order of things in devoting some little time at this point to :—

    (1) A description of the various instruments used in land surveying and levelling ;

    (2) Some illustrations of the purposes for which such instruments will be required ; and

    (3) An explanation of the methods of using such instruments.

For the sake of clearness, I shall give an illustration of each of the instruments described.*

The following is a list of the instruments to which I must refer, given in the order in which they are dealt with. There are many others, but all those usually required by the land surveyor will be noticed. Readers desiring fuller information cannot do better than consult Mr. William Ford Stanley's excellent work on surveying and levelling instruments, in which their construction, qualities, selection, preservation, etc., are fully dealt with.

I shall commence with the more simple instruments, and where there are several which may be used for obtaining similar results, deal with the less complicated first, leaving the more complicated, but also more reliable and complete, to the last. In the list of instruments which follows, I have arranged them in groups according to the purposes for which they are required, and trust this may be found a convenient arrangement, giving as it does at a glance the instruments which may be secured for various purposes.

In the case of many of the instruments having common parts, in order to avoid needless repetition, before commencing to deal with the implements and instruments, I shall describe these common parts, after which they will be merely mentioned by name, it being presumed that the reader is either already familiar with them or will turn to that portion of the work and familiarise himself with the part of the instrument in question.

*The illustrations of the instruments have been reproduced from blocks kindly lent for the purpose by the well-known instrument makers, Messrs. Troughton and Simms, of 138, Fleet Street, E.C.; Mr. William Ford Stanley, Great Turnstile, Holborn, and Victoria Street, Westminster; Mr W. H. Harling, 47, Finsbury Pavement, E.C.; Messrs. Elliott Bros., 101 and 102, St. Martin's Lane, W.C.; Mr. J. H. Steward, 7, Gracechurch Street, E.C.; and Messrs. George Rowney and Co., 64, Oxford Street, W.

(8).—SCHEDULE OF IMPLEMENTS AND INSTRUMENTS.

PURPOSES FOR WHICH USED.	INSTRUMENTS.
The measurement of land	Chain and Arrows and Drop Arrows. Offset Staff. Steel Bands and Linen Tapes. Pedometer. Passometer. Perambulator or Viameter.
For fixing positions and ranging out lines	Pickets or Ranging Poles, Whites, False Pickets, etc.
For setting out right angles, etc.	Cross or Cross-staff. Semi-circumfrenter. Optical Square.
For ascertaining the relative verticle positions of different points in the earth's surface	Water Level. Spirit Level. Reflecting Level. Pocket Telescope Level. Surveyors' Level.
For taking the bearings of lines	Pocket Magnetic Compass. Prismatic Compass.
For measuring verticle angles	Ordinary Brass Quadrant. Clinometer Rule. Abney Clinometer. Troughton's Improved do. Troughton's Clinometer. Prismatic Clinometer.
For measuring angles horizontal or vertical	The Theodolite. The Box Sextant.
For measuring altitude by atmospheric pressure	Barometer.
For filling in (in the field) the details of topographical surveys	The Plane Table.

PURPOSES FOR WHICH USED.	INSTRUMENTS.
For copying, enlarging or reducing drawings	The Glass Table or Frame. The Pantagraph. The Idograph.
For computing areas from plans	Computing Scales. Planimeter.
For plotting plans	Scales of various kinds. Drawing Instruments.
Books, etc.	Field Book. Level Book. Traverse Book. Notebooks. Drawing Blocks, etc.
Materials	Papers, mounted and otherwise. Inks, Colours, etc.

Some Parts Common to various Surveying Instruments.

THE TELESCOPE.

(**9**).—The telescope of surveying instruments consists of the following chief parts :—

    1) Two tubes accurately fitting and sliding the one within the other.

    (2) Rack and pinion.

    3) Ray-shade and shutter.

    (4) Object-glass.

    (5) Eye-piece.

    (6) Diaphragm.

In Illustration No. 5, which represents a longitudinal section through the telescope :

    EP is the eye-piece.

    D is the diaphragm.

    CC´ Capstan-headed screws fixing the diaphragm.

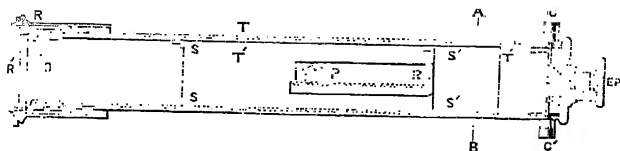T the outer tube.

T´ the inner tube.

ILLUSTRATION No. 5.

SS and S´S´ stops to prevent too much light entering the tube.

R the rack.

P the pinion.

R the ray-shade.

R´ the shutter.

(**10**).—The purposes of these various parts are as follows:—

The sliding tubes are for the purpose of allowing the object-glass and eye-piece to be brought into such relative positions as to focus the telescope for objects at different distances.

The rack and pinion is to enable the tube to be drawn in and out with very slow, smooth motion, by turning the milled-headed screw attached to the pinion, so as to focus the instrument with great accuracy.

The ray-shade, which is a short piece of larger tube, fitting accurately over the outer tube of the telescope, and sliding upon it at the object-glass end, with rims to prevent its being drawn right off or pushed too far on, is intended for use to shut off the sun's rays when necessary by drawing it out so as to project in front of the object-glass.

The shutter, which is merely a piece of flat metal hinged on a swivel, forms a protection to the lens of the objective when the instrument is not in use.

The object-glass and eye-piece are for the joint purpose of magnifying and focusing objects at a distance.

The diaphragm carries the "cross wires" or webs which render it possible accurately to sight points; as, for instance, the divisions on the levelling staff, or points between which angles, horizontal or vertical, are to be read by the theodolite, as will be explained when considering those important instruments. The "cross wires" are generally spiders' webs stretched across the diaphragm, and are seen when looking through the telescope, as shown in Illustration No. 6.
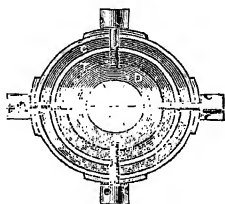

ILLUSTRATION No. 6.

ADJUSTMENTS.

(**11**).—Adjustments of the Telescope.—In order that the telescope may be in proper adjustment, the following conditions must exist :—

    (1) The point of intersection of the "cross wires" must fall accurately within the line of sight, passing centrally through the tube of the telescope from the eye-piece to the object-glass. This line of sight is called the line of collimation.

    (2) The lenses in the eye-piece must perfectly focus the "cross wires."

    (3) The object-glass and eye-piece must be brought into the proper relative positions to focus accurately the object sighted.

CORRECTION FOR COLLIMATION.

(**12**).—For the purpose of enabling the first of these, viz., the correction for collimation, to be secured, the diaphragm, to which the "cross wires" are attached, is
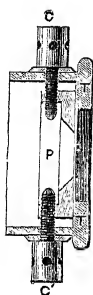
fitted within the tube of the telescope by
capstan-headed screws, accessible from the
outside, so that by screwing one and un-
screwing the other of either opposite pair, the
diaphragm may be shifted with very slow
motion either perpendicularly or laterally, or
both, until the intersection of the "cross
wires" are brought exactly into line with the
line of collimation. Illustration 7 is an
enlarged section through the diaphragm show-
ing how it is fixed and may be adjusted by
the capstan-headed screws.

ILLUSTRATION
No. 7.

CORRECTION FOR PARALAX.

(**13**).—For the purpose of enabling the second adjust-
ment to be obtained, viz., the correction for paralax, the
eye-piece carrying the lenses is made to draw in and out,
sometimes by a slow motion screw, but more often by the
hand, the end of the tube of the telescope, into which it
fits, being sprung for the purpose. Thus the glass may be
pushed in or drawn out until it exactly focuses the 'cross
wires.''

ADJUSTMENT OF THE OBJECT-GLASS.

(**14**).—To enable the third adjustment to be secured,
viz., the adjustment of the object-glass, the rack and pinion
already referred to, which is actuated by turning a milled
head on the side of the telescope, is provided, thus enabling
the object-glass to be brought closer to or taken further
from the eye-piece until the focus is perfected.

THE MAGNETIC COMPASS.

(**15**).—This instrument, as well as being made and used
separately, is attached to and forms part of most of the

larger and more complete surveying instruments. It consists of two principal parts :—

    (1)   A magnetic needle, and

    (2)   A dial.

The needle or bar is accurately and delicately balanced with jewelled bearing on a fine steel supporter, and being magnetised, when swinging freely on its axis, turns one end north or nearly so. The difference between true north and the direction taken by the magnetic needle is technically termed the " declination of the needle," and will be dealt with in its proper place. The present reference is merely to acquaint the reader with the magnetic needle as a part common to many surveying instruments to be described hereafter.
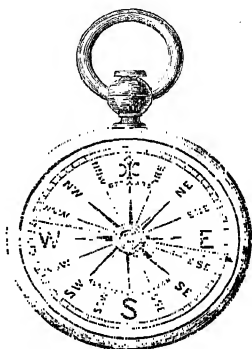

ILLUSTRATION No. 8.

The dial takes one of two forms, and is either a ring of metal on which the needle reads, or is a card or ring of metal attached to the needle, and revolves with it. One of the former class, in the form of a pocket compass, is shown in Illustration No. 8.

In either case, the dial, which is a complete circle, is divided into 360 degrees, and figured at the four cardinal points, North, South, East and West, and usually at each of the three intermediate points between each of these, viz., NNE, NE and ENE ; ESE, SE, SSE ; SSW, SW, WSW ; and WNW, NW, and NNW.

The instrument is enclosed in a dustproof case with a glass top, and is provided with a lever for raising the needle off its bearing when not in use, to prevent un-

necessary wear, and damage. The purpose of the instrument is to enable the bearings of lines to be ascertained.

THE VERNIER.

(**16**).—The vernier has for its object the more accurate reading of the divisions on finely divided scales, either straight or circular. As applied to the theodolite, and many other important surveying instruments, it consists of a circle, or arc, divided into degrees and usually half-degrees, with a short scale working upon it, edge to edge, in a convenient form for reading. The scale and vernier are in the case of the horizontal limb of the theodolite engraved on circular metal plates, which are placed face to face and turn on the same centre, the scales being engraved on the bevelled edges of these plates so that the coincidence of the divisions on the scale and vernier may be easily seen.

The upper plate turns with the motion of the telescope with which the objects, between which the angles are to be taken, are sighted, so that if the arrow head engraved on the vernier is set to zero on the scale when the first point is sighted, and the telescope is turned to sight the second point, the vernier follows its motion, and the arrow head registers the angle which would be formed by lines drawn from these points to a point immediately under the centre of the theodolite.

The present reference to this instrument is only intended just to give some general idea of how the vernier is applied to surveying instruments, it being believed that such an idea of its application will help the description of the contrivance itself, which is as simple as it is valuable.

A description of the vernier, and the method of using it, will now be given. Its application to each particular instrument will be readily understood when those instruments have been dealt with.

Illustration No. 9 shows a vernier as applied to a straight scale in which the main divisions are subdivided into ten equal parts. It will be seen that a space equal to nine subdivisions on the scale has been taken to form the vernier, and that space has been subdivided into ten equal parts; hence the space occupied by the vernier is one-tenth of a division in defect of the distance between the divisions on the scale, and consequently each subdivision
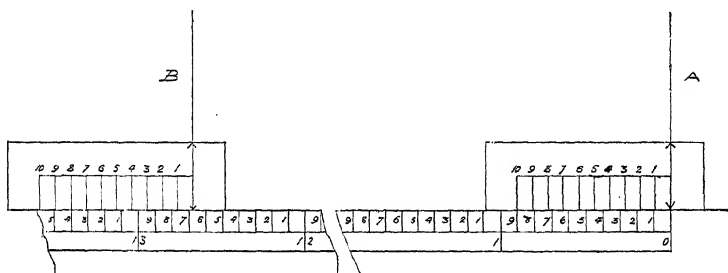

ILLUSTRATION No. 9.

on the vernier is one-tenth of a subdivision in defect of the distance between the subdivisions on the scale. It follows, therefore, that the first subdivisional line on the vernier at *A* is one-tenth of a subdivision on the scale behind the first subdivisional line on the scale, that the second subdivisional line on the vernier is two-tenths behind the second subdivisional line on the scale, the third three-tenths, and so on.

Again, it also follows that, if the arrow head on the vernier at *A* is placed past the zero division on the scale exactly one-tenth of a sub-division, the first subdivisional lines on the scale and vernier will coincide; if it were placed two-tenths of a subdivision past the zero division on the scale, the second subdivisional lines would coincide, and so on.

Once more, suppose we wish to measure the distance between the lines AB. We set the zero division of the scale to the line A and slide the vernier on the edge of the scale until the arrow head exactly cuts the line B. The arrow head is now past the 12th division and 6th subdivision on the scale, and we can, therefore, put down $12\frac{6}{10}$ or 12·6. Then, looking along the vernier we see that the 8th divisional line is the one which coincides with the subdivisional line on the scale, and we may now write down the total measurements $12\frac{6}{10}$ plus $\frac{8}{100}$ or 12·68.

Now, the main divisions on the scale in Illustration No. 9 are divided into ten equal parts, but the scales on surveying instruments are divided into degrees and half-degrees, and the vernier is formed by taking a space equal to twenty-nine or thirty-one half-degrees and dividing it into thirty equal parts; thus each subdivision on the vernier is one-thirtieth in defect or excess of a subdivision on the scale; that is, one-thirtieth of half a degree, or one-sixtieth of a degree, or a minute. Thus angles may be read to single minutes.

· In reading from the scale and vernier attached to surveying instruments, it must not be forgotten, however, that if the arrow head is past the half-degree, thirty minutes must be added to the reading taken from the vernier to give the full number of minutes.

In the larger instruments the degrees on the scale are generally divided into three parts, and in that case the vernier reads to thirty seconds.

The scale given in Illustration No. 9 is a straight scale, but it will be obvious that the vernier may be applied to circular scales of equal parts as readily as to straight ones; and that the readings are taken in exactly the same manner.

THE TRIPOD.

(**17**).—The tripod is merely the stand upon which most of the larger surveying instruments are supported. It is shown in Illustration No. 10, and consists of three (generally mahogany) angular legs bearing the appearance of having been cut out of one round pole. These legs are hinged at the upper end to a round thick plate of brass or gun-metal, which has a wormed neck S to fit into the female screw on the bottom of the parallel plates of the instrument hereinafter described. When not
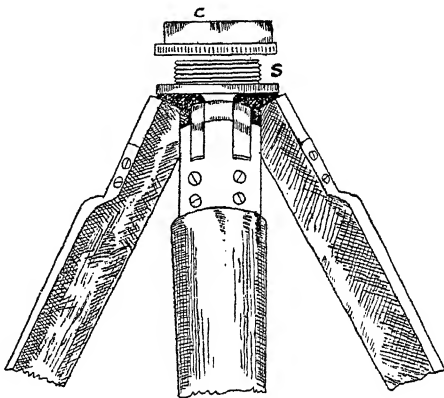

ILLUSTRATION No 10.

in use the legs are kept together by rings of brass, which are slipped on at the ends and pressed with a twisting motion to the thicker part of the legs where they grip them tightly. There should be a cap (C in illustration) to screw on to the wormed neck of the tripod to prevent its getting injured in transit.

PARALLEL PLATES AND SCREWS.

(**18**).—The parallel plates and screws, shown in Illustration No. 11, consist of two brass or gun-metal plates, a conical axis, and four (but sometimes three) congugate screws.

The purpose of this contrivance is to enable an instrument supported on it to be set up with its axis

perfectly plumb or vertical. The conical axis has a wormed neck E on the upper side to fit a female screw on the bottom plate of the instrument. It passes through the upper parallel plate, and accurately fits into the lower plate F.
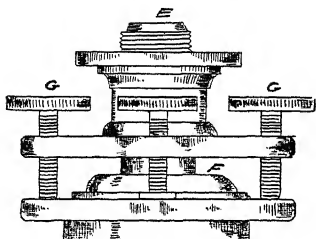
The conjugate screws GG press on the lower and pass through the upper plate, and have milled heads to make them convenient to handle. When the screws are turned so that the parts between the plates are of equal length, the plates are perfectly parallel and the conical axis is perfectly vertical. By screwing the one and unscrewing the other of either pair of opposite screws the upper plate may be brought perfectly level, and consequently the axis, which is made at right angles to it, will be perfectly vertical. The female screw in the collar under the lower plate fits on to the wormed neck of the tripod.

ILLUSTRATION No. 11.

There are various forms of parallel plate arrangements, but the above is intended simply to give a general idea of that part of the instrument, it not being within the province of this work to describe all the different designs or to discuss their particular merits or demerits. The object of the parallel plates in all cases is the same, and the working of them is similar.

CLAMP AND TANGENT SCREWS.

(**19**).—The purpose of this arrangement is to give to the vernier plates of various surveying instruments slow motion for accurate adjustment, without preventing their being turned rapidly by hand for approximate adjustment. We shall first consider this arrangement as applied to the vertical limb of a theodolite, and with a view of making

the explanation as simple and brief as possible, we shall first direct our attention to the tangent screw.

The stem of this screw, as shown in Illustrations Nos. 12 and 13, passes through a boss B and has a collar T
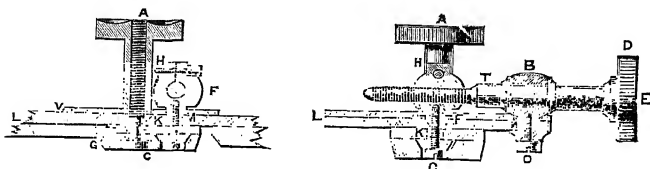

ILLUSTRATION No. 12.

on either side of it, which, without preventing its being turned, prevents it otherwise moving therein. The boss is firmly and permanently fixed to the clipping arm of the transit theodolite, as shown in Illustration No. 13.

The worm of the tangent screw is tapped to fit a nut which takes an external form similar to the boss. This nut is not attached to the clipping arm of the instrument, but to the clamping plates.

The stem of the clamping screw and the nut pass through small openings in the clipping plate which allow them a sufficient field of travel, so that when the milled
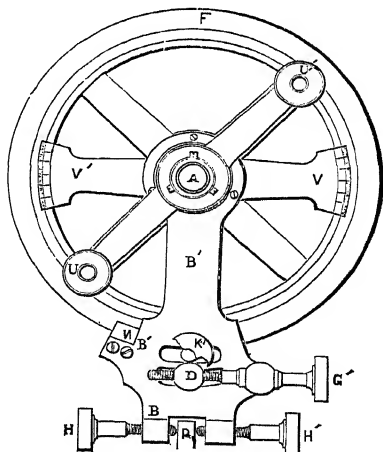

ILLUSTRATION No. 13.

head of the clamp screw is turned the clamp plate is drawn up tightly against the vertical limb of the instrument, into which it usually fits with a fillet and groove to afford steadiness of motion, but not so as to fix it or

prevent its being turned, and the vertical limb becomes connected to the clipping plate by the tangent screw.

The effect of turning the tangent screw, when the instrument is clamped, will be to give the vertical limb a slow, smooth motion.

In Illustration No. 12, which gives two sections at right angles to each other through the clamp and tangent arrangement, A is the clamp screw, B the boss through which the tangent screw passes, O screw connecting the boss to the clipping plate, D the milled head of the tangent screw, E screw fixing the milled head, T the collar of the tangent screw, L the clipping plate, C and K the clamping nut and plate, whilst G shows the clamping plate, grooved into the vertical limb of the instrument which it clamps when the milled head is screwed down.

In Illustration No. 13, B is the clipping plate. The other letters shown will be referred to later on, when the illustration will again be given.
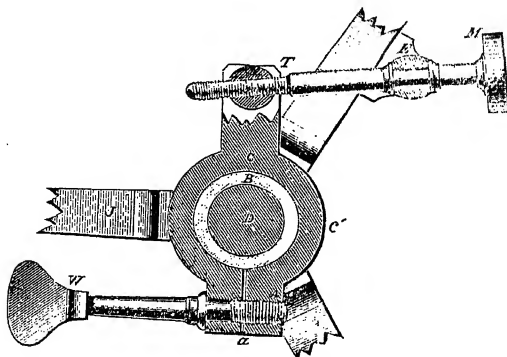

ILLUSTRATION No. 14.

Similar clamp and tangent screws, with certain modification of detail, are applied to the vernier plate of the horizontal limb and to the vertical axis of the theodolite,

and also to various other instruments, but the principle is the same in all cases.

Illustration No. 14 gives a section through the vertical axis of a theodolite, and shows the application of clamp and tangent screw to that part of the instrument. It will be seen that a collar C passes entirely round it, which tightened at a by turning the clamp W, clamps the whole instrument, that is, connects it to a fixed part of the plates by the tangent screw, when, instead of turning freely as before, it can only be moved very slowly for fine adjustment by turning the tangent screw M.

Until the clamp is applied the tangent screw M, the plate E connecting it to the parallel arm J, and the collar T, all revolve together round the centre D, so that the instrument may be turned about freely.

THE VERNIER READER.

(**20**).—This is merely a magnifying lens attached to instruments to enable the vernier to be more accurately read. It is made in various forms, and attached to the instrument in different ways.

CHAPTER III.

Instruments for Measuring Land and Ranging Out Lines.

The Chain — Gunter's Chain — Foot Chain — Metre Chain—Standard Chain—Arrows—Drop Arrows—Offset Staff—Steel Bands—Linen Tapes—The Pedometer — Passometer — Viameter — Perambulator — Pickets or Ranging Poles—False Pickets—Whites.

THE CHAIN.

(**21**).—There are four chains:

    (a) Gunter's chain ;

    (b) 50 or 100 foot chain ;

    (c) Metre chain ;

    (d) Standard chain.

GUNTER'S CHAIN.

(**22**).—The chain in the illustration is that known as Gunter's. It is the one almost always used, except when dealing with building land, when the 100 foot chain is usually employed.
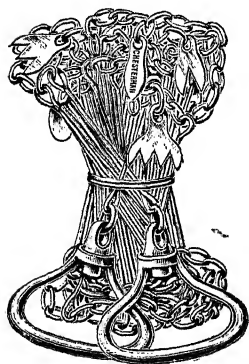
This chain is 66ft. long, divided into 100 equal parts called links, each 7·92in. long, and it has brass handles at each end, which are so made that the chain may revolve in them to prevent kinking when it is twisted.

ILLUSTRATION No. 15.

There is what is technically known as a "teller" at every tenth link from each end, to indicate its position in

the chain. These tellers are of either the suspended or inserted type. In the former case the links are indicated by the number of prongs on the teller, and in the latter case by the number of holes therein, as one for ten, two for twenty, etc. The tellers are of brass that they may be the more readily observed. The centre teller, that is the one at 50 links, is a round disc.

As there are two tellers in every chain indicating ten links, two 20 links, two 30 links, and two 40 links, it is clear that each of these is capable of a double reading, according to the end of the chain from which we start, and that one prong indicates either 10 or 90 links, two prongs 20 or 80 links, three prongs 30 or 70 links, four prongs 40 or 60 links. It is therefore of the utmost importance that this should be borne carefully in mind in surveying.

Chains are made of either iron or steel, the steel chains being the best, because, whilst stronger, they are also much lighter. (See Arts. 179 to 210.)

THE FOOT CHAIN.

(**23**).—This chain is either 100 or 50 feet in length, instead of 66ft. as Gunter's, and its links are each 12in. long, or, as is sometimes the case, 6in. long.

The disadvantage of this chain for ordinary purposes is apparent when we remember that the statute acre contains 100,000 square links (when the link is 7·92in.), and that, therefore, by using Gunter's chain, we may readily reduce square links to acres by simply cutting off five decimal places. In this way we have the advantages of the decimal system of calculation, which we lose in using the foot chain.

Where, however, land is being measured for the purpose of laying it out in building plots, or preparing a plan for

building purposes, in which case both the dimensions and scale are always required in feet, there is an obvious advantage in using the foot chain.

THE METRE CHAIN.

(**24**).—This chain is usually 20 metres long, and is divided into ten equal parts, at each of which it is marked. Thus there are tellers at every two metres, and a plain ring at every other metre. As the tellers are at every other metre, the first one will stand for two metres, the second for four metres, and so on.

These chains are useful where the metric system is in general use; but it would obviously be a mistake to take measurements in such a way that for ordinary purposes they would afterwards need converting into standard measure.

THE STANDARD CHAIN.

(**25**).—These are chains made with great care and accuracy, with brazed joints, and are not used for general work, but kept for the express purpose of testing the accuracy of other chains, or taking very important measurements.

NOTE.—Chains should always be carefully tested before use. A "Standard" may be laid down on or near the ground to be surveyed, at which the chain used may be frequently tested, or it may be compared with any chain known to be accurate or with the "Standard" chain.

It must be remembered, however, that the chain should be 66ft. 2½in. long, the excess being intended to compensate for the fact that the chain is not stretched perfectly tight, which is found impossible in practice.

The Royal Engineers set down a permanent standard for testing measures of various kinds in Trafalgar Square, London, and other standards are to be found in various parts of the country.   (See Art. 185.)

ARROWS.

(**26**).—These are pieces of iron or steel wire formed into a ring at one end and sharply pointed at the other, as shown in Illustration No. 16.   They are usually between 12in. and 18in. long, and about one-eighth of an inch thick.

Steel arrows are much to be preferred, and in view of the very slight difference in cost there is no reason for having iron arrows. Longer arrows than those usually required can be obtained when the nature of the growth on the land makes it desirable   A piece of red cloth is usually attached to the rings of the arrow heads to make them more conspicuous in the field.
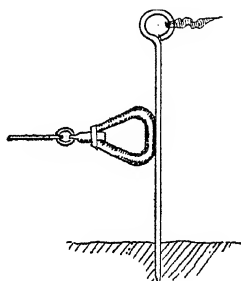
ILLUSTRATION No. 16.

Arrows are usually sold and used in sets of ten.   They are employed in chaining for marking the distance to which each chain length extends, so as to fix the point from which the measurement of the line may be continued when the chain is dragged forward.

In using the arrows one should be passed to the hand holding the chain handle each time as the chain is dragged forward, and should be held outside the brass handle as indicated in Illustration No. 16, so that it may be pressed into the ground at the right time.   It should be put in the ground outside the chain handle, in a direct line with the chain, and perfectly plumb.   (See Arts. 180 and 181.)

DROP ARROW.

(**27**).—This is similar to an ordinary arrow, but weighted, so that when dropped it will fall perpendicularly, point downwards, and enter the ground. It is used when chaining over sloping ground, and the horizontal measurement is required.

In using the drop arrow the surveyor must hold it in the hand by the chain handle, as an ordinary arrow, but instead of being forced into the ground (being above the point at which it should be inserted), it is allowed to fall, when it marks the point. It is generally withdrawn and another ordinary arrow inserted in its place. (See Arts. 189 and 190.)

THE OFFSET STAFF.

(**28**).—This is sometimes a square or flat rod, but more often a round tapering pole, as shown in Illustration No. 17, ten links long, with a hook at the thinner end, and shod at the thicker end. It is divided into ten equal parts, one link in length each, by being painted with white and black bands alternately. Its proper use is to measure offsets or short distances from a given point, in and at right angles to the chain line to objects, the positions of which it is desired to fix, the most usual being the irregular outlines of hedges and ditches. It is also used occasionally temporarily to establish a station from or to which lines are to be run, or for fixing any given point, hence its being shod. It also sometimes serves to drag a chain through a hedge, etc., the hook at the thinner end being intended for that purpose. (See Arts. 167 and 168, and 208 to 210.)

ILLUSTRATION No. 17.

STEEL BANDS.

(**29**).—These are flat thin continuous bands of steel, about three-quarters of an inch wide, are made of standard length, and divided into links, feet or metres, as the case may be, by studs or engraving. They have handles at each end like ordinary chains, and when out of use are coiled on a block and protected by a steel or wooden cross as shown in Illustration No. 18.
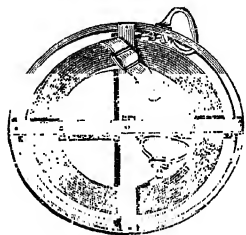
ILLUSTRATION No. 18.

They are very light and convenient in some respects, and have the advantage of being less liable to stretch in wear than ordinary chains. On the other hand they are somewhat delicate, and the readings are not nearly so conspicuous. For ordinary uses a good steel chain is preferable.

The use of steel bands is similar to that of chains, but they are more often used for taking important measurements, owing to their trueness. The method for using them is precisely similar to that described for chains.

LINEN TAPES.

(**30**).—These are almost too well known to need description. They consist of a continuous painted tape measure, made of various lengths, and are usually divided into feet and inches on one side and links on the other. When not in use the tape is protected by a leather case, into which it may be coiled by turning the small brass handle, which causes a block to which the
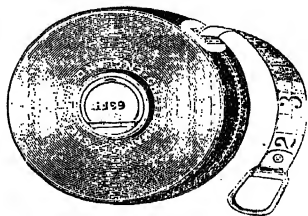
ILLUSTRATION No. 19.

Illustration No. 19 shows the instrument with the handle
closed, whilst Illustration No.
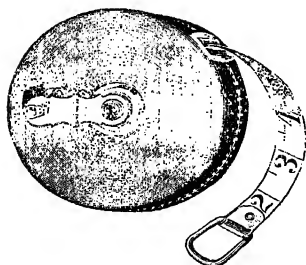20 shows it open for winding
up the tape.

ILLUSTRATION No. 20.

The linen tape is some-
times used for taking offsets,
but the offset staff is far pre-
ferable for the purpose ; first,
because the surveyor can use
the latter without assistance,
and secondly, because the
former would be very unsuited for use in wet weather.
A light 50-link chain is preferable to a tape for taking
offsets. The proper use of the tape is to measure any
building on the land, the block plan of which it is
desired to show, and for this purpose where a Gunter's
chain is being used, the measurements should be taken
in links, for obvious reasons. (See Art. 208.)

THE PEDOMETER.

ILLUSTRATION No. 21.

(**31**).—This is a useful little
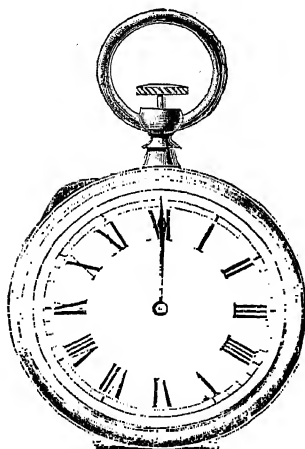instrument when pacing, for
by its aid lines may be roughly
measured by simply walking
over them. It is similar in
appearance and size to an
ordinary watch, the dial being
divided into twelve equal
parts, which indicate miles,
and each of these parts into
four equal parts indicating
quarter miles.

The chief parts are a
horizontal pendulum supported
by a spring, a ratchet, which

is actuated by the movements of the pendulum, and wheels geared and connected with the ratchet and the hands on the dial. There is also a small screw which by being raised or depressed lengthens or shortens the field covered by the pendulum when in motion and so enables the instrument to be regulated roughly to the length of the step of the person using it, a factor which must obviously be considered.

The action of the pendulum is dependent on the jerk of the body caused by each step as it is taken.

The purpose of the instrument is to ascertain the distance walked.

To use the instrument it is placed in the waistcoat pocket, and by means of a clip is fixed in a perpendicular position. The distance paced is read from the dial without calculation. (See Art. 174.)

THE PASSOMETER.

(**32**).—This instrument, although working very similarly to the pedometer, has the great distinction that, whereas the pedometer registers distance travelled, the passometer only registers the number of steps taken. It is very useful to the land surveyor in pacing, who, having trained himself to pace accurately 3ft. at a step, can calculate the length of lines very approximately by its aid. He might, of course, do the same by counting the steps, but he is relieved of the necessity of doing so by using a passometer, and is left free to make other

ILLUSTRATION No. 22.
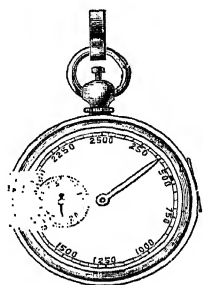
observations while perambulating the land, a very considerable advantage.

The dial has a large and a small hand. The small hand

travels over a face like the counter-sunk face of the second
hand of a watch, but divided into 50 equal parts, each of
which indicates a step, so that the small hand registers up
to 50 steps, and the large hand moves forward one sub-
division on the large face as each 50 steps is registered by
the small hand.

The whole dial is divided into 10 equal divisions, and
these are sub-divided into 50 parts each, and as each
single sub-division equals 50 steps, the instrument is
capable of registering up to 25,000 steps.

The purpose of this instrument is to record the number
of steps taken in walking any distance.

To use this instrument it is merely clipped in an
upright position in the waistcoat pocket as directed for the
pedometer, and the readings are taken from the dial
without calculation. (See Art. 174.)

THE VIAMETER OR PERAMBULATOR.
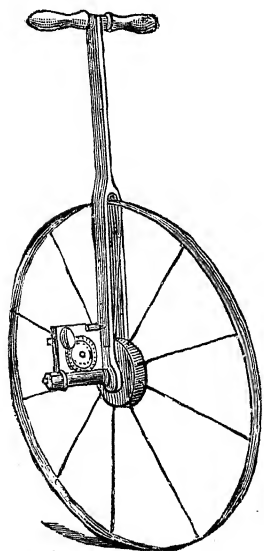
ILLUSTRATION No. 23

(**33**).—This instrument con-
sists of a wheel of exactly 6ft. in
circumference, a handle bar con-
venient for wheeling it along,
and an indicator which registers
the distance travelled. It is
shown in Illustration No. 23.

The instrument differs some-
what according to the fancy of
the manufacturers, but the regis-
tration is effected by rotating
dials actuated by a screw con-
nected with the axle bar of the
wheel, which is extended on one
side thereof and passes through
the case of the register. One
dial registers yards and the other

furlongs and miles. The yard dial is divided into 110 equal parts, each equal to two yards, and numbered for every twenty yards, or one complete revolution of the dial indicates a furlong. The mile dial is divided into 104 equal parts, and is numbered at each eighth part or at each mile.

The dials rotate, and the indicator is fixed. The divisions on the dials are numbered in reverse directions. The furlongs and miles are indicated by the slower rotation of that dial, and the distances are recorded by the amount it consequently gets behind, so to speak, the other dial.

The working parts are so arranged that the wheel must revolve once more to complete the revolution of the mile dial than to complete the revolution of the yard dial, so that each time the yard dial completes a revolution the mile dial is one sub-division behindhand, and in that way indicates a furlong ; this is managed by there being a greater number of teeth by one on the mile dial than there is on the yard dial, and as each rotation of the wheel moves the dials one of their divisions, the wheel must rotate once more to complete the revolution of the one dial than is necessary to complete the revolution of the other.

The instrument is capable of registering thirteen miles without being reset.

The purpose of this instrument is to measure the lengths of roads, particularly in large towns where traffic offers difficulty to other methods of measurement. Provided it is driven quite straight and over smooth ground it accurately registers the distance travelled.

The viameter is used by merely pushing the wheel along in a straight line, and the distances are read from the dials without calculation.

PICKETS OR RANGING POLES.

(**34**).—These are round poles varying from 5ft. to 12ft. in length, and are usually about 1¼in. in diameter at the thicker end, tapering slightly, as shown in Illustration No. 24. They are shod with iron and are painted with alternate bands of different colours, the most favourite being red, white and black. Some pickets have a cross-bar passing through the iron shoe, to enable them to be pressed into the soil with the feet.

It is usual, where the pickets are required to be seen at a great distance, to attach to the top a white or red flag to make them more conspicuous.

Pickets are used for establishing " stations " or points between which lines are to be measured, or from which trigonometrical observations are to be made.

In using pickets they are merely driven into the ground, perfectly plumb, at the point desired to be/ fixed. When they are withdrawn, if

ILLUS. No. 24.

further reference to the stations is likely to be necessary, pegs, known as false pickets, numbered to correspond with the numbers of the stations on the key plan, are put down in their places. (See Art. 169.)

FALSE PICKETS.

(**35**).—These are pieces of wood about 1¼in. in diameter, sharply pointed at one end, and cut off slanting at the other, as shown in Illustration No. 25. They are numbered on the slanting top

ILLUS. No. 25.

to correspond with the number representing the station, the purpose of the bevelled top being to make the number more conspicuous. Sometimes they have a cross cut at the top, the lines forming the

cross being diameters of the circle which would be obtained in a section at right angles to the picket, so that the plummet of the theodolite may be suspended exactly central over the station, the position of which the false picket marks.

False pickets are required to retain the position of a station after the picket has been withdrawn. They are merely put into the hole originally occupied by the picket, and they are therefore the better for being similar in size and shape to the foot of the picket.

WHITES.

(**36**).—These are generally pieces of ordinary lath used for the purpose of what is technically known as "boning out" lines. Thus, where the lines are very long or important, or the stations difficult to see, the lines are more definitely fixed by pieces of lath being put in the ground in a direct line between the stations, at about every chain length. This is for the purpose of helping the chainmen to drive a perfect line.

In "boning out" lines the surveyor stands behind one of the pickets in such a position that it covers the picket at the other end of the intended line, and, by a motion of the hand, directs one of the chainmen where to put the whites in so that they may be perfectly in line with each other and the two pickets.

CHAPTER IV.

The Cross or Cross Staff—The Semi-circumfrenter—The Optical Square.

THE CROSS OR CROSS STAFF.

(**37**).—This instrument, in its most simple form, consisted merely of a perfectly square piece of board, with grooves cut on it from angle to angle, and therefore crossing each other at right angles, supported on a staff, generally the offset staff, on to the top of which it was
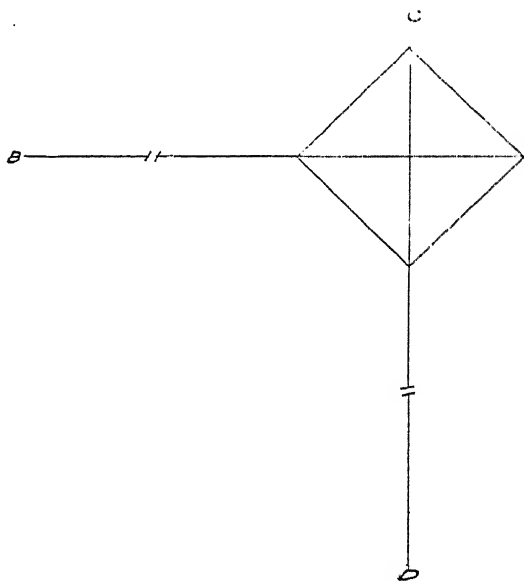

ILLUSTRATION No. 26.

made to fit with a socket. It will be seen from Illustration No. 26 that, if this instrument is put down at

the corner of a field, the sides of which are at right angles to each other, and the board turned until looking from *A* to *B,* the groove is in perfect line with the boundary of the field ; looking from *C* to *D,* the groove in that direction will also be in a right line with the adjoining boundary, and if such lines are not formed it will prove that the sides of the field are not at right angles.

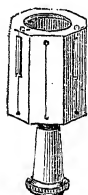The instrument is now made in a more elaborate form, as shown in Illustration No. 27, although the·principle is the same. Slots in the head, with fine wires as sights, have taken the place of the grooves on the flat board, and some are made with a double head with clamp and slow-motion screws, divided cylinder and vernier, as shown in Illustration No. 28, so that they may be used for taking horizontal angles.
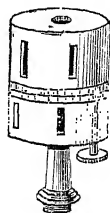
ILLUSTRATION
No. 27.

ILLUSTRATION
No. 28.

Owing, however, to the short distance between the sights, the instrument, no matter how much elaborated is a failure, and only fit for the most approximate purposes. If it is used at all the head should be of a large size, to give plenty distance between the sights, and as no amount of elaboration can make it a satisfactory instrument, it is better in its simple form, the best being that made similarly to the semi-circumfrenter, shown in Illustration No. 29, only with the sights fixed at right angles to each other instead of being movable.

This instrument, as will be understood from its description, is used to ascertain whether the adjoining sides of fields are at right angles with each other, or to find at what point in a chain line, a line at right angles to it, dropped from an opposite angle of the field, will cut. This is generally required to be ascertained to enable base lines and perpendiculars to be measured for the purpose of

computing the area of growing crops.   It has likewise
been very improperly used to assist in taking long offsets,
setting out buildings, etc.   (See Arts. 5, 38 and 39.)

TO USE THE CROSS STAFF.

To use the cross to ascertain the point in a chain line
which will be cut by a perpendicular from an opposite
angle of a field, move the staff along the chain line until
you find the point, where, looking through the opposite
slots, the hair lines accurately cut, respectively, a picket
at some distance in the said chain line and the corner of
the field from which the perpendicular is to be measured.

The method of using the instrument to ascertain whether
the adjoining sides of fields are at right angles with each
other, setting out rectangles, etc., is given in the
description of the instrument.

THE   SEMI-CIRCUMFRENTER.

(**38**).—The semi-circumfrenter is capable of performing
all that the more elaborate cross staff claims to perform,
and is a far more satisfactory instrument.   It consists of a

brass semi-circle, divided into
degrees and minutes, with
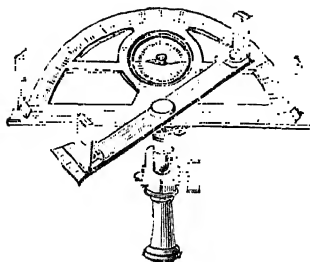sights at the extremity of its
chord or diameter, as shown
in Illustration No. 29.

Revolving on the centre
of the circle of which the
graduated limb is half, there
is another plate or strip
of brass, which also has
sights at its extremities, and

ILLUSTRATION No. 29.

bevelled ends, on which verniers are formed.   The semi-
circumfrenter is mounted on a staff shod at the end for
entering the ground.

The whole instrument may be turned on the staff to adjust it roughly for use, but is fitted with clamp and slow-motion screws for accurate adjustment.

The purpose of this instrument is to measure horizontal angles, and the method of using it is apparent from the illustration, and description of the cross staff already given.

THE OPTICAL SQUARE.

(**39**).—This instrument, which is shown in section in Illustration No. 30, is a great favourite with land surveyors, owing to its simplicity, absence of working parts to get out of order, portability and approximate accuracy.

We may regard it as consisting of two parts :—(1) The mirrors ; and (2) the round metal box enclosing them.

The illustration will serve to convey a general idea of the instrument and its principles. It consists of two small mirrors fixed at an angle of 45 degrees with each other, and perfectly vertical to the base or bottom of the box. There is a window in the rim of the box opposite the face of one of the mirrors and sight holes at right angles thereto. The mirror, which is in the line of

ILLUSTRATION No. 30.

direct vision through the sights, is only silvered half way up so as not to obstruct the view. The upper part of the box turns on the lower, so as to close all openings when the instrument is not in use.

For its accurate working the instrument depends on the principle that the angle of incidence equals the angle of reflection, or, in other words, that at whatever angle the image of an object incides on a mirror with perfectly flat surface, it will be reflected at an equal angle, and, there-
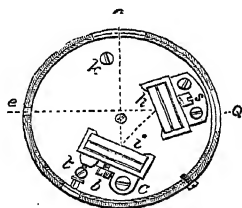
fore, by having two mirrors fixed at an angle of 45 degrees
with each other, as shown in Illustration No. 30, the line
of incidence *o i* from the object to the mirror is at right
angles with the line of sight *e Q* to the reflected object in
the other mirror at *h*.   Therefore, if we sight an object
through the sights by direct vision, any other object, a
line from which to a point immediately under the centre
of the instrument would be at right angles with the line of
sight, will be reflected in mirror *h*, and seen to stand
immediately under the sighted object.

The screws marked *t, k, c, b* and *s* in Illustration No. 30
fix and adjust the mirrors.

PURPOSES FOR WHICH THE OPTICAL SQUARE MAY BE USED.

(**40**).—If the foregoing description of the instrument is
understood, the following purposes for which it may be
used will be apparent :—

> (1) To range out a perpendicular from a given point
>   in a chain line ;
> (2) To ascertain the point at which a chain line
>   will be cut by a perpendicular let fall from a
>   given point on either side of it.

The instrument is much used for laying out right angles
in the field, particularly in taking offsets at right angles to
the chain line, and also in connection with the valuation
of growing crops, in which case the measurement of base
lines and perpendiculars is the most simple means of
arriving at the quantities for that purpose.   (See Arts. 5,
167 and 168.)

TO ERECT A PERPENDICULAR TO A GIVEN LINE AND AT A GIVEN POINT THEREIN, WITH THE OPTICAL SQUARE.

(**41**).—Stand with the centre of the instrument over
the given point and sight a pole at a considerable distance
in the given line, and in the direction which will turn the

window of the instrument on the side of the line on which the perpendicular is to be laid out. Send a man with a pole out into the field some distance, and cause him to move backward and forward until the reflection of the pole he holds is seen in mirror h, immediately under the pole sighted by direct vision, which will give the station to which the line must be run in order that it may be at right angles with the given line.

TO ASCERTAIN THE POINT AT WHICH A CHAIN LINE WILL BE CUT BY A PERPENDICULAR LET FALL FROM A GIVEN POINT ON EITHER SIDE OF IT.

(**42**).—Fix two poles in the line on which the perpendicular is desired to be dropped, at a considerable distance from where it is likely to cut, and in such a position that when, looking in the direction of the line which they will give, the window of the instrument will be turned towards the given point from which the perpendicular is to be let fall, which should also be marked by a picket. Walk along the line, all the time sighting the station in it, until the pole at the given point from which the perpendicular is to be dropped is reflected in mirror h, immediately under the one seen by direct vision; put in a pole on the line immediately under the centre of the instrument, which will be the point at which the line will be cut by the perpendicular.

Note.—A plummet may be used if considerable accuracy is required, to fix the point immediately under the centre of the box.

CHAPTER V.

LEVELLING INSTRUMENTS.

*The Water Level—Spirit Level—Pocket Telescope Level
—Reflecting Level—Y Level—Dumpy Level—Tests
—Adjustments — Collimation — Parallax — Levelling
Staff — Purpose — Use — Method of Practice —
Directions to Staff Bearer—Staff Plate—Staff Pad.*

The action of gravity is the soul of all levelling instruments, the most simple representatives being the plummet, and the position which liquid naturally assumes when unaffected by counteracting agencies.

THE WATER LEVEL.

(**43**).—This consists of a length of flexible or inflexible tube, with an open glass vessel at each end, sufficiently charged with coloured water that it may be in
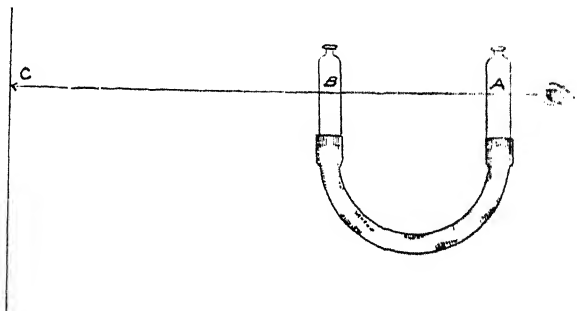

ILLUSTRATION No. 31.

view in the glass vessels. By placing the tube in the direction of the line on which it is desired to ascertain the levels the water line in the vessels AB (Illustration 31)

will give the level line, by which the graduations on a staff C, held at the desired points, may be read, and by which means their relative heights above or below an assumed base or datum may be ascertained. The instrument is held by the glass vessels, one close to the eye and the other at arm's length, and a line is sighted through the points assumed by the surface of the liquid, to the staff which is held at the various points at which the levels are to be taken.

THE SPIRIT LEVEL.

(**44**).—The spirit level consists of a glass tube nearly filled with spirit, so as to leave a small bubble of air in the tube. It is too well known to need description.

Provided the tube itself is very true, and that where it is mounted, as it almost always is as a means of protection, this is done with great accuracy, so that the bubble is in the centre of its run when the level lies on a perfectly level surface, it is a useful instrument for levelling walls, floors, steps, etc.; but, apart from other instruments and contrivances, it cannot be applied to the levelling of land on a large scale.

THE POCKET TELESCOPE LEVEL.

(**45**).—This is an application of the spirit level or bubble tube, in a form to make it more serviceable for levelling on a larger scale. It consists of a telescope with
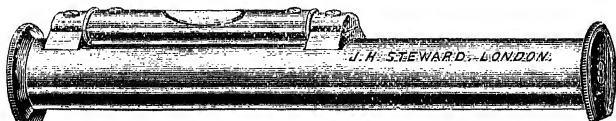

ILLUSTRATION No. 32.

a small bubble tube fixed on it, parallel with the line of collimation. (See (1) Art. 11.) There is a small hole on

the top of the tube of the telescope, under the bubble tube, through which the image of the bubble is reflected into a silvered reflector fixed on an inclined plane with the tube of the telescope, and extending only half way across it, and there is a line marked on the reflector which will bisect the reflected image of the bubble when the bubble itself assumes a perfectly central position, or, in other words, when the telescope is held perfectly level.

The telescope is provided with cross webs, or a pointer, which takes the place of them, and the object-glass is focused by means of a rack and pinion actuated by a milled head. (See Arts. 9 and 10.) By this contrivance the operator at the same time as looking to a levelling staff can see whether he is holding the instrument in a perfectly horizontal position, and by this means a truly level line from the eye to the object sighted is secured, by which the staff may be read.

This instrument may be used in any case where the difference of level of two or more points in the earth's surface is required, but it would not be sufficiently accurate for important work. In taking a series of levels a staff would be required. (See Art. 60.)

TO USE THE POCKET TELESCOPE LEVEL.

(**46**).—Hold the instrument steadily in the left hand, turn the milled head until the telescope perfectly focuses the levelling staff or object sighted, move the hand to bring the level perfectly horizontal, as evidenced by the reflected image of the bubble being cut by the line on the reflector, and note at the same time the reading of the cross webs or pointer on the staff. A description of the methods of simple and compound levelling will be found in Chapter XXII. (See Arts. 292 and 293.)

THE REFLECTING LEVEL.

(**47**).—This little instrument, which is shown in Illustration No. 33, consists of a piece of plate glass silvered half way across, and so balanced as to hang perfectly plumb from a swivel and ring by which it is handled when in use. The purposes for which the instrument may be used are similar to those applicable to the pocket telescope level. (See Art. 45.)
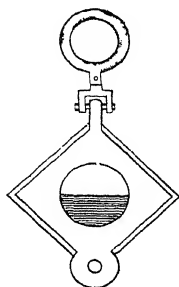
TO USE THE REFLECTING LEVEL.

The operator at the same time as looking forward to the levelling staff, held at the points at which it is desired to ascertain the levels, observes in the

ILLUSTRATION
No. 33.

silvered glass that the reflected image of his own eye is exactly bisected by the meeting line of the plain and silvered parts of the glass, and by this means is assured that the line from the eye to the staff is perfectly level.

THE LAND SURVEYOR'S LEVEL.

This is the one thoroughly reliable instrument for levelling operations on a large scale, and where great accuracy is required. There are many forms of this level, each having certain advantages and disadvantages, but it will be quite unnecessary to describe more than two, viz., the Y level and the Dumpy level. The others are all more or less similar and generally take their names from their inventors. The object, too, of each is similar, viz., to supply an instrument which will give a level line of sight through a telescope, which, being directed to a graduated staff held at the various points at which it is desired to ascertain the levels, enables the distance from

that level line of sight to the surface of the earth at each
of those points to be taken

The subject of levelling is one of great importance,
and is fully dealt with in Chapter XXII. We are at
present only concerned with a description of the
instruments and the method of adjusting and using them.

Although there are many points of similarity between
the Y and Dumpy levels, and the manner of effecting the
adjustments which it is within the power of the operator
to make, yet there are differences which make it advisable,
for the sake of clearness, to keep their descriptions quite
separate, and that course will therefore be followed.

THE Y LEVEL.

(**48**).—This level, which is shown in Illustration No.
34, takes its name from the fact that the telescope
is supported in what are known as Y's, the chief
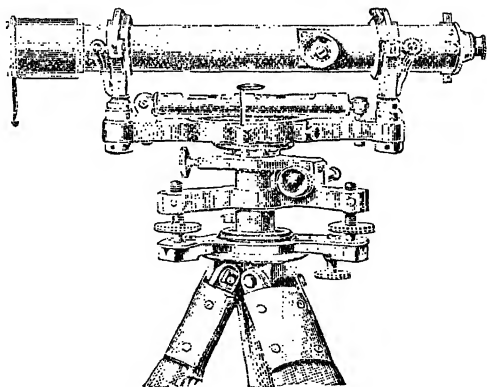advantage of which is that the telescope may be revolved

ILLUSTRATION No. 34.

or reversed end for end in the Y's or supports, which
enables the instrument to be more easily and accurately
tested for collimation and axis.

It consists of a telescope, which finds its bearing in Y-shaped supports, fixed with capstan-headed screws into a stout horizontal plate, which usually carries a magnetic compass, and a long and very sensitive bubble tube or spirit level, hinged at one end and capable of being raised or depressed at the other by turning the screw underneath the plate. There is a reading glass to the compass box, and means for throwing the needle off its bearing when not in use. The whole instrument is mounted on parallel plates, and these in turn upon a tripod. In the case of the instrument shown in Illustration No. 34, the axis is provided with clamp and tangent screws, but this refinement is very frequently dispensed with.

The various parts of the level, viz., the telescope, magnetic compass, clamp and tangent adjustments, parallel plates and screws, and tripod have already been fully described and illustrated separately under the head of " Parts Common to Various Surveying Instruments" in Chapter II., and anyone who has carefully followed it will experience no difficulty in tracing each of those parts in the illustration.

TO SET UP THE INSTRUMENT READY FOR USE.

(**49**).—Open the legs of the tripod to make a stand, screw the parallel plates on to the head of it, open the Y's and put the telescope into them, locking the Y's after so doing. Push two of the feet of the tripod firmly into the ground, opened to an extent to suit your height, move the free leg from right to left and from and towards you until the bubble stands in approximately the centre of its run when the telescope is turned over each pair of conjugate screws, and press the third leg firmly into the ground.

It will be observed that the bubble rises to the highest point. In getting the third leg into position, allowance must be made for the fact that the leg will be lowered when pressed into the ground, the extent of which will depend greatly on the condition and nature of the soil.

The instrument is now roughly set. To finely adjust; place the telescope over one pair of screws and turn them in opposite directions (unscrew one and screw up the other) until the bubble is central. Now turn the telescope over the other pair of screws and do likewise, and repeat the operation until the bubble maintains its central position in whatever direction the telescope is turned, when the instrument will be perfectly set up, and, assuming it is in accurate adjustment, ready for use.

ADJUSTMENTS.

(**50**).—There are four conditions which should exist in order that the instrument may be in adjustment, apart from those involved in "setting up." They are as follows :—

(1) The eye-piece must perfectly focus the webs and the object-glass must perfectly focus the object sighted, to avoid error arising from paralax.

(2) The intersection of the webs or cross wires must be accurately in the line of collimation.

(3) The bubble tube must be at right angles to the vertical axis, so that, when levelled over each pair of screws, the bubble may maintain its central position during a complete revolution on the vertical axis.

(4) The optical axis of the telescope must be at right angles to the vertical axis of the instrument, otherwise when the telescope is turned round, the line of collimation, which is set level

by the bubble tube, will not maintain a
perfectly level line.   This is equivalent to
saying that the line of collimation and the
bubble tube must be perfectly parallel with
each other, and each must be at right angles
to the vertical axis of the instrument.

The method of testing the accuracy of the instrument,
and making the adjustments in each of the foregoing cases,
will now be described.   They will be dealt with in the
order in which they have been set out above, which is that
in which they should be made.

PARALAX.

(**51**).—Paralax may be described as an apparent move-
ment in the cross webs in the diaphragm of the telescope,
enabling the operator to see over and under them, whereby
their purpose as a means of reading accurately on the
levelling staff is depreciated, in that they do not cut any
particular division definitely but may be read to cut any
one of several divisions.   The error arises from the eye-
piece not being accurately focused to the cross webs.

TO TEST FOR PARALAX.

(**52**).—Sight some well-defined object and observe
whether the horizontal web cuts it definitely or whether it
is uncertain as to exactly at what point it cuts.   If it does
not cut definitely, when the object sighted has been
properly focused by adjusting the object-glass, adjustment
for paralax is necessary and may be made as follows :—

TO ADJUST FOR PARALAX.

(**53**).—Turn the milled head until the object-glass is
drawn right out, as far as it will go, so as to be quite out
of focus.   Move the eye-piece in and out, with a slightly
twisting motion until the lense clearly focuses the webs.
The correction is then complete.   (See note to Art. 69.)

Collimation.

TO TEST WHETHER THE INTERSECTION OF THE CROSS
WEBS FALLS ACCURATELY IN THE LINE OF COLLIMATION.

(**54**).—The line of collimation is the line of sight which
passes centrally down the tube of the telescope through
the centre of the eye-piece and object-glass respectively ;
therefore, to test whether the intersection of the cross
webs falls accurately in that line of sight, as it should do,
twist the telescope round in the Y's whilst sighting some
well-defined object at a great distance, and note whether
the intersection of the webs cuts it accurately throughout
a complete revolution.   If it does, the intersection of the
webs falls accurately in the line of collimation, and no
adjustment is necessary ; but if not, correct as follows :—

TO ADJUST THE CROSS WEBS SO AS TO BRING THEIR
INTERSECTION WITHIN THE LINE OF COLLIMATION.

(**55**).—Turn the capstan-headed screws which fix the
diaphragm, screwing and unscrewing opposite pairs to
raise or lower the webs and to move them right or left
until the intersection is perfectly central.   (See Art. 12.)

TO TEST WHETHER THE BUBBLE TUBE IS AT RIGHT
ANGLES TO THE VERTICAL AXIS.

(**56**).—Level the tube over each pair of plate screws, so
that the bubble remains in the centre of its run when
placed over either, turn it a complete revolution on the
vertical axis and if the bubble maintains its central position
during the complete revolution, the tube is in perfect
adjustment, but if not it needs adjustment as follows :—

TO ADJUST THE BUBBLE TUBE AT RIGHT ANGLES WITH
THE VERTICAL AXIS.

(**57**).—Level the tube over one pair of screws, turn it
half a revolution on the vertical axis, so that the object-.

glass is where the eye-piece was and *vice versa*, and note
the difference in the position of the bubble. Turn the
capstan-headed screw, raising or depressing the level tube,
until the bubble returns one-fourth of the difference, and
then the parallel plate screws for another fourth. Place
the level over the other pair of plate screws and repeat the
process above described exactly as before. Now turn the
telescope a complete revolution on the vertical axis, noting
if the bubble remains absolutely stationary in the centre
of its run, and if not, again repeat the whole process until
it does.

TO TEST WHETHER THE LINE OF COLLIMATION IS AT
RIGHT ANGLES TO THE VERTICAL AXIS.

(**58**).—The bubble tube being the means by which the
telescope is set level, and the line of collimation being the
line of sight which extended to the staff enables it to be
read, it follows, in order that the line of collimation may
be set perfectly level, it and the bubble tube must be
perfectly parallel with each other. Set up the level,
adjusting finely by the parallel plate screws as already
directed, read to the levelling staff or any well-defined
object at a considerable distance, reverse the telescope end
for end in the Y's, turn the instrument half a revolution
on its vertical axis and again read to the same object, and
if the cross webs cut accurately as before, no adjustment
is required, but if not, correct as follows :—

TO ADJUST THE LINE OF COLLIMATION AT RIGHT ANGLES
TO THE VERTICAL AXIS.

(**59**).—Having ascertained that the instrument is out of
adjustment in the manner already indicated, note carefully
whether the point cut by the cross webs on taking the
second reading to the staff is higher or lower than the point
previously cut. Raise or depress, as required, the end of
the telescope by the capstan-headed screws under the Y's.

The adjustments require very careful making and one or two trials may be needed before perfect accuracy is obtained.

Note.—A slight variation in the method of adjustment will be necessary where, as is sometimes the case, the bubble tube is attached to the telescope, but if the instrument is understood, no difficulty will be experienced.

THE LEVELLING STAFF.

(**60**).—The staff which, as regards both form of construction and reading, is most generally preferred is shown in Illustration No. 35. It consists of two rectangular or semicircular tubular mahogany cases and a solid slide, each slightly tapering to its upper end and sliding telescopically within the section below it.
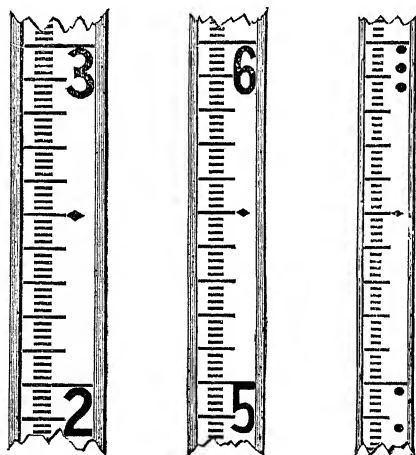

ILLUSTRATION No. 35.

When the slides are all drawn out, the staff is usually 14ft. high, although longer staffs may be obtained if required. When closed its dimensions are: length 5ft. 6in., face 3½in., and depth 2in. There are brass shoes and caps at the end of each section, and usually two brass bands round the lower one to strengthen it.

The slides of the staff, when extended, are held in position by brass spring catches on the back of each of the sections.

The face of the staff is divided (either on varnished paper slips or by painting) into feet, each foot is sub-

divided into ten equal parts, and each sub-division is again divided into ten equal parts, by black lines, which are each a one-hundredth part of a foot thick, so that the whole staff is divided by alternate black and white divisions. These lines do not extend right across the staff, which leaves room for the feet to be numbered from the bottom of the staff upwards by large red figures. The lines at each one-tenth foot are drawn longer than the rest, whilst that marking the half-foot is distinguished by a star, to render the readings of fractions of a foot as plain as possible without crowding the staff with figures. In some staffs the sub-divisions are numbered.

The readings on different staffs differ considerably, but a few minutes' observation will acquaint one with any of them.

THE PURPOSE OF THE STAFF.

(**61**).—The purpose of the staff is to supply a conveniently graduated measure from which readings may be taken with the level, to enable the difference in altitude between any number of points in the earth's surface to be ascertained. The exact way in which this is accomplished is fully explained in Chapter XXII. on "Levelling." (See Arts. 292 and 293.)

TO READ THE STAFF—METHOD OF PRACTICE FOR BEGINNERS.

(**62**).—Most levels reverse, and the surveyor has therefore to get accustomed to reading the staff upside down. This may be practised without the level by standing the staff upside down and moving a thread to different parts of it at random, taking the readings each time.

HOW TO USE THE STAFF—DIRECTIONS TO THE STAFF-
BEARER.

(**63**).—The only directions which need be given are
the following :—

(1) Open out the staff, taking care that the catches
at the back are locked properly, or one of the
slides may slip down a little way unnoticed,
causing erroneous readings.   Do not draw
out the third slide unless required.

(2) Hold the staff with its face at right angles to the
line of sight from the telescope.

(3) Keep the staff perfectly upright.

(4) Where the ground is soft, use a staff plate,
or where there is turf or other vegetation on
the land, tread it down well before planting
the foot of the staff.   (See Art. 64.)

(5) When turning the staff round for backsight
readings take care that it occupies the exact
same position as it did when the foresight
was taken.

(6) Stand behind the staff and so hold it that your
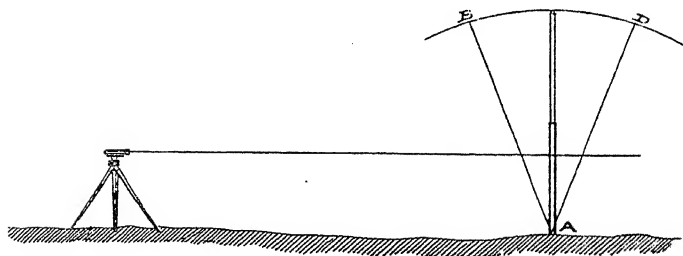fingers do not cover any part of the face.

ILLUSTRATION No. 36.

Note.—A very good plan to guard against inaccurate
readings being taken through the levelling staff not being
held upright, is to have it first rocked forward slightly,

as shown exaggerated by line *AB*, Illustration No. 36, and then slowly backward, as line *AD*. The surveyor with the level sees the horizontal web apparently run down the staff a little way and then commence rising again, and by taking the lowest reading secures that on a perpendicular staff.

<div align="center">STAFF PLATE.</div>

(**64**).—In levelling where the ground is soft, a small triangular iron plate with corners turned down and formed into prongs, with chain and ring attached, is sometimes used to form a base for the staff to rest on, so that there may be no alteration in its level value when it is turned round for back and foresight readings. (See (5) Art. 63.)

<div align="center">STAFF PAD.</div>

(**65**).—When the levelling staff is not in use, the closed staff and the tripod of the level are usually strapped together, the tripod resting against the face of the staff. This is found to scratch the readings, and a spring pad to go between the staff and tripod, which presses on the frame of the former, so protecting its face, will be found very desirable A pad of the kind suggested is shown in Illustration No. 37. D is a section through the tripod, E the spring pad. A B C sections through the slides and case of the levelling staff, and F the leather strap which buckles at S on the back of the staff where it will do no damage.
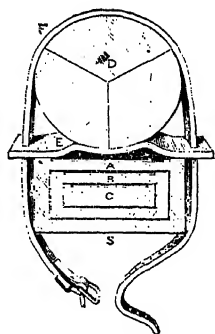
ILLUSTRATION No. 37.

<div align="center">THE DUMPY LEVEL.</div>

(**66**).—The Dumpy is the instrument most generally approved of by surveyors, because, although not pos-

sessing all the advantages of the Y level as regards provision for making adjustments, it is smaller, lighter, and a more handy instrument, at the same time possessing an equally powerful telescope. It is shown in Illustration No. 38, and consists of the following parts:—An
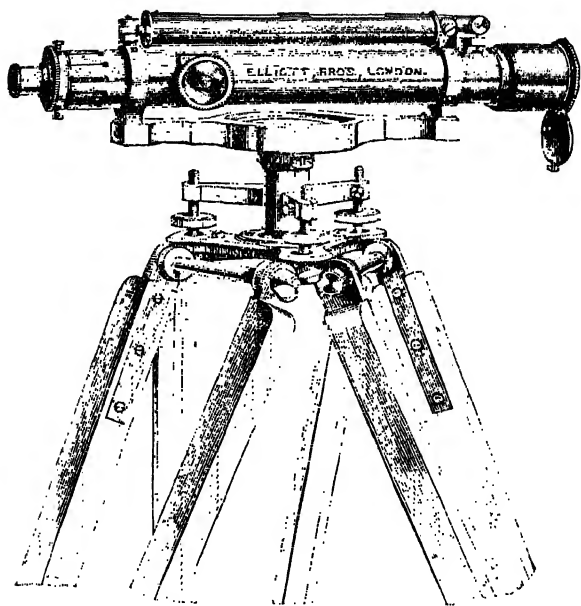

ILLUSTRATION No. 38.

achromatic telescope, a long and very sensitive bubble tube, a short bubble tube fixed at right angles to the long tube, a magnetic needle, parallel plates and screws, and tripod.

It will be noticed that the bubble tubes are, in the case of the Dumpy, fixed on the top of the telescope. The short cross bubble tube is a most convenient arrangement, as it enables the instrument to be set up with approximate accuracy by the tripod without turning the instrument

about, as by watching the two bubbles its horizontality may be ascertained.

Sometimes a small mirror is supplied with the instrument, which clips on to the top of the telescope and being set at an angle overlooking the bubble tube, enables the operator to note the position of the bubble when taking the readings, and so to guard against error arising from the instrument getting accidentally out of adjustment unnoticed.

The chief difference between the Dumpy and the Y level is that the telescope is permanently fixed in the supports, which are made in one piece with the bar carrying the magnetic needle, and consequently the adjustments for the two instruments are not always identical. (See Art. 48, Y level.)

ADJUSTMENTS.

(**67**).—There are three adjustments in addition to those involved in setting up and using the instrument, which may be made by the surveyor in the field:—

    (*a*) The adjustment of the eye-piece for paralax.

    (*b*) The adjustment of the bubble tube at right angles with the vertical axis.

    (*c*) The adjustment of the cross webs for collimation.

The instrument should be tested and the adjustments made in the order given above.

In "setting up," preparatory to testing, it should be noted that the parallel plates are as near as can be seen parallel ; and the instrument should be set up as level as possible, by adjusting the tripod legs, before the plate screws are applied. The screws will turn with perfect ease if the plates are kept parallel and the screws are not turned unequally. Great damage is done to the instrument by continuing to turn the screws when they are working tight.

The method of testing the accuracy of the Dumpy and making the adjustments will now be described.

TO TEST FOR PARALAX.

(**68**).—This is the same as for the Y level. Sight some
well-defined object and observe whether the horizontal
web cuts it definitely or whether it is uncertain as to
exactly at what point it cuts. If it does not cut definitely,
adjustment for paralax is necessary and may be made
as follows :—

TO ADJUST FOR PARALAX.

(**69**).—Turn the milled head until the object-glass is
drawn right out as far as it will go, so as to be quite out of
focus, move the eye-piece in and out with a slightly
twisting motion until the lense clearly focuses the webs :
the adjustment is then complete.

Note.—If when this adjustment has been accurately
made, you still appear to see under and over the webs
when reading to the staff, this indicates that the object-
glass is not accurately focused.

TO TEST WHETHER THE BUBBLE TUBE IS AT RIGHT
ANGLES TO THE VERTICAL AXIS.

(**70**).—Level the tube over each pair of plate screws so
that the bubble remains in the centre of its run when
placed over either, turn the telescope slowly a complete
revolution on the vertical axis, and if the bubble main-
tains its central position during the complete revolution,
the tube is in perfect adjustment, but if not it needs
correction as follows :—

TO ADJUST THE BUBBLE TUBE AT RIGHT ANGLES TO THE
VERTICAL AXIS.

(**71**).—Level the tube over one pair of screws, turn it
half a revolution on the vertical axis so that the object-

glass takes the position previously occupied by the eye-piece and *vice versa,* and note the difference in the position of the bubble. Turn the capstan-headed screw, raising or depressing the level tube until the bubble returns one-fourth of the difference, and then the parallel plate screws for another fourth. Place the telescope over the other pair of plate screws, and repeat the process above described exactly as before. Now turn the telescope slowly a complete revolution on the vertical axis, noting whether the bubble remains absolutely stationary in the centre of its run, and if not, again repeat the whole process until it does so.

DISTINCTION BETWEEN THE Y AND DUMPY LEVELS IN THE MATTER OF ADJUSTMENT.

(**72**).—The telescope of the Dumpy being fixed perma-nently in its supports, it cannot be so simply or accurately tested for collimation by the surveyor in the field ; but the parallelism of the line of collimation as given by the cross webs, with the bubble tube, may be tested and corrected by a method based on the following con-siderations.

If two staffs are held on either side of, and at exactly equal distances from, the level, and the vertical axis on which it turns is and remains vertical, no matter how far out of the horizontal the line of collimation may be, a straight line drawn through the points cut on the staffs will be level, because when the instrument is turned half round on the vertical axis to read the second staff, the angles formed by the lines of sight with a level line will be equal. But if the staffs are not at equal distances from the instru-ment, or the axis is not vertical, then a line drawn through the points cut on the staffs will not be level. From these facts we have means by which the line of collimation may be tested and corrected.

METHOD OF TESTING AND CORRECTING THE DUMPY LEVEL
TO COLLIMATION.

(**73**).—Set up the level and perfectly adjust by the
parallel plates, measure three chains on each side of the
level and drive down a peg firmly at each of these points,
have the levelling staff held on each of these pegs, as
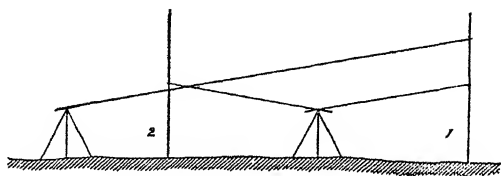shown at 1 and 2, Illustration No. 39, read to each of the

ILLUSTRATION No. 39.

staffs, and carefully note the readings. Now move the level
to, say, a chain length past staff No. 2 and again adjust;
read from the staff held on each of the pegs, and once
more carefully note their readings. If the difference of
the readings taken with the level in the second position,
and the difference between those taken with the level
in the first position, are identical no correction is required;
if they are not identical correct for half the error by
turning the capstan-headed screws fixing the diaphragm,
unscrewing one and screwing the other so as to raise or
lower the horizontal web. (See Art. 12, Diaphragm.)

Illustration No. 39 represents the positions occupied by
the level and staffs in testing for collimation, and assumes
a case where correction is required. It will be seen that
owing to the position of the instrument the line of sight
in the second reading cuts staff No. 2 at a point *lower* than
the first reading on that staff, whereas the point cut in the
second reading on staff No. 1 is considerably *higher*
than in the first reading, thus instancing an extreme case
which illustrates the principle.

Illustration No. 40 represents the instrument and staffs in similar positions, but assumes a case where the instru-
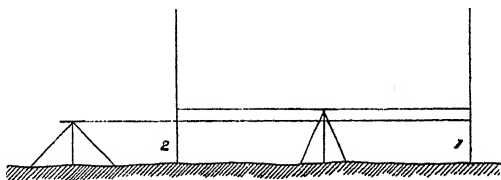

ILLUSTRATION No. 40.

ment is in perfect adjustment, the difference between the first and second readings on each staff being identical.

The Dumpy is " set up " and used in the same way as the Y level. (See Art. 49.)

The levelling staff has already been described in connection with the Y level. (See Art. 60.)

INSTRUMENTS FOR TAKING THE BEARINGS OF LINES
AND ANGLES OF ACCLIVITY AND DECLIVITY.

The Pocket Magnetic Compass— Prismatic Compass—
Brass Quadrant, its purpose and use—The Abney
Clinometer, its purpose and use—Troughton's Improved
Abney Level — Troughton's Clinometer — Prismatic
Clinometer—The Combined Clinometer and Prismatic
Compass.

THE POCKET MAGNETIC COMPASS.

(**74**).—This instrument consists of a magnetic needle, as

ILLUSTRATION No. 41.

described in Art. 15, but in this case forms a separate instrument in itself. It is generally either sunk in a solid wooden square case with hinged top, or in a round case similar to and about the size of a small watch, as shown in Illustration No. 41. The pocket magnetic compass can be obtained in various sizes and designs, and is so well known that further description is unnecessary. It is useful for taking the bearings of lines with approximate accuracy.
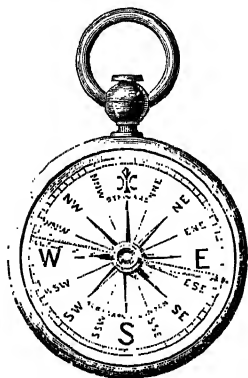
THE PRISMATIC COMPASS.

(**75**).—This instrument, which consists of a magnetic compass with the addition of a prism for reading the card, and a sight-vane, reflector and sun-glass is shown in Illus-

tration No. 42. The one shown has an open top revealing
the whole card, but some have the top closed in, with the
exception of a small space immediately under the prism.
The size varies from 2½in. to 6in. in diameter. In each
case the card or ring of
metal is attached to the
magnetic needle, and re-
volves with it. The sight
and prism fold down out
of the way for protection
when not in use, and the
prism may be raised or de-
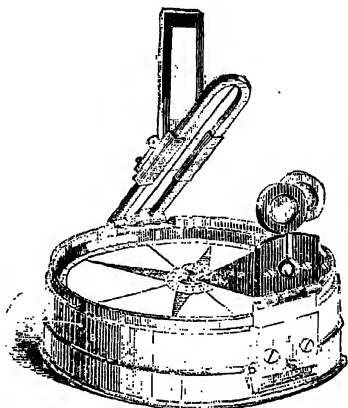pressed sufficiently to ac-
curately focus the card.

The object of the prism
and sight-vane is to enable
the operator, at the same
time as accurately sighting
objects marking the direc-

ILLUSTRATION No. 42.

tion of lines the bearings of which are required, to take
the reading from the card. He does not need first to get
the instrument into position and then turn the eye to
take the reading, whilst he was doing which it might
get moved. The prism, too, magnifies the reading, thus
considerable accuracy is obtained by the use of this
ingenious arrangement.

TO USE THE PRISMATIC COMPASS.

(76).—Open out the sight-vane and prism, raise or
depress the prism until it focuses the card to suit your
vision, hold the instrument quite horizontal in the hand
so that the card swings freely, turn it until the sights
give the correct direction, steady the card by pressing the
button under the sight-vane, and when it has ceased to
vibrate take the reading direct from the card.

The south pole of the needle being at zero or 360 deg. on the card or metal ring, and the degrees being numbered in that direction, all readings are taken as east of north.

THE ORDINARY BRASS QUADRANT.

(**77**).—This simple little instrument, which is shown in Illustration No. 43, consists merely of (1) an ordinary brass quadrant or protractor, with its circular edge
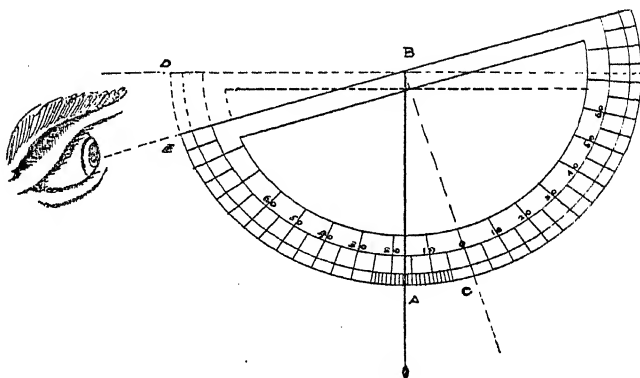

ILLUSTRATION No. 43.

divided into degrees and half-degrees in each direction from its centre, and (2) a small plummet suspended by a fine silken thread from a notch in the straight-edge of the quadrant.

It is very handy for taking angles of acclivity or declivity, for enabling lines measured on the slopes of hills to be reduced to horizontal measure, or for any other case in which vertical angles are desired to be known, and where approximate accuracy is all that is required. (See Art. 190.)

HOW TO USE THE QUADRANT AS A CLINOMETER.

(**78**).—Erect two poles of equal length or marked with equal heights, one at the top and the other at the foot of the incline, the angle of which is to be taken ; sight along

the straight-edge of the quadrant from and to the points
of equal height on the poles, in doing which it will be
brought parallel with the sloping surface of the land.
While the instrument is in this position, hold the silken
thread of the plummet by the thumb and finger of the
disengaged hand on the quadrant, and read the angle *ABC*
from the graduated edge, which, being equal to the angle
DBE, gives the angle of acclivity or declivity.

THE CLINOMETER RULE.

(**79**).—This little instrument, as will be seen from
Illustration No. 44, resembles a two-fold measuring rule,
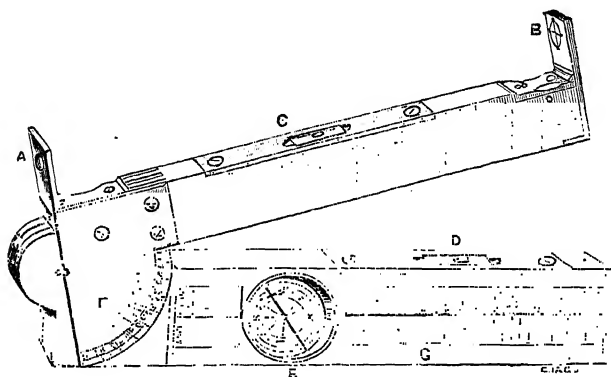
ILLUSTRATION No. 44.

but it is stouter in section. It consists of two rules hinged
at one end, the quadrant of the hinge-plate F being
divided into degrees and parts. The zero division is
accurately cut by the upper edge of the lower rule when
the instrument is quite closed, and consequently the angle
formed by the rules when open may be read from the
quadrant. A bubble tube is let into the upper edge of
each limb (C and D in illustration), so that when the

rules are closed and the instrument is on a truly level surface the bubbles are in the centre of their runs.

There is usually a magnetic needle (E) in the side of one of the rules, which is useful for taking the bearings of lines. The upper limb has a sight at each end (A and B in illustration), in which there are fine cross wires, for enabling objects to be sighted with the greater accuracy, and these sights are hinged to fold down flush with the edge of the limb when not in use.

A table of the deductions which must be made from hypotensual measurements of different angles to reduce them to horizonal measurements, is stamped on the side of the lower rule at G.

The whole instrument is enclosed in a leather case, and its size is about 6¾ins. by 1¾in. by ½in.

PURPOSE OF THE CLINOMETER RULE.

(**80**).—The purpose of the clinometer rule is the same as that of the simple brass quadrant, but it is a more perfect and reliable instrument. (See Arts. 77 and 190.)

HOW TO USE THE CLINOMETER RULE.

(**81**).—This instrument may be used in either of two ways :—(a) by setting the bottom limb on some level surface, or by holding it level in the hand and raising the upper limb as necessary to enable the object sighted to be accurately cut by the sights, or (b) by setting the bottom
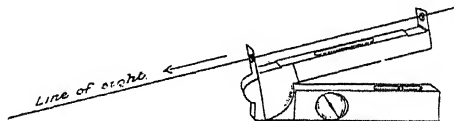

ILLUSTRATION No. 45.

limb on a straight-edge on the sloping surface, the gradient of which it is required to ascertain, and raising

the top limb until the bubble assumes a position in the centre of its run. The angle is then read from the quadrant.

The former method is employed when finding the angle of slope of lands, the operator standing at the top or bottom of the hill and sighting to a picket, as shown in Illustration No. 45 ; the latter is the way in which the
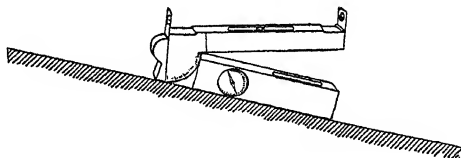
ILLUSTRATION No. 46.

instrument is used in ascertaining the inclination of a drain, etc., from which its gradient may be calculated. Illustration No. 46 shows this method of using the instrument.

THE ABNEY CLINOMETER.

(**82**).—This is another instrument for measuring vertical angles. It is shown in Illustration No. 47, and

ILLUSTRATION No. 47.

consists of a small telescope, a brass graduated quadrant carrying a bubble tube actuated by a milled head, to which is attached an arm with bevelled edge formed into a vernier.

When the bubble tube is perfectly horizontal, as shown by the bubble taking a position in the centre of its run, and the line of sight or collimation which is parallel with it therefore level, the arrow of the vernier plate and the zero of the quadrant co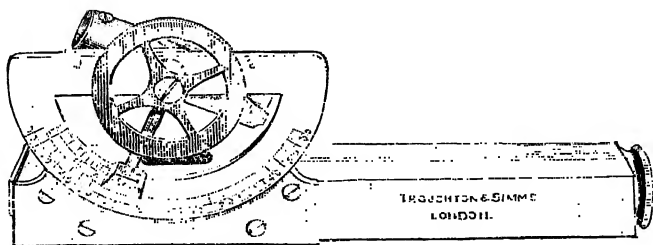incide accurately, so that all angles formed by the line of sight through the telescope and a level line may be read from the quadrant and vernier.

The carriage of the bubble tube is open both on its upper and under side, and there is also an opening in the upper side of the tube of the telescope immediately under the centre of the level and over a silvered reflector which is fixed at an angle in the tube of the telescope. This reflector extends only half way across the tube, so that the line of sight through it is not obstructed. On this reflector a line is drawn, which exactly bisects the reflected image of the bubble of the level. when the telescope is held perfectly horizontal, so that, while the operator is looking through the telescope sighting the object in the distance, he can also see when the bubble has assumed its central position, and the reading from the quadrant and vernier may then be taken.

THE PURPOSE OF THE ABNEY CLINOMETER.

(**83**).—The purpose of this instrument is similar to that of the clinometer rule. (See Arts. 79 and 190.)

TO USE THE ABNEY CLINOMETER.

(**84**).—Sight from and to poles or pickets marked with similar heights above ground, one fixed at the top and the other at the foot of the incline, at the same time turning the milled head which moves the bubble tube and vernier arm, until the bubble is seen in the reflector in the telescope accurately bisected by the line drawn thereon, indicating that the bubble is in the centre of its run. Read the angle of acclivity or declivity from the scale and vernier.

For a description of the vernier and how to read it see Chapter II., Art. 16, on "Parts Common to Surveying Instruments."

TROUGHTON'S IMPROVED ABNEY LEVEL.

(**85**).—This is a great improvement on the original form of Abney Clinometer. Instead of the level and vernier arm being directly actuated by the turning of the milled head, the vernier is fixed on a plate attached to the telescope, the bubble tube is carried by the quadrant, which is actuated by a pinion which gives slow and smooth motion and secures greater ease as well as greater accuracy in adjustment.

TROUGHTON'S CLINOMETER.

(**86**).—This is a very similar instrument to Troughton's Improved Abney Level, but has no vernier plate. It is
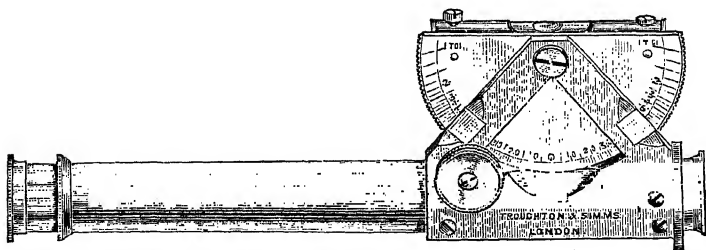
ILLUSTRATION No. 48.

represented in Illustration No. 48, and needs no further description.

PRISMATIC CLINOMETER.

(**87**).—There are two forms of this most useful instrument. The watch form is shown in Illustration No. 49, and the ordinary form in Illustration No. 50.

The instrument consists of a circular card (C in Illus-

ILLUSTRATION No. 49.

tration 49, A in Illustration 50) divided into degrees and
half-degrees, turning on a centre in a circular box which is

ILLUSTRATION No. 50.

provided with sight-vane D, and prism similar to the
magnetic compass already described. (See Art. 75.)

The card is so weighted on one side that when the instrument is held edgewise with the line of sight as given by the sight-vanes perfectly level, it assumes a position giving the reading at zero, and therefore when the line of sight is not level, but on an incline, the number of degrees it is out of the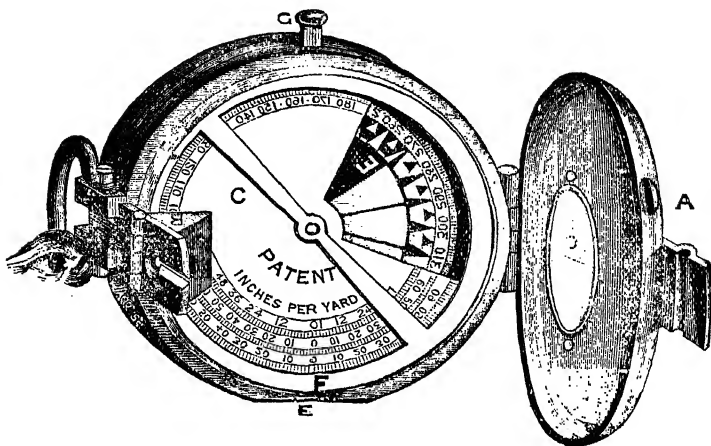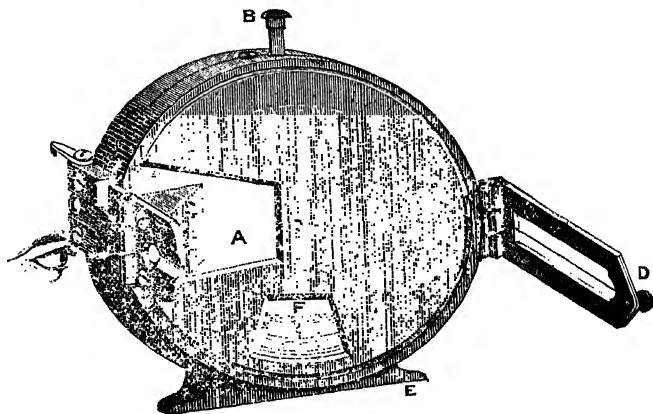 horizontal may be read direct from the card by the aid of the prism at the same time that the distant object is sighted.

In the ordinary form the underside of the rim of the box is made flat at E that it may the better rest on a wall or stand; and there is a scale (F) of deductions for reducing hypotensual to horizontal measurements printed on the face of the card.

THE COMBINED CLINOMETER AND PRISMATIC COMPASS.

(**88**).—The prismatic clinometer and prismatic compass are often combined, as shown in Illustrations Nos. 49 and 50, and, in that case, a part of the clinometer card is cut away so as to expose the compass card when required for taking bearings. (See Illustration 49.) The clinometer card may be turned by pressing a stop which brings the opening in it immediately under the prism, so enabling the readings on the compass card which is under it to be taken.
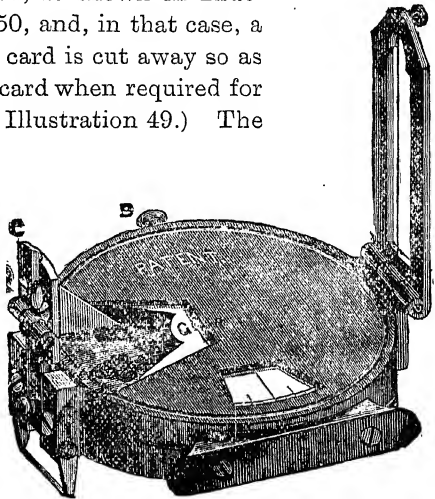
ILLUSTRATION No. 51.

Illustration No. 51 shows the combined form being used as a compass

for taking the bearings of lines. The knob B is for turning the clinometer card (G) so as to expose the part of the compass card under the prism. C is the sight-vane and prism.

This is probably one of the most useful pocket instruments the land surveyor can possess, for, combined with the prismatic compass, which is much employed in filling in details in large surveys, traversing through woods, etc., etc., we have the clinometer, an instrument almost as frequently requisitioned for taking the angle of slope.

There is no doubt that, when considerable accuracy is required, the prismatic compass employed should be one of fair size, say six inches in diameter, and mounted on a tripod, but the pocket instrument of which I have been speaking, used in the hand, will be found to give results sufficiently accurate for all practical purposes where approximation is all that is necessary.

CHAPTER VII.

INSTRUMENTS FOR MEASURING ANGLES, VERTICAL OR HORIZONTAL.

The Theodolite—Y Theodolite—Transit Theodolite—Parts of which the Instrument consists—Its Practical Uses —Testing Accuracy of a Chain Survey—Measuring Angles to enable the Lengths of Lines to be calculated —Ascertaining the heights or distances of inaccessible objects—Finding the Angle of acclivity or declivity of Land—Ranging out Straight Lines—Levelling Operations—Taking the Bearings of Lines—Travers-ing—Setting out Railway Curves—How to use the Theodolite—To take a Horizontal Angle—To take a Vertical Angle — The Box Sextant — Description— Adjustments—To Test and Adjust for Verticality of Mirrors—To Test and Adjust for Index—Points Important to be borne in mind—How used.

THE THEODOLITE.

(**89**).—This is an instrument for measuring both horizontal and vertical angles. I might, perhaps, have said it is *the* instrument, since it is the only one which does the work with the greatest possible accuracy. Although it is more complicated than most of the other instruments employed in land surveying, it is very simple in use.

It is a difficult instrument to explain on paper, and my object being to give a description, which, with the aid of illustration, may be readily understood, I will first try and picture an imaginary, primitive instrument, which may represent the theodolite stripped of all its refinements.

We will suppose it to consist of a tripod stand, on the top
of which is fixed two round plates of metal, placed face to
face, with bevelled edges, and turning on the same centre,
as shown in Illustration No. 52.   The edge of the lower

Elevation.

Plan.

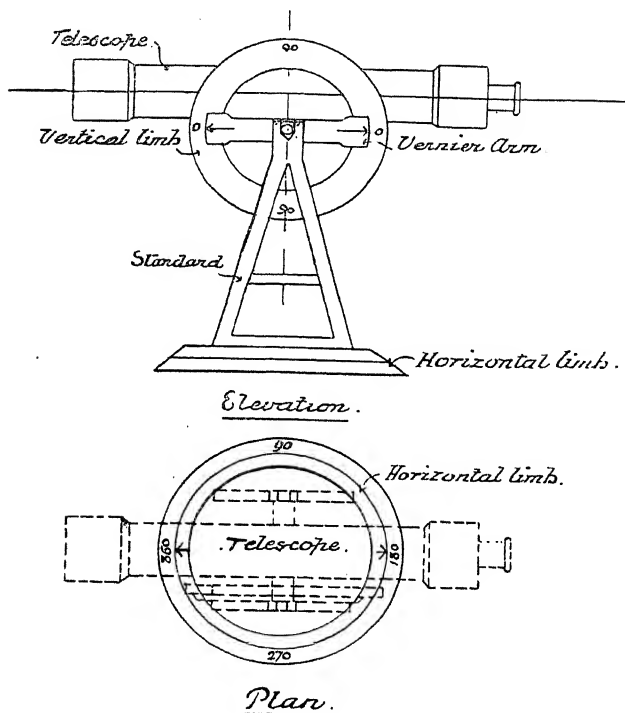ILLUSTRATION No. 52.

plate is divided into degrees and parts numbered from 0
to 360, and an arrow head is engraved on the edge of the
upper plate, from which two standards rise to support a
telescope on a horizontal axis.   To the side of the
telescope is attached a circle of metal, which is divided
into degrees and parts, and numbered from 0 at the
extremities of its horizontal diameter, to 90 at the

extremities of its vertical diameter; and arms marked
with arrow heads, which coincide with zero on the circle
when the telescope is horizontal, project from the
standards.

To measure the horizontal angle *ABC* (Illustration
No. 53) we adjust the instrument with its centre over *B*,
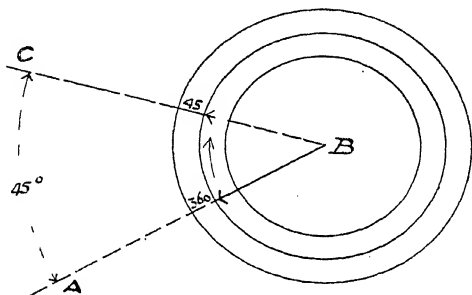
ILLUSTRATION No. 53.

set the arrow head on the upper plate to zero on the lower
plate, and turn the whole instrument on the tripod until *A*
is sighted. We then turn the upper plate only until *C* is
sighted. The arrow head points out the angle through
which the telescope has travelled.

To measure the vertical angle *EDF* (Illustration No.
54), set the telescope horizontal (when zero on the circle,
which moves with the telescope, will coincide with the
arrow head on the vernier plate, which is fixed, as shown
by the inner ring in illustration), turn the telescope until
E is sighted, and read angle of elevation *EDO* from the
circle (as indicated by the middle ring). Again turn the
telescope until *F* is sighted, and read the angle of
depression *ODF* (as indicated by the outer ring), and
finally add together the angles of elevation and depression
for the total angle.

It is believed that the foregoing descriptions set forth the
main features of the theodolite with greater simplicity and

clearness than can be done in a full and proper description
of the instrument in its complete form, and with its many
refinements, without which, however, it would be useless.

It will be readily seen that for the instrument to be
efficient there must be means for adjusting it accurately
over the station, setting the plates horizontal, finely
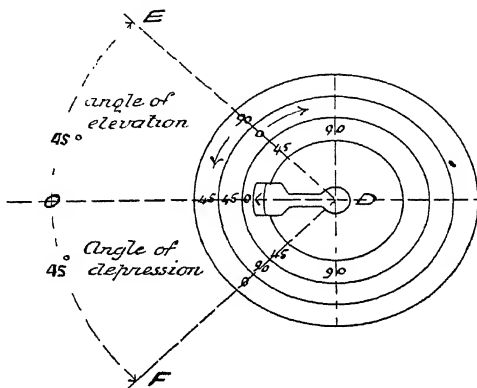
ILLUSTRATION No. 54.

sighting, accurately reading the angles, adjusting the tele-
scope perfectly horizontal, etc., which are respectively
secured by the plummet, bubble tubes, three sets of clamp
and slow motion screws, the vernier, the vernier reader,
clipping plate, etc. Most of these parts have already
been explained in Chapter II., under the head of "Parts
Common to Surveying Instruments." (See Arts. 9 to 20.)

Having thus cleared the ground, I shall now proceed
to give a full and complete description of the theodolite,
its purposes and adjustments, as well as the method of
using it.

(**90**).—There are two forms of theodolite, the Y, as
shown in Illustration No. 55, and the transit, as shown in
Illustration No. 56, the chief difference being that, whilst

the former has only part of a circle for the vertical limb, the latter has a complete circle, which is an advantage.

Slight variations in construction will be found in instruments by different makers, but no difficulty will be
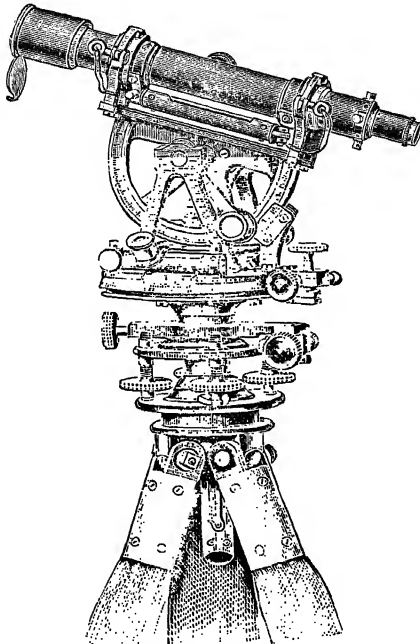
ILLUSTRATION No. 55.—THE Y THEODOLITE.

experienced in working any of them when once the nature of the contrivance is properly understood.

(**91**).—The instrument in its complete form consists of a large number of parts, and may be divided up in various ways for description, but I think the better order will be to consider it in the parts into which it actually divides for packing, and to explain what each of these parts consists of, building it up as it would be put together in the field when required for use. They are as follows :—

PARTS OF WHICH THE THEODOLITE IS COMPOSED.

(1) The Tripod.—This has already been explained in Chapter II., Art. 17.
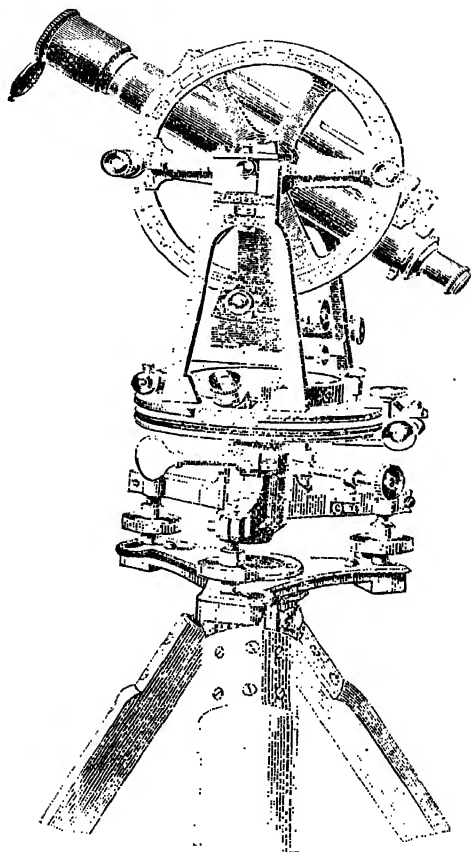
ILLUSTRATION No. 56.—THE TRANSIT THEODOLITE.

(2) The Parallel Plates and Screws.—These are similar to those which have also already been described in connection with the surveyor's

level, but have connected with them a clamp and tangent screw, which enable the whole instrument to be turned on its vertical axis, first by hand for approximate sighting, and secondly by the tangent screw for fine adjustment. The application of the clamp and slow motion screw has been described in Chapter II., Art. 19, under the head of " Parts Common to Surveying Instruments."

(3) The Horizontal Limb comprises two perfectly flat circular metal plates, placed face to face, with bevelled edges, turning on the same centre. The edge of the lower plate is divided into degrees and half-degrees, the upper plate has two verniers engraved on its bevelled edge, opposite each other. This part of the instrument is provided with a clamp and tangent screw, which enables the upper or vernier plate to be finely adjusted both for setting the vernier and also for directing the telescope to the object sighted, when reading an angle. The upper or vernier plate also carries a compass box, two standards which support the telescope, and two spirit levels fixed at right angles to each other, to enable the instrument to be set perfectly horizontal by the parallel plates and screws. (For Vernier, see Art. 16 ; Clamp and Tangent Screws, Art. 19 ; Magnetic Compass, Art. 15 ; Telescope, Art. 9 ; Parallel Plates and Screws, Art. 18.)

(4) The Vertical Limb comprises a telescope mounted on a horizontal axis, to which it is firmly attached, the vertical limb or circle, usually divided into degrees and half-degrees, and numbered from 0 at the extremities of its

horizontal diameter, to 90 at the extremities
of its vertical diameter. Turning freely
on the same axis, and not attached to the
telescope or vertical circle, is the clipping
plate, shown in Illustration No. 57, in one
piece, with which are the vernier arms. There
is also an arm carrying two vernier readers.
The telescope is mounted by a spirit level
which enables it to be set perfectly horizontal.
There is a clamp and slow motion screw ar-

ILLUSTRATION No. 57.

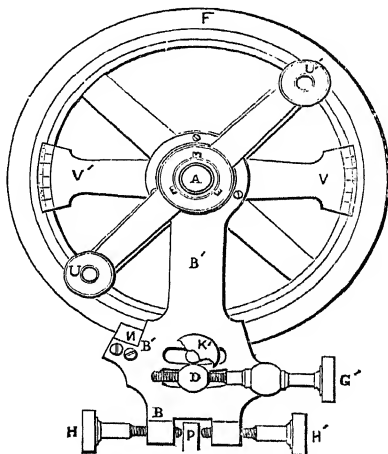rangement connected with the clipping plate,
which enables the telescope (not shown in
Illustration No. 57) and vertical limb to be
finely adjusted for making the cross webs
accurately cut the object sighted when taking
vertical angles. In Illustration No. 57, F is
the vertical limb, V and V′ the verniers, A the
axis, U and U′ the vernier readers, B the
clipping plate, H H′ the screws to the forked

end, P the projecting piece on the standard, K´ the clamp screw, G´ the tangent screw, and D the boss through which the tangent screw passes. It may help matters to repeat here that, whilst in the case of the horizontal limb, it is the upper or vernier plate which is turned in taking angles, in the case of the vertical limb it is the circle itself which follows the motions of the telescope, the verniers remaining stationary.

(5) The plummet, which is simply a brass weight of suitable form. It is suspended from a hook between the tripod legs by a silken cord. Some theodolites are provided with means for adjusting the plummet accurately over the station.

THE PRACTICAL USES OF THE THEODOLITE.

The purpose of the instrument, it has already been pointed out, is to enable horizontal and vertical angles to be accurately measured in the field; its practical uses in land surveying will be better understood by the aid of one or two simple examples.

TO TEST THE ACCURACY OF A CHAIN SURVEY.

(**92**).—First, let us suppose we have made a chain survey, and that Illustration No. 58 represents the lines chained.

We (1) accurately set up the instrument over each of the stations *A*, *B*, *C*, *D*, in turn, making the plummet hang precisely over the intersection of the cross lines on the top of the false picket, which we will assume has been inserted in the hole made by the picket which formed the station ; and (2) adjust the instrument, and

carefully read the angles formed by the lines at the various stations.
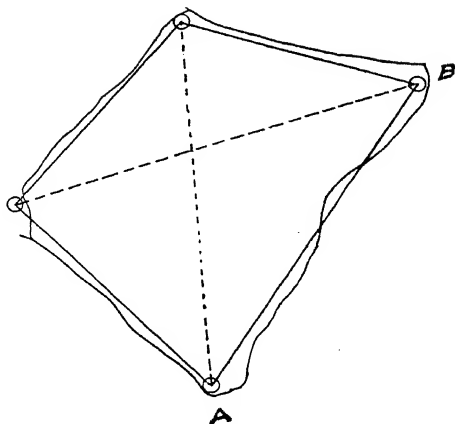
Having thus obtained the angles in the field, the vernier protractor is adjusted on the plan which has been plotted from the chain survey, and the angles formed by the lines on paper carefully ascertained, and if these are found to correspond with those measured in the field, the accuracy of the work is proved. In practice, however, the plotting and the testing of its accuracy would proceed simultaneously, so that any error would be discovered as the work of plotting advanced.

TO MAKE A TRIGONOMETRICAL SURVEY.

(**93**).—Again, suppose we measure only the base line AB in the field represented by Illustration No. 59, and then with the theodolite measure the angles ABD, DBC, DAC and CAB, the remaining three lines, BC, CD, DA, may be calculated by trigonometry, and the lines may be plotted to scale, and checked with the vernier

protractor. Thus, by the aid of the theodolite, we are enabled to make what is known as a trigonometrical
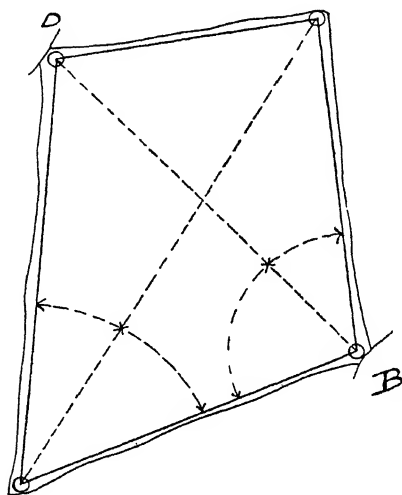
ILLUSTRATION No. 59.

survey, of which the foregoing is only intended to be a very simple illustration. ("Solution of Triangles," see Chapters XXVI. and XXVII.)

TO FIND THE HEIGHT OR DISTANCE OF AN INACCESSIBLE OBJECT.

(**94**).—Once more, suppose it is desired to find the height of any inaccessible object. By measuring the vertical angles to the top of that object from two points, and the horizontal distance between those points, the perpendicular height may be found by trigonometry. Thus, suppose it is desired to find the perpendicular height of the cliffs in Illustration No. 60. By measuring the line *CD* accurately, setting up the theodolite over the

stations C and D, and measuring the angles ACB and ADB, the remaining parts of the triangle may be
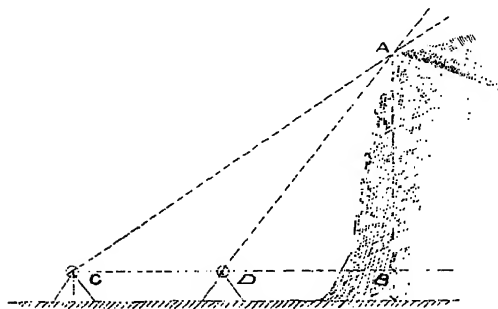
ILLUSTRATION No. 60.

calculated and the height of the cliffs obtained by adding the height of the telescope above the ground to the height AB.

The distance of an inaccessible object from a given point may likewise be obtained by measuring one side and two of the three angles of a triangle, one of whose sides is the shortest distance, or, in other words, a straight line, from the inaccessible object to the given point. (See Arts. 198 to 202.)

TO ASCERTAIN THE ANGLE OF ACCLIVITY OR DECLIVITY OF LANDS.

(95).—The theodolite, too, may be used for ascertaining the angle of acclivity or declivity of land for the purpose of
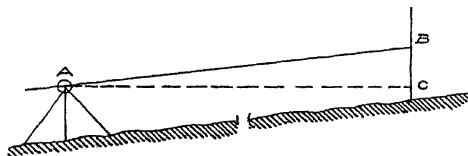
ILLUSTRATION No. 61.

reducing hypotensual to horizontal measure, by having a picket marked with the height of the telescope above the

ground held at the top or bottom of the incline and sighting to it, as shown in Illustration No. 61, when the angle of elevation BAC may be read. (See Arts. 77 to 88.)

TO RANGE OUT A PERFECTLY STRAIGHT LINE OVER A HILL.

(**96**).—The theodolite may also be used for ranging out a straight line over a hill, as by setting up the instrument in such a position that stations on either side of the hill may be seen through the telescope, sighting one of the stations, then turning the telescope right over on its axis, and having a pole put in to fix the station on the other
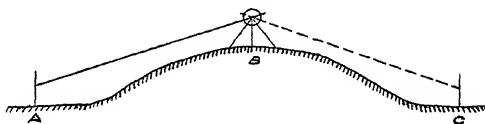
ILLUSTRATION No. 62.

side, and also one at the point indicated by the plummet of the theodolite, we have three stations, all in a direct line, as indicated by A, B, C in Illustration No. 62, and the chaining may proceed from either side of the hill to the station established at the top of it, and thence to the station on the other side. (See Art. 194.)

LEVELLING OPERATIONS.

(**97**).—Again, with the aid of a levelling staff, levelling operations may be conducted with a theodolite, but its extra weight and size over an ordinary "dumpy" level makes the latter far to be preferred for all ordinary cases. When using the theodolite in levelling operations, the telescope is, of course, kept set perfectly level, and the whole instrument is set horizontal by the parallel plate screws, as the level would be, each time it has to be moved. (See Art. 49; also, Y and Dumpy Levels, 48 to 72, and Levelling, 265 to 306.)

TO TAKE THE BEARINGS OF LINES.

(**98**).—Once again, the theodolite, having a compass box, is very useful for taking the bearings of the main lines of a survey. The instrument is simply set up over one of the stations in the line, and the telescope directed to another station therein, at a great distance, when the bearing of the line may be read from the needle. (See Art. 15.)

In using the theodolite for this purpose, it must not be forgotten that the needle does not point true north, and proper allowance for declination must be made. (See Chapter XXIV., Art. 326.)

TRAVERSING.

(**99**).—Traversing, too, where accuracy is required, is always performed with the theodolite. (See Chap. XXIV.)

TO SET OUT RAILWAY CURVES.

The theodolite is also much used in setting out railway curves, the method of doing which is described in Chapter XXV.

HOW TO USE THE THEODOLITE.

(**100**).—Open the tripod stand, screw the parallel plates on to the top of it, and the horizontal limb on to them, open the sockets of the standards and place the axis of the vertical limb in them, and close same; also turn the screws of the forked end of the clipping plate until they bite the projecting piece on the standard. Suspend the plummet from the hook between the tripod legs. The instrument is now put together ready for use.

TO TAKE A HORIZONTAL ANGLE.

(**101**).—Adjust the instrument over the station by moving one leg of the tripod at a time until the plummet

hangs exactly over the intersection of the cross on the false picket inserted in the hole made by the ranging pole in forming the station.

Now turn the opposite pairs of parallel plate screws until the bubbles of the level tubes on the horizontal vernier plate maintain a position in the centre of their run during a complete revolution of the instrument.

Clamp the horizontal plate and unclamp the horizontal vernier plate, and turn the latter until the broad arrow of the vernier as nearly as possible coincides with the 360 on the horizontal limb. Clamp the horizontal vernier plate and turn the slow motion screw attached thereto until the broad arrow of the vernier and the 360° of the horizontal limb accurately cut. Use the magnifying glass to ascertain this.

Unclamp the whole instrument and turn the telescope round until it approximately cuts the extreme left-hand object between which and a point on the right the angle is to be measured. Clamp the whole instrument, and apply the slow motion screw until the cross webs in the telescope accurately cut the object sighted.

Unclamp the horizontal vernier plate and turn the telescope to the right until the second object is approximately cut by the cross webs of the telescope, clamp the vernier plate and turn the tangent screw until fine adjustment is obtained. The angle may now be read from the horizontal limb and vernier.

(**102**).—To obtain greater accuracy take the readings from each of the verniers, add them together, subtract 180 degrees from the sum and divide the result by two for the correct angle.

(**103**).—It may be well here to emphasise the fact that it is only angles less than half a degree which are read from the vernier, that is to say, degrees and half-degrees are read from the circle, and the vernier records quantities

between degrees and a half-degree, or degrees and a half-degree and the complete degree. Thus if the arrow head of the vernier is between 20 and 20½ degrees, 20 is read from the circle; but if the arrow head is between 20½ and 21 degrees, 20½ degrees, or 20 degrees 30 minutes, is read from the circle, and the fraction of a degree beyond from the vernier. (See Vernier, Art. 16.)

TO TAKE A VERTICAL ANGLE.

(**104**).—Adjust the instrument over the station as before described, set the broad arrow of the vernier accurately to zero on the circle, at the same time that the telescope is perfectly horizontal as shown by the bubble tube, using the clamp and tangent screws, the screws connected with the forked end of the clipping plate, and the vernier reader for the purpose. Read the angles of elevation and depression from the circle and vernier. These angles added together give the total angle. (See Art. 16.)

THE BOX SEXTANT.

(**105**).—This instrument in some respects resembles the
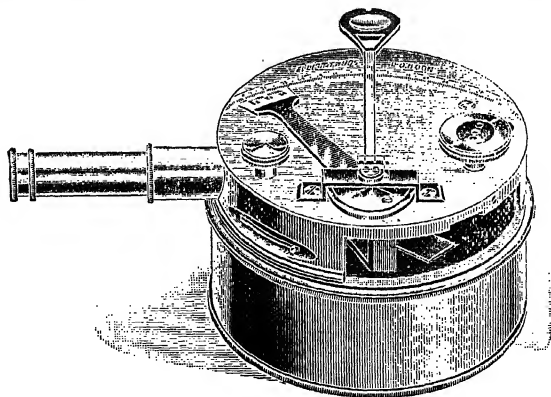
ILLUSTRATION No. 63.

optical square, but is far more complicated and complete,

and capable of a much wider field of work. A general view of the instrument is shown in Illustration No. 63.

DESCRIPTION.

It consists of a flat circular metal box, about three inches in diameter, and one and a half inches deep, the top of which takes off and fits on to the bottom to form a handle when in use, a small telescope with dark glass for the eye-piece, two sun-glasses, two mirrors, one fixed, the other capable of being turned on its axis, a rack and pinion actuating the moveable mirror, a vernier which follows the movements of the mirror and reads to single minutes on a graduated arc on the top of the box, and a vernier reader.

Its optical parts consist of the two mirrors, and its principle depends on the relationship which exists between the line of incidence and the line of reflection. Instead of the mirrors being fixed, however, as in the optical square, one is moveable for adjustment only, and the other is actuated by a rack and pinion, and is attached to an arm and vernier which reads upon the graduated arc on top of the box. The movements of the index glass, as it is called, are therefore accompanied by movements of the vernier, and the angles formed by lines drawn from the objects, seen respectively by direct vision and reflection, to the point of sight, which the relative positions of the horizon and index glasses imply, are thus recorded.

Illustration No. 64 represents a section through the box, in which I is the index mirror with rack and pinion attached, the pinion being turned by a milled head on the top of the box. C is the horizon-glass with screw Q, which may be turned by a key from outside the box, through the hole P, to adjust the mirror parallel with the index-glass at the same time that the latter is in that position which gives the reading of the index arm at zero.

The two small screws *c c* fix the carriage of the mirror which may be rocked by screwing up one and unscrewing the other, so as to bring the faces of the mirrors perfectly parallel vertically.   The same key which fits the head of
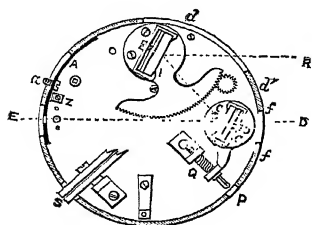
ILLUSTRATION No. 64.

the screw *Q* also fits the heads of these screws   It is like a watch key, and, when not in use, screws into the face of the instrument.   *d* is the window to the index-glass, *f f* the window to the horizon-glass.   *E* is the sight, the telescope having been with-drawn and the shutter closed over the opening, and *a* is a small screw sliding in a groove to prevent the shutter being drawn too far either one way or the other.   *S* indicates the nibs of the sun-glasses which are shown broken off to prevent confusion.

There is a slit-like opening in the bottom of the box (not shown) with a shutter which draws over it.   When the sun-glasses are not required, this shutter should be opened before the top of the box is fitted on to form the handle, so that the nibs of the sun-glasses may be pressed up, when the glasses will pass through the opening into the top which now forms the bottom of the box, and so be out of the way.

The line *ED* indicates the line of direct vision to an object sighted at a great distance, and the line *RI*, although not technically so, may be regarded as the reflected ray from the same object.   This ray impinging on the mirror at *I* is, provided the faces of the mirrors are perfectly parallel, again reflected to the horizon-glass immediately over the object seen by direct vision.   The instrument is so adjusted that when the faces of the mirrors are perfectly parallel the index reads at zero.   Consequently, when

the object seen simultaneously by direct vision and reflection is one and the same, and therefore no angle exists, the index registers zero ; and when the objects seen by direct vision and reflection are not one and the same, the angle formed by lines drawn from those objects to the centre of the instrument is correctly registered.

This arrangement, although not absolutely accurate, is sufficiently correct for all practical purposes, for, although lines drawn from objects in the lines R and D to the centre of the instrument would form an angle, the distance between the line of vision and the line of reflection is so small, compared with the distance at which objects are usually sighted, that no appreciable angle is formed, and when one and the same object is sighted, and therefore no angle subtended, the vernier registers zero. The slight error which does exist, however, known as error from parallax, may need to be taken into account in some extreme instances, but from a technical rather than a practical point of view.

ADJUSTMENTS.

(**106**).—There are only two adjustments which it is within the power of the operator to make, viz. :—

     (*a*) The adjustment for verticality of mirrors ;

     (*b*) The adjustment for index.

With regard to the former, it is necessary, in order that the laws of incidence and reflection, as applied to the instrument, may not be counteracted, that the mirrors should be perfectly vertical and their faces perfectly parallel with each other ; and, as from various causes the instrument is liable to get out of adjustment in this respect, means are provided whereby any error may be readily corrected. This is done by turning the screws *c c*.

With regard to the latter, when the mirrors are perfectly parallel horizontally, the index must read to zero. Means

of adjusting the instrument in this respect, viz., the adjustment for index, are provided in the screw Q.

TO TEST AND ADJUST THE INSTRUMENT FOR VERTICALITY OF MIRRORS.

(**107**).—Apply the dark glass to the eye of the telescope, sight the sun, turn the milled head of the index right and left of zero until you see the sun also reflected in the horizon-glass, note whether in the course of its passage it forms an unbroken orb with the half seen by direct vision. If it does, no adjustment is required. If it does not, turn the screws c c on the top of the box, until the sun may be seen as one unbroken orb, half by direct vision and half by reflection, and the adjustment will be perfected.

This adjustment must always be made before the adjustment for index.

TO TEST AND ADJUST FOR INDEX.

(**108**).—Set the vernier accurately at zero, sight the sun, when, if the halves seen by reflection and direct vision form one perfect orb, the instrument is in adjustment. If not, correct as follows :—Sight the sun, turn the index until it is seen as one orb by reflection and direct vision, and note the reading, reverse the instrument, that is, turn it upside down, again sight the sun and again note the reading, which will be on the opposite side of zero, add these readings, and correct for half the result by turning the screw Q. Again test the instrument, sighting the sun with the index set to zero, and if not this time seen as an unbroken orb, again adjust until perfection is obtained.

THE USE OF THE BOX SEXTANT.

(**109**).—The instrument may be used :—

> (*a*) For any of the purposes for which the optical square may be employed, by setting the vernier at 90 degrees ;

(b) For measuring horizontal angles, such as the angles which chain lines form with each other, so affording an additional check on the accuracy with which the lines comprising a system of triangulation have been ranged out and chained ;

(c) For taking vertical angles for the purpose of enabling the height of some inaccessible object to be ascertained by trigonometrical calculation ;

(d) Measuring oblique angles.

POINTS IMPORTANT TO BE BORNE IN MIND IN USING THE BOX SEXTANT.

(**110**).—There are two considerations which must not be lost sight of :—

(a) That the sextant measures actual angles ;

(b) That parallax may affect the readings.

Usually in land surveying we require to take either vertical or horizontal angles, and it is necessary, therefore, in order to do this with the sextant, that the objects selected for sighting should be, in the case of horizontal angles, as nearly as possible level with each other, and in the case of vertical angles over each other, to avoid the necessity of reducing oblique to horizontal angles.

To avoid appreciable error from parallax, the object chosen for reflection should be at a considerable distance from the point of view, but the error is so slight that no notice need probably be taken of it, except when great accuracy is desired, in which case the theodolite had better be employed.

HOW TO USE THE BOX SEXTANT.

(**111**).—Hold the instrument in the left hand, open the shutter, allowing the sun-glasses to be turned out of the way if not required, remove the cover, screw it to the

bottom of the box, pull out and fix the telescope, if required, or remove it and close the sight shutter, sight one of the objects, between which the angle is to be read, by direct vision (the right hand object), turn the milled head of the index-arm with the right hand until the second object is reflected in the silvered part of the horizon-glass accurately over the object seen by direct vision ; and read the angle from the graduated arm and vernier.   (See description of Vernier, Art. 16.)

In taking vertical angles, the process is similar, but the box is held edgewise.

The instrument may be used right side up, upside down, or edgewise, as the exigencies of the case demand.

CHAPTER VIII.

INSTRUMENT FOR MEASURING ALTITUDE BY ATMOSPHERIC PRESSURE.

The Aneroid Barometer—Description—Purposes of the Instrument—How Used.

THE ANEROID BAROMETER.

(**112**).—This is perhaps hardly a surveying instrument, strictly speaking, but is very useful for ascertaining altitudes approximately, and is often used for that purpose where great accuracy is not essential.

It consists of a corrugated metal box, a strong spring, a set of levers, and a dial with a hand reading upon it, the whole enclosed in a metal case with a glass front, as shown in Illustration No. 65.

It depends for its action upon the pressure of the atmosphere on the corrugated metal box from which the air has been exhausted ; the tendency of the

ILLUSTRATION No. 65.

vacuum within the box to cause it to collapse, and the action of nicely-arranged springs to keep it from collapsing, being so finely balanced that the box forms so sensitive a medium as to be affected by even the variation in air pressure occasioned by a difference of altitude of only two feet. This extreme sensitiveness also implies the peculiar delicacy of the instrument.
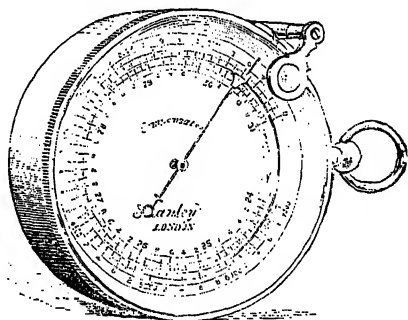
By aid of levers and very delicate and nicely-regulated mechanism, the movement caused by the expansion or contraction of the corrugated box is so multiplied and transferred as to cause a hand to travel over a dial, which is carefully graduated. In this way the altitude is registered with something like accuracy by the better instruments, and useful approximation by those of less expensive manufacture.

Dials are variously divided and figured, but generally the circumference of the dial is divided into two distinct rings, the outer ring showing feet in hundredths and thousandths, and the inner ring registering units and tens, the former being denoted by small, and the latter by larger figures. The readings are taken direct from the dial without calculation.
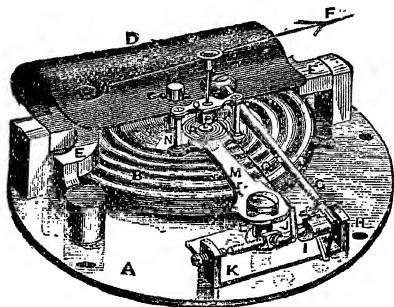
Illustration No. 66 shows the working parts of the aneroid barometer, and almost speaks for itself. B is the corrugated box, D the spring, and G the levers which transfer all movements in the box to the hands F, which read on the dial, not shown in this illustra-

ILLUSTRATION No. 66.

tion, its removal being necessary to disclose the working parts. The plate A, frame E, etc., need no explanation.

THE PURPOSE OF THE INSTRUMENT.

(**113**).—The instrument is used, as already pointed out, for taking altitudes where great accuracy is not essential; chiefly in contouring or ascertaining the approximate difference in level between two or more points which may

be required to be known for various engineering and surveying purposes, which it is not possible to stop here to discuss. (See Chapter XXIV. on " Contouring.")

TO USE THE ANEROID BAROMETER.

(**114**).—In use, the instrument is simply carried in the hand, the glass face being gently tapped with the finger nail occasionally to make sure that the working parts are in operation, when it will register the altitude, which may be read from the dial without calculation.

CHAPTER IX.

Instruments for Filling in in the Field the Details of Topographical Surveys.

The Plane Table — Description — Purposes of the Instrument — How Used.

THE PLANE TABLE.

(**115**).—This instrument (a general view of which is given in Illustration No. 67) consists of a drawing-board supported on a framed tripod stand, a ruler, *a* in illustration, technically known as an " allidade," which has either sights at each end or is surmounted by a small

ILLUSTRATION No. 67.

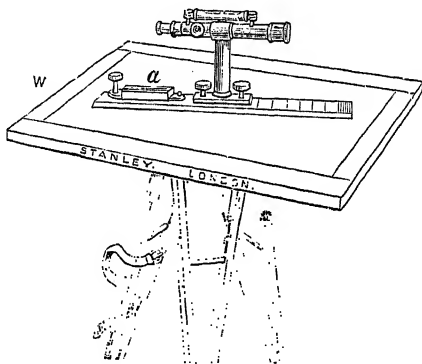telescope provided with a spirit level ; a separate spirit level ; and a trough compass. The drawing-board has generally a countersunk margin round the edge, so that the drawing pins fixing the paper shall not get in the way of the ruler. Some boards are made for continuous paper, and these have slit-like openings at each end, nearly across their full width, for the paper to pass through ; and

there are rollers underneath the board, so that the paper is rolled off one on to the other as it is used. This is a very convenient arrangement where the survey is of a long strip of country, as for a railway or canal.

The best stands are provided with three screws, one at each of the angles of the triangular head formed by the three legs *s s s*, as shown in Illustration No. 68, so that the board may the more readily be set perfectly level. It is screwed down in the centre, and presses on the heads of these screws. In some tripods, one of the legs is made to shorten, when

ILLUSTRATION No. 68.

necessary, to enable the board to be set level on very hilly ground.
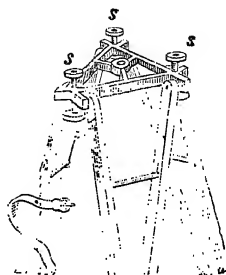
PURPOSE OF THE PLANE TABLE.

(**116**).—The purpose of the instrument is to enable details of surveys to be drawn in the field without the necessity of chaining and plotting, or taking angular measurements, and thus to save much time. It should not be supposed, however, that work done with the plane table is more than approximately accurate, or that it will bear comparison with a trigonometrical survey performed with the theodolite. That it is, nevertheless, sufficiently accurate for the purpose for which it is intended is shown by the fact that the topographical details of most of the great surveys have been mapped by its aid.

TO USE THE PLANE TABLE.

(**117**).—Set up the tripod at a convenient spot for sighting all the corners of fields, etc., as station No. 1, Illustration No. 69, clamp the board upon it, set the edge

marked W truly north and south, and stretch the paper
upon the board.   From a point on the paper convenient
for taking in the piece of land to be plotted, having regard
to the scale required, make a fine dot with a fairly hard
pencil, direct the ruling edge of the allidade from this dot
to each of the angles, etc., required to be mapped, and
draw fine pencil lines to the station on the paper.   These
are shown in hard lines on the board at station 1,
in Illustration No. 69.
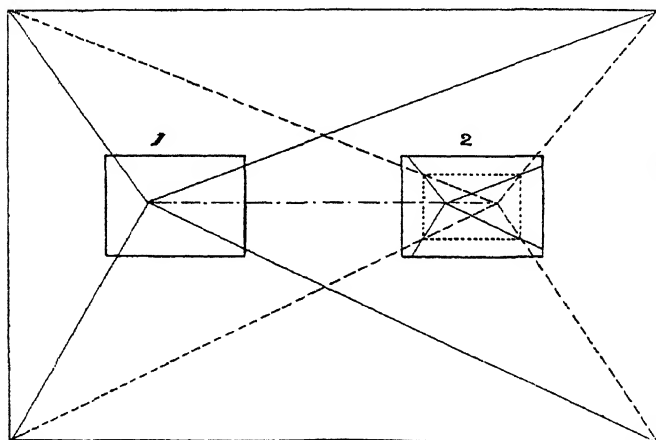
ILLUSTRATION No. 69.

Next choose another station from which all the points
previously sighted may be seen, which must be at a
distance from station 1, which will allow of its being
plotted to the desired scale on the sheet of paper.   Have
a picket put in at station No. 2, and, before the board is
moved from station No. 1, sight to it with the allidade,
and draw a line on the paper shown ——·——·——·—— in
illustration.   Measure the distance between station 1 and
2 on the ground, and very accurately plot station 2 to
scale on the paper.   Now move the whole instrument,

and adjust it over station No. 2, again directing the edge of the board marked *W*, north and south, and also sighting with the allidade to a picket put in at station 1, observing that the line on the paper representing the distance between stations Nos. 1 and 2 lies perfectly between them. This done, sight to all the points before sighted, and again draw lines on the paper represented — — — — in the illustration. The intersection of these lines with those previously drawn from the same points will give their positions on the plan, and the boundaries, etc., may be drawn in, as shown by fine dotted lines.

For the sake of simplicity and clearness, a mere rectangle has been chosen for illustration, but it will be readily seen that a figure of any number of sides may be treated in the same way.

It seems hardly necessary to observe that the accuracy of the work will depend on the perfect horizontality of the board and ruler, the careful sighting of the objects, the correct measurement and scaling of the distance between stations Nos. 1 and 2, and the accuracy with which the setting up of the board at the second station is performed.

CHAPTER X.

THE GLASS TRACING TABLE.

(**118**).—This consists of a sheet of good plate glass in a
frame, sunk in a rabbet so as to make a perfectly level
surface ; the frame being raised on tressels so as to admit
the light to the under side of the glass ; sometimes a
reflector is provided between the tressels for the purpose
of increasing the light.

Drawings, copies of which are required, are laid upon
the glass with a sheet of drawing paper over them, and
both are pinned to the frame of the table. The light
under the glass is sufficient to enable the drawing to be
traced on to the paper over it, and thus an exact
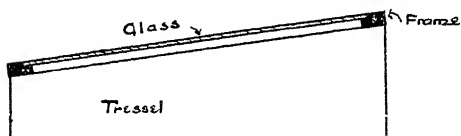
ILLUSTRATION No. 70.

copy is obtained without pricking or otherwise marking
the paper or the plan to be copied.

The reflector may be so supported as to be capable of
being raised or depressed and fixed to suit the angle of the
light in which it is being used.

The glass tracing board in its simplest form is shown in
section in Illustration No. 70.

THE PANTAGRAPH.

(**119**). *Description.*—The pantagraph, shown in Illustration No. 71, although usually described as an instrument for reproducing, enlarging, or reducing drawings, would more properly be described as an instrument for reducing
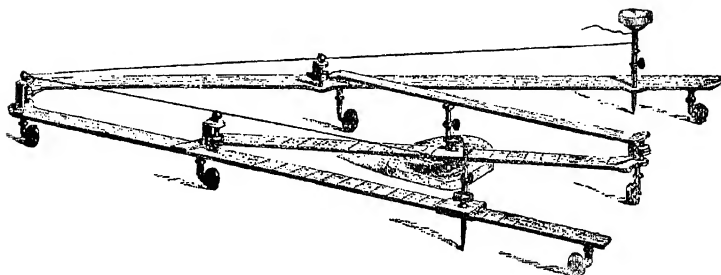
ILLUSTRATION No. 71.

only, that being all it is capable of performing with a sufficient degree of accuracy for practical purposes.

The nature of the instrument will be best understood by a reference to the illustration. The following are its chief parts :—

(1) Two pair of square brass tubular arms, so jointed together as to allow of the free movement of the instrument in a lateral direction. One of the shorter arms and half one of the longer arms is graduated and marked $\frac{1}{2}$, $\frac{1}{3}$, etc., and the whole instrument is mounted on castors to give free, smooth motion.

(2) The fulcrum or axis upon which the whole instrument moves. It consists of a heavy weight with some fine needle points projecting slightly from the underside to prevent slipping, and a pin piece projecting from its upper side, which fits accurately into a socket on a sliding head or index.

(3) Two sliding heads which clamp to the arms of the instrument by means of screws and plates, and carry a tube-like socket to receive either the pin piece of the fulcrum, a pencil, or the tracer. The socket for the pencil is mounted by a small brass cup, which being filled with shot, bears sufficiently on the pencil to ensure its marking without pressure from the hand.

(4) A tracer, merely a dummy pencil, which may be carried over the lines of a drawing without marking or injuring it in any way.

(5) A thin silk cord passing from the tracer to the pencil holder over pulleys, to enable the pencil to be raised off the paper by the hand using the tracer.

METHOD OF SETTING THE PANTAGRAPH.

(**120**).—Before the pantagraph can be used, it has to be set so that the drawing produced may be in the desired proportion to the original from which the copy is to be made.

There are two ways of setting the pantagraph, technically known as the " erect " setting and the " reverse " setting.

The " erect " setting is that by which the copy is produced erect or the same way up as the original. The
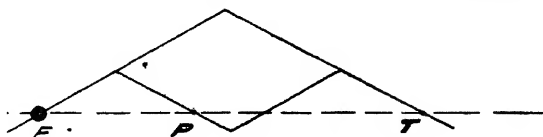
ILLUSTRATION No. 72.

fulcrum is in this case under the graduated long outside arm, and the pencil is on the graduated short inside arm, as shown in Illustration No. 72. With this setting

drawings may be reduced to one-half or any smaller proportion.

The "reverse" setting is that by which the copy is produced upside down or the reverse way up to the original. The fulcrum is in this case under the short central graduated arm, and the pencil on the long outer graduated arm, as indicated in Illustration No. 73.
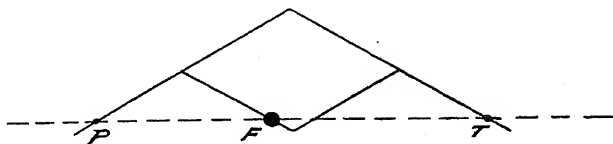
ILLUSTRATION No. 73.

It will be seen that with the fulcrum at F in Illustration No. 72, a downward or upward movement of the tracer T would be accompanied by a corresponding downward or upward movement of the pencil P; but with the fulcrum at F, in Illustration No. 73, an upward movement of the tracer T would be accompanied by a downward movement of the pencil P; hence the copy is reversed in the latter case.

(**121**).—*To set the pantagraph according to erect setting* :—

> (1) Where the numerator of the proportion is unity as 1 to 3, 1 to 5, one-third, one-fifth, etc. Rule—Simply clamp the heads for the fulcrum and pencil at the divisions marked with the proportion required, which cannot exceed one-half the original.

> (2) Where the numerator of the proportion required is greater than unity as 2 to 3, 3 to 5, etc., two-thirds, three-fifths, etc. Rule—Set the heads at the divisions on the arms marked with the same numerator as the proportion required, but with a denominator equal to the denomi-

nator of the proportion required minus the numerator of the proportion required. Thus, if you require the proportion two-fifths, set the head at the division marked two-thirds, etc.

(**122**).—*To set the pantagraph according to reverse setting :—*

(1) When the copy required is to be in any proportion to the original in which the numerator is greater than unity. Rule—Simply clamp the heads for the fulcrum and pencil at the divisions on the arms marked with the proportion required.

(2) When the copy is required to be in any proportion in which the numerator is unity. Rule—Clamp the heads at the divisions on the arms marked with a proportion which has the same numerator and a denominator less by one than the denominator of the proportion which it is desired the copy should bear to the original. Thus, to get a proportion of one-seventh, set the heads at divisions marked one-sixth; to get one-eighth, set the heads at one-seventh, and so on.

Note.—In all cases when the instrument is properly adjusted, the fulcrum, pencil and tracer are in the same straight line.

From the foregoing rules it will be seen :—

(1) That by the erect setting, we may adjust the instrument without calculation, for any proportion in which the numerator is 1, up to one-half full size ; and similarly, by the reverse setting, the instrument may be adjusted without calculation for any proportion in which the numerator is greater than 1, up to the full size.

(2) That in all other cases, the slight calculation mentioned in the rules must be made.

(3) That the instrument only reduces to different scales where those scales bear to each other one of the proportions engraved on the arms.

(4) That the tracer remains in the same position in all cases when producing reduced copies. The pencil may change places with the tracer when it is desired to produce an enlarged copy, but as already pointed out, the instrument is not satisfactory for the purpose and the work cannot be relied upon.

(5) If the copy is to be in a proportion greater than one-half, the reverse setting must be employed.

(6) The erect or reverse setting may be employed in other cases.

(7) In the reverse setting the fulcrum stands between the two drawings, by which means more room is gained, and this is, therefore, usually the more convenient manner of using the instrument.

TO USE THE PANTAGRAPH.

(123).—The manner of using the instrument when once set is very simple. The drawing of which a reduced copy is required is pinned on the right-hand side of a large drawing board, and a clean sheet of paper on the left-hand side. The instrument is then placed in position so that the operator may be able to trace as much of the drawing as possible (the whole, if possible), without its being removed. The tracer is then guided over the lines by a steady hand, a flat ruler being used to guide it in the case of straight lines, and the pencil simultaneously produces the reduced copy on the clean sheet of paper. When it is desired to remove the tracer from one part of the drawing to another without tracing, the silk cord is pulled, and the pencil raised to prevent false lines being produced on the copy.

If it is necessary to remove the drawing before the copy is finished, owing to its size or other reason, all that is necessary in resuming operations is to take care that the drawings are so fixed that the tracer and pencil occupy identical positions on the original drawing and copy respectively. Copies of drawings to same scale as original may be simply made by aid of the glass tracing table. (See Art. 118.)

THE EIDOGRAPH.

(**124**). *Description.*—The eidograph, like the pantagraph, is an instrument for producing copies of drawings, and is better for reducing than enlarging. It is probably the better instrument of the two, and has the great advantage of reducing from one scale to any other, instead of only in fixed proportions. The instrument is shown in Illustration No. 74, from which a good idea
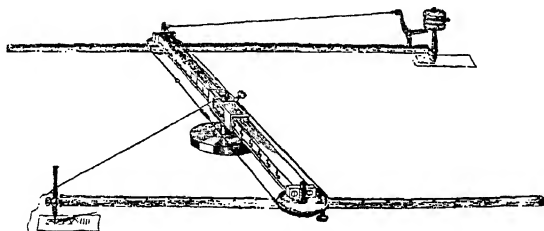
ILLUSTRATION No. 74.

of its form and character may be gathered. Its chief parts are as follows :—

    (1) A fulcrum similar to that of the pantagraph.

    (2) A centre beam and two arms each divided and numbered so as to give a hundred equal parts each side of a central line.

    (3) Two wheels of equal size turning on axles in the ends of the centre beam, connected by steel belts attached to the circumference, and provided with means of lengthening and

shortening to enable them to be adjusted to exactly the same length which brings the arms absolutely parallel with each other. Each of these wheels has a box-like head on the under side through which the arms pass, and an opening in the upper side exposing them to view. One edge of each of the openings is bevelled and formed into a vernier, which reads into the graduations on the arms. (For description of Vernier see Art. 16.)

(4) A head or index through which the centre beam passes, with a socket into which the fulcrum pin accurately fits. This head has an opening formed in its upper side with one edge bevelled and formed into a vernier similar to those on the wheels. This head is provided with a clamp screw for making it fast to the beam, as also are the boxes on the wheels.

(5) Tube-like sockets to receive the pencil and tracer.

(6) A silk cord, like that on the pantagraph, for enabling the pencil to be raised by the hand engaged with the tracer.

(7) A loose weight to slide on the centre beam and balance the instrument when, in order to get the required proportion, the fulcrum has to be placed far out of the centre of the beam.

TO SET THE EIDOGRAPH.

(**125**).—Before the eidograph can be used it has to be set, according to the proportion the copy is to bear to the original. This is simply done as follows :—

RULE.—To the difference of the terms of proportion which the copy is to bear to the original, add two cyphers and divide the result by the sum of the terms of the proportion. Then set the verniers to the number corre-

sponding. It will be noticed that there are two places
on each arm where the verniers may be set to the same
reading. For reducing, the centre beam is to be set at
the reading nearest the pencil and the arm carrying the
tracer to the reading on the side farthest from the tracer.
For enlarging, the setting would be the reverse.

EXAMPLE.—Suppose the proportion to which it is
desired the copy shall be drawn be as 5 is to 7, the
difference is 2, which with two cyphers added becomes
200, $5+7=12$, and $\dfrac{200}{12}=16{\cdot}666$.   The index must there-
fore be set to $16{\cdot}67$, by the vernier.

Suppose it is desired to reduce a plan from 40ft. to an
inch to four chains to an inch, the scales must be reduced
to like terms in order to get their proportion.   Four chains
to an inch equals 264ft. to an inch, so the proportion
in this case is as 40 is to 264, and their difference is
224, which, with two cyphers added, becomes 22,400.
$264+40=304$, and $\dfrac{22400}{304}=73{\cdot}68$, which gives the index
reading for that proportion.

TO TEST THE INSTRUMENT.

(**126**).—The instrument should always be tested before
use, and, if necessary, accurately adjusted. This may
be done as follows :—

> (1) With the indexes set accurately at zero simul-
> taneously make two lines with the pencil and
> tracer at right angles to the centre beam ; turn
> the whole instrument round on the fulcrum
> and again make lines, which should fall exactly
> on those previously made. If this is not so,
> correct for half the error by shortening one
> belt and lengthening the other, and again test
> and adjust until perfect accuracy is obtained.

(2) When the adjustment already referred to is accurately made, observe whether the fulcrum, tracer and pencil are all in the same straight line. If not, the instrument must be corrected by the manufacturers.

TO USE THE EIDOGRAPH.

(**127**).—The eidograph is used similarly to the pantagraph, that is to say, the tracer is merely guided over the lines accurately, and the pencil simultaneously draws the copy to the scale required on the clean sheet of drawing paper provided to receive it.

THE COMPUTING SCALE.

(**128**). *Description.*—The computing scale is a simple, ingenious instrument for computing areas from plans or maps. It may be said to consist of three parts :—(1) A boxwood rule about 20in. long ; (2) an index, or reader ; (3) a piece of transparent paper divided by parallel lines.

The instrument is shown in Illustration No. 75, from which a general idea of its character may be gathered, but the details of its construction will be better understood after the very simple principles upon which it
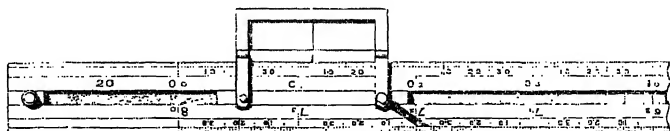

ILLUSTRATION No. 75.

depends are known, and, therefore, before further describing it, I will state them as clearly as possible.

Since there are 100,000 square links in an acre, a rectangular strip of land one chain or 100 links deep, and ten chains or 1,000 links long, will contain an acre, consequently, if a plan of land is divided up into strips

one chain deep, according to the scale to which it is drawn, and the total length of those strips in chains is divided by 10, the result will be acres. This is what the computing scale does, only by it the dividing of the plan into strips is simply accomplished by the transparent paper being laid over it, and the length of the strips or acreage is read direct from the scale, without calculation.

After this explanation of the principles upon which the instrument depends, the following detailed description of it will be easily followed :—

THE RULE.—The rule, which is about 20in. long, is engine-divided on each edge, so as to give two different scales. The distance between the main divisions equals ten chains according to the scale, and the divisions are marked 0, 1, 2, 3, etc., representing acres. These main divisions are subdivided into four equal parts, and numbered 1, 2, 3, etc., representing roods ; and the subdivisions are again divided into ten equal parts, each division consequently being equal to four poles.

There is an undercut groove down the centre of the rule throughout its entire length, in which the carriage of the index slides, and small metal stops, at each end of the groove, prevent its being drawn right out.

THE INDEX. — The index is for the purpose of measuring lines. It consists of a metal frame with a piece of glass or other transparent material inserted in it, which is divided by a line engraved thereon.

The length of the carriage, the tail piece of the rule, beyond the divided portion, and the stops at the ends of the groove, are so adjusted that, when the index is brought right back, the line on the glass exactly corresponds with zero on the scale. There is a small head screwed into the slide of the index for convenience in using it.

THE TRANSPARENT PAPER; technically, HORN PAPER.—
This is simply a sheet of hard transparent paper divided
by parallel lines, the distance between the lines corre-
sponding with the scale of the computor. Where the
computor has a different scale on opposite edges, as is
usual, separate pieces of horn paper ruled to the respective
scales will be required.

TO USE THE COMPUTING SCALE.

(**129**).—In using the computor, the horn paper of
proper scale is laid over the plan from which the area is
to be computed, and fixed with weights or pins.

As lands are rarely bounded by straight lines, in order
to divide the land into rectangular strips it is necessary
to imagine give and take lines. This is very easily and
accurately done by the eye if the horn paper is used
diagonally, as shown in Illustration No. 76; the eye
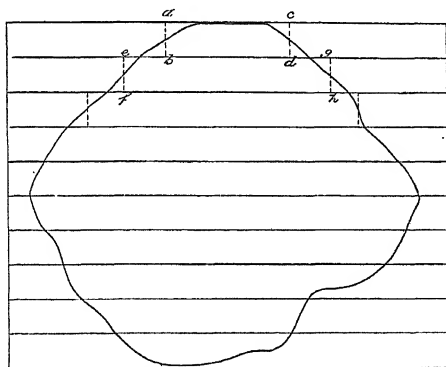
ILLUSTRATION No. 76.

will divide the small pieces at the ends of the lines
with remarkable precision. The give and take lines are
not marked on the paper or plan; the dotted lines in the
illustration merely represent the position of the line on
the indicator of the instrument.

The scale, with the reader or index set to zero,
is laid with its edge along the lower parallel line
bounding the uppermost strip, and with the line of the
index in the position indicated by the give and take
line *a b* in illustration. The index is then moved to
the position indicated by the give and take line *c d*
on the right. The rule, without moving the index, is
then placed with its edge on the second parallel line,
and with the line of the index in the position indicated
by the give and take line *e f*. The index is then
moved to take the position of the give and take line
g h, and the scale, without shifting the index, is
again moved to the third line, and this process is
repeated until the index reaches the end of the scale,
where it reads a certain number of complete acres.
The index is then once more set to zero, and the
process is repeated for the remaining lines until they
have all been scaled.

A careful record of the number of times the index
reaches the end of the scale must be kept by making a
mark on a piece of paper each time the index has to be
reset to zero.

When the index reaches the end of its run in the course
of scaling any line, a mark from which the computation
may be resumed must be made with a soft pencil on the
horn paper.

When all the lines have been scaled, the number of
acres represented by the scale must be multiplied by the
number of times the index has been reset at zero, and the
reading on the scale, when the last line was completed,
must be added for the total area.

The stops at the ends of the groove can be removed and
the index reversed when the scale on the other edge of
the rule is required to be used. (See Chapters XVI. and
XVII.)

THE FIXED PLANIMETER.

(**130**). *Description.*—The planimeter is an instrument for computing areas from drawings, and is probably more generally useful to the mechanical engineer than to the surveyor, it being more serviceable in computing the areas of very small surfaces than of considerable ones such as the land surveyor usually has to do with.

The computing scale, already described, is by far the more generally useful instrument for our purpose; but where it is required to compute the area from a map drawn to a small scale, the planimeter will, no doubt, give a more accurate result.

It must not be forgotten, however, that maps and plans of land drawn to a very small scale cannot show detail, and the boundaries, therefore, cannot always be accurately defined. For instance, in the case of the small scale
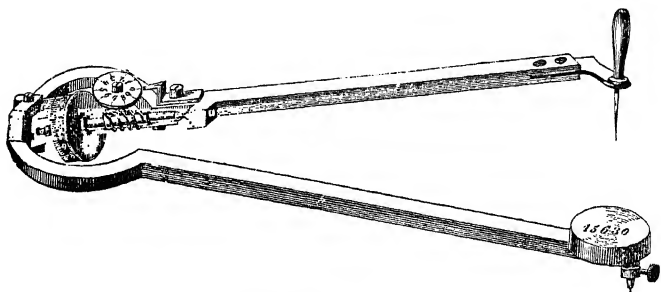
ILLUSTRATION No. 77.

ordnance maps from which areas are so frequently computed for approximate purposes, boundaries are indicated by single lines, and where, therefore, the boundary consists in a hedge and ditch, or ditch only, the exact extremity of the estate cannot be judged.

The instrument consists of two metal arms, which, for the sake of distinction, I will call respectively the axis arm and the tracing arm. The "axis arm" has a fine

needle point at one end, with a weight fitting on to a pivot over it, the purpose of the latter being to keep the needle from riding out of the puncture made by it in the paper, when the instrument is in use. I have called this the axis arm because the needle point referred to forms the axis upon which the whole instrument turns in operation. The other end of the arm is hinged to the " tracing arm " with pivots, allowing free horizontal motion. There is a constant number engraved on one of the arms, to which we shall refer hereafter. The " tracing arm " carries a tracer at the free end, and what may be spoken of as the working parts of the instrument at the other. These consist of :—

(1) A roller which revolves on a horizontal axis, and is divided on its edge into 100 equal parts. This roller turns either backwards or forwards with all horizontal movements of the instrument, except those in the direction of its axis, by friction on the paper as it rolls over it. There is a vernier reading into the divisions on the roller to one-tenth of a division or one-thousandth part of a revolution.

(2) A dial which rotates with the movement of the roller ; it is divided into ten equal parts, each part indicating a complete revolution of the roller. A fixed indicator shows the movements of the dial.

TO USE THE INSTRUMENT.

(**131**).—(1) Press the needle point of the axis bar into the paper in a convenient position for the tracer to follow the outline of the figure to be computed, and place the weight on the pivot over it.

(2) Carefully note the reading of the dial, roller, and vernier before commencing to trace. The dial registers integers, and the roller and vernier decimals.

(3) Carefully trace the outline of the figure to be computed with the tracer, noting or marking the point of commencement.

(4) Having traced the outline, again read the dial, roller and vernier.

(5) In cases where the needle of the axis arm has had to be placed within the boundary line of the land to be computed, in order that the tracer might reach all parts of it, add the constant number engraved on the instrument to the reading, and deduct, from the resulting sum, the reading taken before commencing to trace.

(6) When the needle of the axis arm has been placed outside the boundary line of the figure to be computed, deduct the reading taken before the tracing was commenced from the reading taken after it has been completed.

(7) Where the dial has gone through more than one complete revolution, add or deduct the number of units represented by each revolution multiplied by the number of revolutions, to or from the reading arrived at as before directed. The number is added if the dial has revolved forwards, and deducted if it has revolved backwards.

(8) Multiply the resulting area, which is square inches, by the square of the scale to which the plan is drawn, to obtain the true area.

For " Computations " see Chapters XVI. and XVII.

THE POLAR PLANIMETER.

(132).—There is another form of this instrument, known as the polar planimeter, in which the axis arm may be lengthened so as to give the unit in square inches, metres, etc., but there is little if any advantage in it over the fixed kind, from a land surveyor's point of view, and it will not therefore be further referred to here.

CHAPTER XI.

DRAWING INSTRUMENTS.

*Scales—Drawing Boards—T-Squares—Straight-edges—
Set-Squares—Parallel Rulers—Rolling Parallels—
Compasses—Spring-Bows—Beam-Compasses—Pens—
Stencils—Printing Pens—Drawing Pins—Pencils—
India-rubber — Erasers — Inks — Colours — Brushes
— Saucers—Books and Pads—Drawing Papers.*

It would serve no useful purpose to give a lengthy
description of the numerous drawing instruments which
may be procured. The most common, and those which it
is really necessary to have, will be familiar to everyone
who is likely to be interested in the subject under con-
sideration. It will, however, be handy to some readers to
have a brief description of the instruments which everyone
engaged in the work of plotting land surveys will require.
These are as follows :—

SCALES.

(**133**).—The best scales for the land surveyor's purpose
are undoubtedly those which have
only one scale on each edge, and

ILLUSTRATION No. 78.

are in section, as shown in Illus-
tration No. 78.
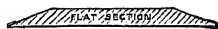
In these scales the divided edge is quite on the
paper, which enables distances to be plotted with great
accuracy, and the confusion caused by an attempt to crowd
a number of scales on to one rule is avoided. A scale of
the kind advised is shown in Illustration No. 79.

Each scale should be accompanied by an off-set piece,
viz., a short scale about two inches long. This off-set
piece is used when plotting off-sets, which are short

distances measured at right angles to and from a given
point in a chain line. Thus, laying the scale with zero at
the commencement of the chain line, the off-set piece may
be slided to the various points at which the off-sets occur,
and their distances from the chain line may be set off at
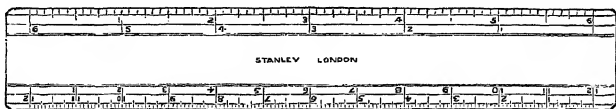
ILLUSTRATION No. 79.

the given points, and at right angles to it, in one operation,
instead of having first to mark off the distances on the
chain line, and then to set off the off-sets at each of the
various points separately.

Two fairly heavy paper-weights, one placed at each end
to steady the scale while the off-set piece is being used
upon its edge, will be found very useful.

DRAWING BOARDS.

(**134**).—The boards almost universally used by surveyors
for office purposes are of the battened kind, as shown in
Illustration No. 80, which gives a back view. They are

made of well-seasoned pine and
have a slip of ebony let into
one edge, which affords a very
true surface for the head of the
T-square to work upon.
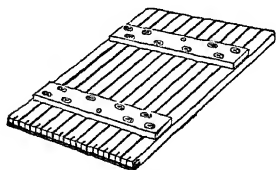
They are made in sizes 31in.
by 23in., 42in. by 29in. and 54in.

ILLUSTRATION No. 80.

by 33in., these dimensions suit the sizes of the ordinary
drawing papers, but any size may be had to order.

Boards of special kinds are obtainable, such as metal
protected boards for hot climates, portable boards, plate-
glass tracing boards, etc., etc.

TRESSELS.

(**135**).—Tressels constructed of hard wood, to suit the sizes of the different boards, are also supplied ; but, as a general rule, a fixed horizontal bench or table, with a slant for each board to be used on it, will be found by far the more convenient arrangement, as by this means a flat table upon which instruments, etc., may be laid, is obtained, whilst the board is on an incline, as it must always be for convenience in use. Of course, a very large specially-made board on tressels, with smaller boards and slants, with space between them, will fulfil this requirement.

T-SQUARES.

(**136**).—The T-square is shown in Illustration No. 81. The best kinds are those made in mahogany with an ebony edge. They should be of a size to suit the drawing board, and may be obtained from 12in. to 54in. long. Some T-squares are made with a shifting head. This is a very convenient arrangement for ruling a number of parallel lines at an angle with the edge of the board,

ILLUSTRATION No. 81.

but they will not be found nearly so handy for ordinary use as the simple T-square. It is better to have them as distinct instruments. Medium-sized instruments are, of course, always more handy to use than extra large ones, hence the desirability of having more than one set, the medium-sized instruments for use in general, and larger boards, squares, etc., for special cases.

STRAIGHT-EDGES.

(**137**).—Straight-edges are simply flat rules, the best being made of mahogany with bevelled ebony edges. They are very handy for ruling in long lines, such as the chain lines in a land survey and are, therefore, essentially part of the land surveyor's tools. Flat or square rules with metal edges are also most useful for trimming drawings, for which purpose the ordinary straight-edge should never be used, for obvious reasons.

SET-SQUARES.

(**138**).—The transparent celluloid set-square is the kind I prefer, as it is so frequently a convenience to be able to see the part of the drawing covered by the square. Set-squares may be obtained of various sizes, degrees and kinds; such as pear wood, framed mahogany, vulcanite, aluminium, etc., the latter being very nice but costly.

There are also what are known as lettering set-squares, which give the correct angles for Roman and block printing, and are most useful for their purpose.

PARALLEL RULERS.

(**139**).—The parallel ruler consists of two flat (usually ebony) rules so connected by pieces of brass at each end that they are always kept parallel with each other. They are used for drawing a number of parallel lines, by holding the lower rule still, while the upper rule is shifted to the various points from which it is required to draw the lines. They may be obtained in various sizes and qualities.

ROLLING PARALLELS.

(**140**).—The rolling parallel is shown in Illustration No. 82, and consists of a rule (usually ebony) with ivory or metal edge. It is also made in solid brass, gun-metal, electrum and ivory. The rule has two openings

formed in it through which milled rollers protrude to
its underside.   These rollers turn on pivots, and are
made of exactly equal size, so that when the rule is

ILLUSTRATION No. 82.

rolled upon them, lines drawn by its edge are perfectly
parallel.   The bevelled edges of the rule may be divided
to any useful scale.

A glance at the illustration is all that is necessary to
acquaint the reader with the instrument, but it is so
familiar that in most cases no introduction will be
necessary.

COMPASSES.

(**141**).—The ordinary half-set of compasses, a good
ruling pen, and a set of spring-bows, are probably all
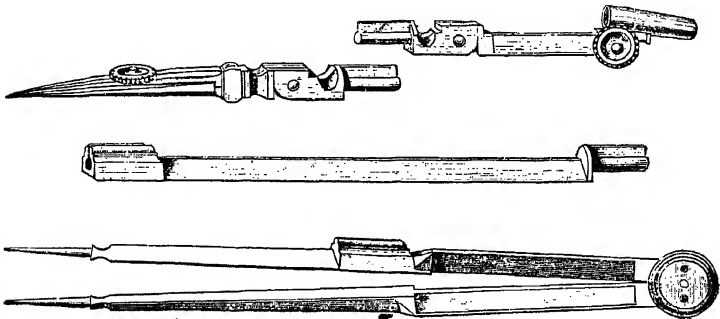
ILLUSTRATION No. 83.

the drawing instruments which it is necessary the
student should possess, although the beam-compasses, to
which I shall refer later on, will be found very useful.

The compasses are too well known to need description. The best instruments are usually made in electrum, have sector joints, needle points, and turn-up nibs to the pens ; but instruments of good manufacture, without these refinements, are to be preferred to poor ones possessing them. The best advice that can be given on the choice of instruments is to recommend their being purchased from a good manufacturer whose name is sufficient guarantee of the quality of the goods supplied.

SPRING-BOWS.

(**142**).—A set of spring-bows will be found extremely useful in turning very small circles, which could not be done with equal neatness with larger instruments.

BEAM-COMPASSES.

(**143**).—The beam-compasses are extremely useful to the land surveyor in plotting the main lines of a survey ; indeed, they are almost essential in large work. They consist of three parts : (1) a beam or lath, (2) a head carrying a needlepoint, (3) a head with pencil and pen.

The beam may be a long rule divided to any scale, or the heads may be used on the straight-edge or laths kept for that purpose. When the beam is a scale, the heads may be set and finely adjusted to any distance apart directly ; otherwise the distance must first be plotted on the paper, and the instrument set from it.

The compasses consist of two heads or metal clamps carrying the needlepoint and pen and pencil holder. Inside the heads there are loose plates, which are prevented coming right out by two small screws, whilst there is a large milled head which, when screwed down, presses on this plate, so that the beam is held firmly between it and the side of the box. In this way the heads are so set on the beam that the distance between the point and the pencil

equals the radius of the arc to be described. One of the heads is provided with a fine adjustment screw, by which means great accuracy in setting may be obtained.

The object of the beam-compasses is to enable arcs to be struck, the radius of which could not be taken in the ordinary compasses, hence the usefulness of the instrument in plotting the constructional lines of a large survey.

DRAWING PENS.

(**144**).—Of drawing or ruling pens there are many kinds. Illustration No. 84 shows four pens useful for
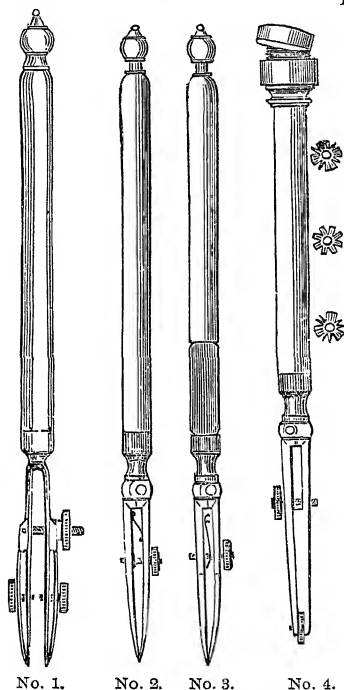
No. 1.   No. 2.   No. 3.   No. 4.
ILLUSTRATION No. 84.

different purposes. No. 1 is a road or double pen for drawing two parallel lines simultaneously; Nos. 2 and 3

are ordinary ruling pens; and No. 4 is a pen for making dotted lines, and contains a set of four wheels.

STENCILS, LETTERING, CORNERS, NORTH-POINTS, ETC.

(**145**).—The best advice I can give with regard to the use of stencils cn important drawings is that they should be avoided. There are, however, circumstances where the expenditure of much time in printing is not justifiable, and in those cases stencils may be found very serviceable. Stencils in words such as "Longitudinal Section," "Cross Section," "Section," "Plan," "Datum," "Bench Mark," and also north-points, corners for border lines, and the like, will be found useful; but mere alphabets are not easy to use, and not much time is saved by employing them.

PRINTING PENS.

(**146**).—There are probably no nibs better suited for the purpose of printing than those manufactured by Messrs. Gillott. What the draughtsman needs to do is to find a nib which suits his particular style.

Printing will be referred to in connection with plotting and finishing plans.

DRAWING PINS.

(**147**).—Little need be said with regard to drawing pins. Those are best in which the pin does not come right through to the upper side of the head, or in which it has a shoulder, so preventing the possibility of its coming right through and entering the finger when being pressed into the board, an occurrence which does not conduce to that undisturbed state of mind which is helpful when engaged on important work.

The heads of the best pins are made of electrum and should be fairly large, not too thick, and taper well from

the centre to the outer edge so that they may not be constantly catching the T-square.

PENCILS.

(**148**).—It is very important to the draughtsman to have good pencils, and these he can best secure by obtaining them from one of the well-known manufacturers.

For our work it is not really necessary to have more than three grades, F, H and HH. The harder pencil is used for plotting and the softer for drawing in the undulating boundaries, division fences, etc., etc., and for sketching in the printing.

Messrs. Rowney and Co. supply several different classes of good pencils. The fine Cumberland graphite, hexagon in shape, with round lead, is that which I prefer. They are made in nine degrees, as follows : —

H	Hard.
HH	Harder.
HHH	Very hard, for architects.
HHHH	Extra hard for drawing on wood.
HB	Hard and black.
B	Black.
BB	Softer and very black.
BBB	Extra soft and black.
F	Firm for ordinary drawing.

INDIA-RUBBER.

(**149**).—India-rubber must be of good quality, white, and when rubbed on clean paper must leave it quite free from greasiness, stickiness, or mark. If it is too hard it will injure the surface of the paper and the colour will not take well. It will likewise mar the clearness of the inked-in lines.

Rubbing-out should be avoided as much as possible. Mere cleaning should be done with bread crumb, but the necessity for cleaning may be minimised by keeping

the drawing covered up, with the exception of just the space being worked on.

Properly speaking, it never should be necessary to remove a line which has been inked in, but unfortunately slight mistakes will sometimes happen. The best way to remove lines is to wet the part to be removed with a camel hair pencil, blot up the water at once with quite clean blotting paper, and then gently rub with soft india-rubber. This injures the paper less than scratching with a knife or eraser. On tracing cloth, however, lines are best removed with ink-eraser, as the cloth cannot be wetted without seriously defacing it. On tracing paper remove as directed for drawings. The disc erasers are the most convenient and best which can be obtained.

INKS.

(**150**).—The stick is the best form in which to have Indian ink, but then a quantity must be rubbed up fresh on each occasion when it is required. Liquid Indian ink is mostly used in drawing offices, as it can be obtained of excellent quality, and its use saves much time.

Where a drawing must be coloured directly after it is inked in, waterproof drawing ink should be used, or the lines will wash up to an extent which will at any rate be sufficient to mar their sharpness, if not to entirely spoil the drawing.

COLOURS.

(**151**).—The hexagonal colours are undoubtedly the best for our purpose, the size and shape so thoroughly adapting them to the requirements of draughtsmen.

Illustration No. 85 shows a box of these colours which will be found to contain all that may be required.

In addition to these colours it will be found very convenient to have a bottle or two of liquid colour, which is particularly suited to showing coloured lines upon
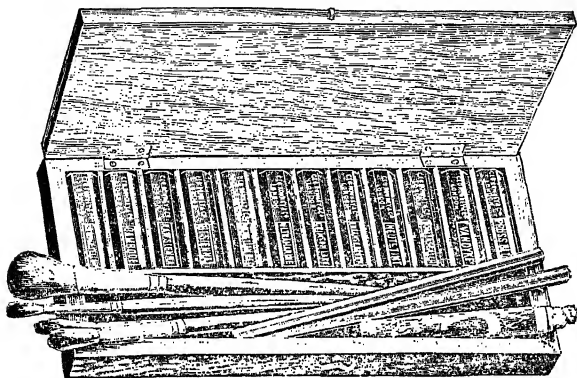
ILLUSTRATION No. 85.

plans, such as the lines upon which sections have been taken, etc., etc., datum lines on sections, and the like.

The colouring of plans is dealt with in Chapter XV., Art. 212.

BRUSHES.

(**152**).—The necessity of having good brushes cannot be too strongly urged. It is practically impossible to satisfactorily colour a drawing with a bad brush. Again, the size of the brush must be suited to the work to be performed.

Generally speaking, it is well to use a fairly large brush, having regard to the work to be done, which helps to secure evenness and flatness of tint. The red sable brushes in seamless plated ferrules (round) are those recommended. What are known as "stripers" will be found very useful when it is required to carry

a border of darker colour round a boundary line, which is frequently done.

There are a number of different brushes suitable for particular purposes, but it is not thought necessary to refer further to them here.

SAUCERS, WATER GLASSES, ETC.

(**153**).—The " cabinet nest " of saucers and the tinting saucers are those best suited to the requirements of the land surveyor. An Indian ink stone will also be found useful.

Two water glasses will be required ; one for washing the brushes in, and the other to contain clean water for mixing purposes.

BOOKS.

(**154**).—The following are the chief books a land surveyor is likely to require for field use :—

　　(a) Field Book ;
　　(b) Level Book ;
　　(c) Traverse Book ;
　　(d) Sketch Book.

The field book should be about seven inches long when closed and about five inches wide, in order that there may be plenty of room for sketches on either side of the centre column. The book opens lengthwise, and each page is ruled either with a single line down the centre or two lines about five-eighths of an inch apart, thus forming a central column.

The first entries are made on the bottom of the last page, and are continued from the bottom to the top of each page, from the end to the beginning of the book. The force of this mode of entry will be seen when, in a later chapter, the method of booking field notes is dealt with. The books are usually bound in basil leather, and have an

elastic band to protect them. (See "Example Field Notes," Chapters XII. and XIV.)

THE LEVEL BOOK.

(**155**).—Given below is the headline of the most generally used level book, showing the columns into which it is divided. (See Chapter XXII. on "Levelling.")

Back-sight.	Inter-mediate.	Fore-sight.	Rise.	Fall.	Reduced Level.	Distance.	Remarks.

THE TRAVERSE BOOK.

(**156**).—Traverse books are variously ruled; the following will be found efficient :—

Line.	Dis-tance	Ver-nier.	Latitude.		Departure.		Total Depart-ure. East.	Double Longitudes.	Double Areas.	
			N'th.	S'th.	East.	West			North.	South.

The purposes of the various columns will be understood when the subject of traversing has been dealt with. (See Chapter XXIV.)

SKETCH BOOKS AND BLOCKS.

(**157**).—It frequently occurs in surveying that a good deal of detail has to be sketched; and sometimes in surveying small pieces of land, especially when laid out as ornamental grounds, and the detail has to be shown, it is better to make a good sketch of the land and put down the dimensions on it, rather than booking in the ordinary way. In such cases a sketch block of fair size should be used.

These sketch blocks are made of Whatman's drawing paper, which, together with their increased size, renders them far more suitable for this particular purpose than the ordinary field book.

DRAWING PAPERS.

(**158**).—For taking colour perfectly, and for durability, Whatman's papers are recommended   There are, however, cases in which a less expensive paper will answer the purpose.

Of Whatman's various papers, that usually used for plans is known as "Not" or "Medium." It has a finely-grained surface and takes colour perfectly.

The following tables of different papers, giving the name, surface, size, price per sheet and per quire, will probably be found particularly useful.*

DRAWING PAPERS.—HAND-MADE.

REFERENCE.—H. P. signifies hot-pressed and has a smooth surface. N. signifies not hot-pressed, and has a finely grained surface. R. signifies rough, and has a coarsely grained surface. Hot-pressed paper is mostly used for pencil drawing; not paper is used for water colour drawing, and general purposes; rough for very bold drawing and sketching.

WHATMAN'S DRAWING PAPERS.

Name.	Surface.	Size.	Per Sheet.		Per Quire.		
			s.	d.	£	s.	d.
Demy . . .	H. P. & N.	20in. by 15½in.	0	1½	0	3	0
Medium . .	H. P. & N.	22in. by 17½in.	0	2½	0	4	6
Royal . . .	H. P., N. & R.	24in. by 19½in.	0	3	0	5	9
Super Royal .	H. P. & N.	27in. by 19½in.	0	4	0	7	3
Imperial . .	H. P., N. & R.	30½in. by 22in.	0	5	0	9	9
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CHAPTER XII.

DEFINITIONS AND GENERAL PRINCIPLES.

Technical Terms—Main Purposes for which a Survey may be Required—Order in which Surveying Work should be Conducted—Key-Plan—Triangulation — General Principles of Construction—Examples of Triangulation—Field Notes—Examples.

All that need be said about the more important instruments was concluded in the last chapter.

The next subjects to which we have to address ourselves are the general principles and methods by which we should be guided. Before this, however, I shall considerably clear the ground by defining the technical terms which will be made use of, and which it would be inconvenient to stop to explain as they occur.

DEFINITIONS.

(**159**). *Land Surveying* may be defined as a branch of mathematics applied to the measurement of lands, and chiefly depends for its principles on geometry and trigonometry.

(**160**). *Chain Survey.*—A survey made with the chain without the aid of angular instruments or trigonometrical observations and calculations.

Trigonometrical Survey.—A survey made chiefly with angular instruments and depending on trigonometrical observations and calculations.

(**161**). *Combined Chain and Trigonometrical Survey.*— A survey combining the two last-mentioned.

(**162**). *Triangulation.*—That system of imaginary lines by which we divide lands into triangles, trapeziums, etc., in order to enable us by measuring such lines to plot on paper a true representation of the land in question and to prove the accuracy of our work.

(**163**). *Base Line.*—The main line or lines in the system of triangulation and upon which it depends, as *AB* in Illustration No. 86 ; also, in a trigonometrical survey,
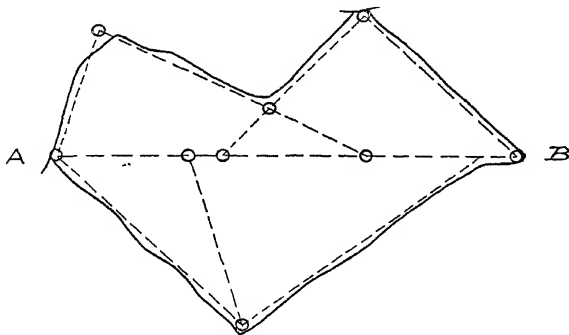

ILLUSTRATION No. 86.

the line or lines actually measured on the ground, and from which angles are read to other points, and the lengths of other lines calculated by trigonometry.

(**164**). *Chain Line.*—Any line measured with the chain.

(**165**). *Subsidiary Line.*—A line run and measured for the purpose of taking up fences, etc., and enabling us to show them accurately upon the plan, as *AB* in Illustration No. 87. This line, it will be noticed, acts as a " tie " also.

(**166**). *Tie or Proof Line.*—A line measured from an ascertained point in a survey line to an ascertained point in some other survey line, for the purpose of enabling us to test the accuracy of the work. Thus, if the line *AB* in

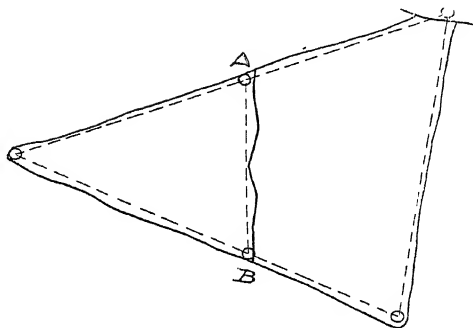Illustration No. 87 is found to scale accurately on the plan

ILLUSTRATION No. 87.

what it measures in the field, it is a proof of the accuracy
of the work.

(**167**).   *Off-sets.*— Short lines measured from ascertained
points in and at right angles to a chain line to some object
the position of which it is required to delineate on the plan,
such as *a b, c d, e f, g h, i j* and *k l*, in Illustration No. 88,
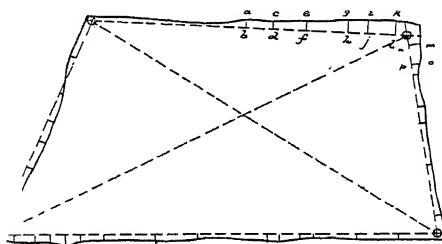

ILLUSTRATION No. 88.

which represents off-sets from a chain line to a crooked
fence, to enable the outline thereof to be correctly
delineated on the plan.

(**168**).   *In-sets.*—Similar to off-sets, but measured
from a line outside the boundary.

Stations.—Points from, to and between which lines are

measured in surveying, usually indicated thus $\odot$ on plan. See previous illustrations.

Chain Angles.—The angles formed in surveying a wood, lake etc., etc., which, owing to our inability to pass through
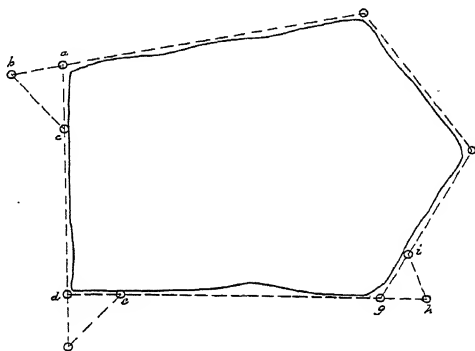

ILLUSTRATION No. 89.

it, has to be surveyed by circumscribed lines, as *a b c, d e f* and *g h i,* in Illustration No. 89, thus enabling us to ascertain and accurately plot on plan the circumscribing lines, and by the aid of insets to delineate the lake, etc.

(**169**).—*Key-Plan.*—*(a)* A small sketch plan made at the commencement of a survey, on which the system of triangulation to be employed in the survey may be fixed ; *(b)* a plan of small scale showing the relative position of a site or lands to towns, railway stations, or any known centre, to aid in its identification, generally accompanying a large scale plan of the site, lands, etc.

(**170**).   *Plan.*—An accurate representation of the land surveyed, according to measurements taken on a horizontal plane, and showing more or less detail, according to the requirements of each particular case.

(**171**).   *Rough Plan.*—A plan on which the outline of the lands surveyed and objects connected therewith are correctly plotted, and on which the system of triangulation

used in surveying is shown, but not embellished, coloured or otherwise finished.

(**172**). *Finished Plan.*—A plan which, in addition to being an accurate representation of the land, is carefully finished, embellished, coloured and printed in a suitable manner.

Note.—The technical terms applicable to levelling and their definitions are given in Chapter XXII., which deals with that branch of our subject. (See Arts. 265 to 290.)

(**173**). *Stepping.*—The method of chaining lines over hilly ground, so as to get the measurements on a horizontal plane.

(**174**). *Pacing.*—The method of roughly measuring land by training the step to a certain distance, and counting the steps taken in covering the line desired to be measured.

SURVEYS GENERALLY.

(**175**). *The Main Purposes for which a Survey may be made.*—It is now convenient to consider the main purposes for which we may be required to make a survey. They are briefly as follows :—

(1) To prepare an accurate plan of lands ;

(2) To ascertain without preparing a plan the area of lands on a horizontal plane, such as would be obtained by scaling from an ordinary map or plan ;

(3) To ascertain the actual surface area of land for computing the quantities of growing crops for valuation purposes, etc. ;

(4) To rectify boundaries, as, for instance, where fields are divided by crooked fences, etc.; and it is desired to straighten them, giving each owner his just quantity of land ;

(5) To apportion lands between different owners either in equal or unequal shares and on the basis of either quantity or money value ;

(6) To lay out plots of land of given quantity in various forms of stated relative proportions;

(7) To separate from lands of various forms given quantities of land by lines or fences to be run from fixed points on the land;

(8) To stake out land in building plots of given frontage and depth;

(9) To lay out estates, defining the course of roads, etc.;

(10) To determine the ownership of lands, fences, hedges, ditches, etc.

(11) To obtain true sections through given points in land, showing its natural slopes and irregularity of surface; and

(12) To find the difference in level between two or more given points on land.

Armed as we now are with a general knowledge of what we have to do; for what purpose we are doing it; what implements and instruments we have at our disposal, their character and purpose, and the way to use them; the technical terms which will be made use of; and the most common purposes for which we may be required to make a survey; we may at once launch out upon our study of principles, methods and rules with a considerable amount of assurance.

THE ORDER IN WHICH SURVEYING SHOULD BE CONDUCTED.

(176).—Let us suppose we have been engaged to prepare an accurate plan of land, and that the case is one for a chain survey. In such a case our first steps would be:—

(a) To visit the land and make a reconnoitre of it, carefully noting the positions of fences, hedges, etc., etc.;

(b) To make a key-plan in the first leaf of the
      field book ;

(c) To consider carefully the best system of
      triangulation for the survey and the positions
      for the lines and stations ;

(d) To sketch with care on the key-plan the
      system of triangulation decided on ;

(e) To fix the order in which the various lines
      are to be measured, and number them
      accordingly on the key-plan, as shown in
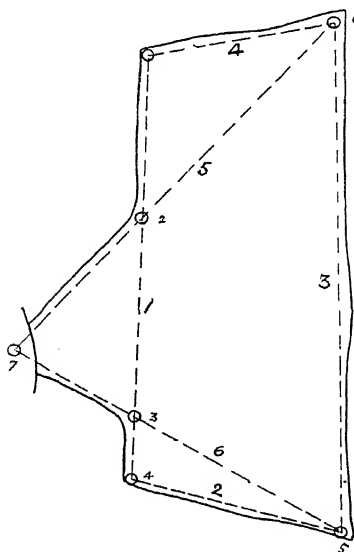   Illustration No. 90 ;

ILLUSTRATION No. 90.

(f) To note on the key-plan, in addition to
      the date of the survey, the names of adjoining
      owners ;   the names of roads, rivers, etc.;
      cultivation of the different fields comprised in

the land; the locality of the land; and any other particulars likely to be of service when finishing the plan to be prepared from the survey about to be made.

THE KEY-PLAN.

(**177**).—An excellent method for securing a sufficiently accurate key-plan is to "pace" the lines decided on, and then roughly plot the plan from the measurements thus obtained. By this means the relative proportions may be ascertained and shown with sufficient accuracy. An easier and perhaps better way, however, especially if the acreage is considerable, is to get the ordnance sheet showing the lands. Although drawn to a scale too small to aid in the preparation of an accurate plan, this ordnance sheet gives the outline of the land in true proportion, and is very useful for the purpose.

In laying down on the key-plan a good system of triangulation on which the survey is to be based, there are some general points to be kept in mind, the chief of which are briefly as follows.

TRIANGULATION.

(**178**). *General Principles of Construction.*—(1) The principles on which all surveys should be based are accuracy and simplicity; and it may be worth noting that the former is greatly helped by the latter; (2) and these principles are best secured by observing the following points:—

> (*a*) That the base line should usually, except when the constructional lines form a single triangle, be somewhere about the centre of the land,

and run in the direction of its longest dimensions, as AB in Illustration No. 91 ;

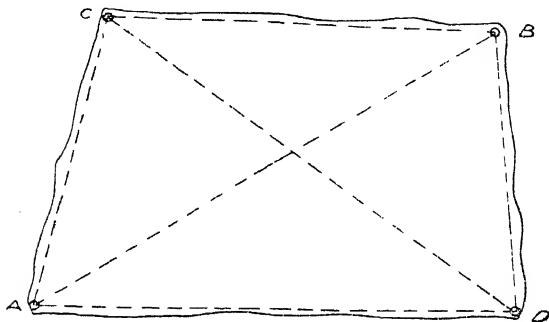ILLUSTRATION No. 91.

(b) That the figure X is a very good form for base lines to take, because in this way the work is well proved. Thus, if we plot the four-sided figure, shown in Illustration No. 91, by means of the diagonal AB, and then on scaling from the plan so plotted find that the diagonal CD
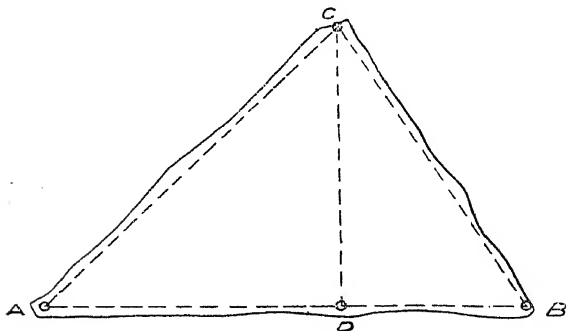

ILLUSTRATION No. 92.

scales exactly what it measured in the field, the work is proved ;

(c) That we must always have sufficient lines
   (1) To plot the figure on paper ; and
   (2) To prove the work when plotted.
  Thus, in the triangle in Illustration No. 92, if
  we measure the three lines, AB, BC and CA,
  the figure will plot on paper; and if we also
  measure CD, and find on scaling that the
  length of that line on paper is precisely what it
  measured in the field, the work is proved ;

(d) That we must also measure sufficient lines to
  enable us to show on the plan accurately the
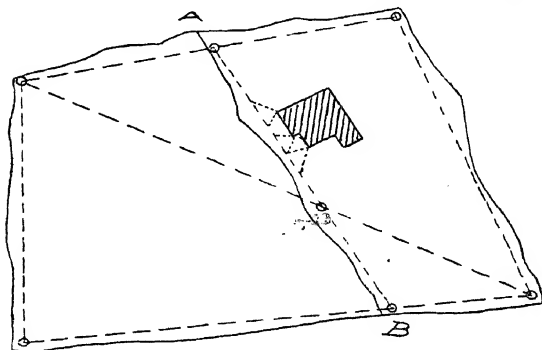
ILLUSTRATION No. 93.

  position of all fences, hedges, buildings, etc., as
  the line AB in Illustration No. 93, taking the
  fence ;

(e) That often a line may serve both as a tie line,
  and also to show the positions of fences, etc.,
  but no line which is necessary for plotting the
  main system of triangulation can be also a tie
  line. The line AB in the last illustration serves
  both to tie the triangles and also to show the
  positions of the fence on one side and the
  buildings on the other ;

(*f*) That where good tie lines can be obtained in positions in which they may also be used for showing the position of fences, etc., such should be done ; and

(*g*) That the lines should be so arranged as to keep the off-sets as short as possible.

(*h*) Very acute or obtuse angles should be avoided.

Illustrations Nos. 94 and 95 show the correct and incorrect methods of surveying the piece of land represented. Many cases will occur in practice requiring the

ILLUSTRATION No. 94.

ILLUSTRATION No. 95.

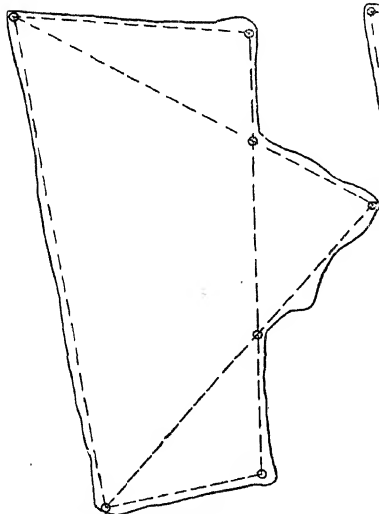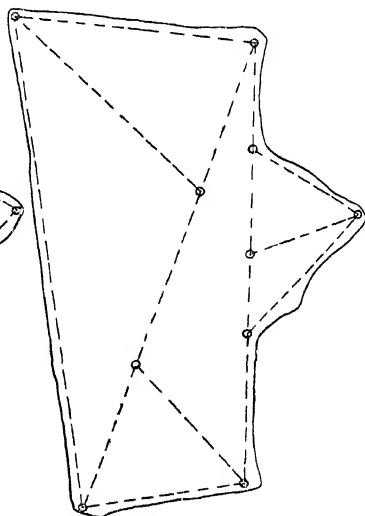exercise of considerable ingenuity. It is not necessary to point out the advantages of the correct method over the incorrect one, as they are self-evident. The fewer lines, the shorter off-sets, and the greater simplicity, the better ; clearness and accuracy, of course, not on any account being endangered.

When we have finished the key-plan, and laid down a

system of triangulation upon it, we should be in a position practically to do in the field what we have already done theoretically on paper.

EXAMPLES OF TRIANGULATION.

I shall now devote some space to a consideration of the application of these principles in particular cases.

In the case of a mere four-sided figure, such as that shown in Illustration No. 96, clearly we cannot do better
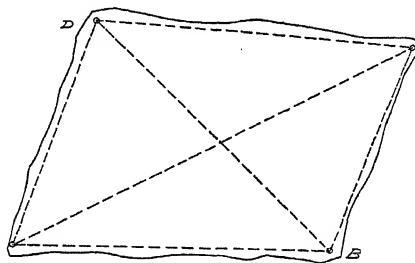
ILLUSTRATION No. 96.

than measure four lines, *AB*, *BC*, *CD*, *DA*, comprehending the boundaries and the two diagonals *AC*, *BD*, one of which will enable us to plot the figure on paper, and the other act as a proof line. It is not difficult, however, to imagine even a four-sided field in another form, which consequently would be dealt with in a different way.

Suppose we had to survey a four-sided field in the form shown in Illustration No. 97. Here again we must measure four lines comprehending the boundaries, *AB*, *BC*, *CE*, *EA* ; but the diagonals taken in the last case are not applicable, and by continuing line *AB* to *D* we have both a plottable and provable system of triangulation. The triangle *ADE* may be plotted, the line *ED* continued to *C*, and then if the line *BC* exactly

fits in according to scale, the accuracy of the work
is proved.

Whatever the form may be, we must measure sufficient
lines to comprehend the boundaries and to plot and prove
the figure. In Illustration No. 96 we should need to
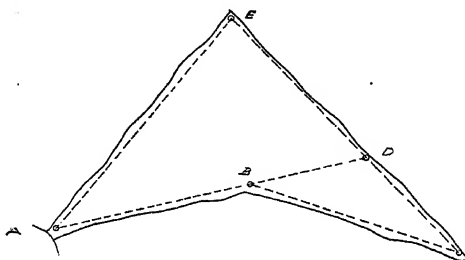
ILLUSTRATION No. 97.

measure the four lines and two diagonals; in Illustration
No. 97 we should only need to measure the four lines
and continue line *AB* to *D*.

Again, we might have a five-sided figure of the form
shown in Illustration No. 98. We should measure the
five lines *AB*, *BC*, *CD*, *DE*, *EA*, to take up the
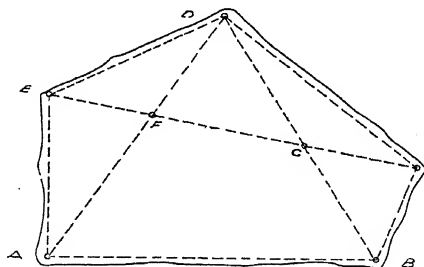
ILLUSTRATION No. 98.

boundaries; the diagonals *AD*, *BD*, to enable the figure to
be plotted, and the line *EF*, *FG*, *GC*, to prove each of the
three triangles.

But here again the five-sided figure might take many
forms, requiring to be dealt with in as many different ways.

Suppose the case of a five-sided field in the form given
in Illustration No. 99. In this case we should only
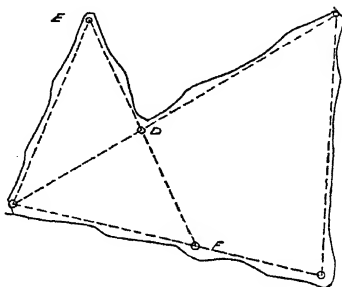
ILLUSTRATION No. 99.

need to measure the five lines *AB*, *BC*, *CD*, *DE*, *EA*,
and continue line *ED* to *F* and line *CD* to *A*, when
each of the triangles *ABC* and *AEF* is, of course,
plottable, and the line *AD* acts as a proof line.

Let us now consider a six-sided field, which, we will
assume, takes the form indicated by Illustration No. 100.
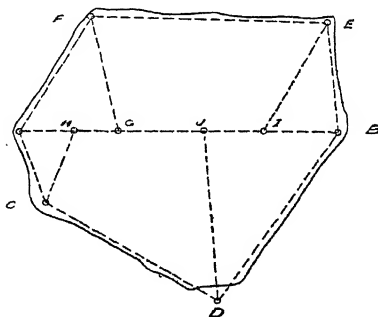
ILLUSTRATION No. 100.

Here we measure the base line *AB* and the lines following
the boundaries *AC*, *CD*, *DB*, *BE*, *EF*, *FA* ; but so far
the figure is not plottable or provable.   We therefore
measure the line *FG*, which enables the figure on that

side of the base line to be plotted, and the line EI, which
is a proof line to that figure. We also measure the
lines CH and JD, which enable the figure on the other
side of the base line to be plotted and proved in a similar
manner. Of course the field might take many forms and
each case must be dealt with on its merits.

Suppose we have a six-sided figure taking the form
shown in Illustration No. 101. Here we measure the base
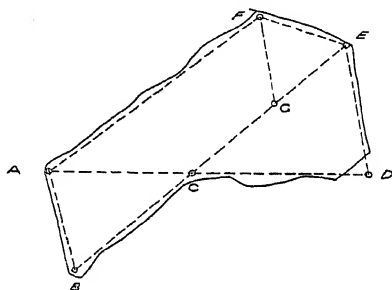
ILLUSTRATION No. 101.

line AD, and the lines taking up the boundaries AB, BC
(which is continued to E), CD, DE, EF, FA, and the line
FG, and thus have established a system of triangulation
which is both plottable and provable. Here it will be
noticed that the lines AD, BE are in the X form, which,
it has been pointed out, is a good one for lines to take.

The line AD being laid down to scale, the triangle ABC
may be plotted, and the line BC continued to E, when, if
the work has been accurately carried out, the line ED will
exactly fit in, or, in other words, the distance between E
and D will scale on plan exactly what it measured in
the field, the lines AF, FE will be plottable, and the
figure $ACEF$ proved by line FG.

We will now consider one or two figures of a somewhat
more complicated character, and see how the simple
principles already referred to may be applied to them.

Let Illustration No. 102 represent an estate of which a plan is required.   Here is a case in which there are no internal fences to influence us in the selection of the
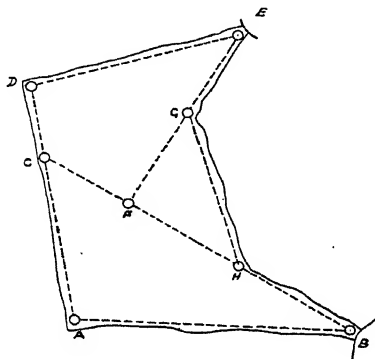
ILLUSTRATION No 102.

position for our chain lines.   By adopting the system of triangulation indicated in the sketch every line serves to take up one or other of the boundaries, and at the same time the cutting of hedges by the chain lines is avoided. The triangle ABC is, of course, plottable, and the line AC is continued to D.   Then the figure $CDEF$ is plottable from D and F at E, and if the work has all been accurately done the distance GH will scale on the plan what it measured in the field, hence the system of lines is plottable and the line GH proves the whole figure.

In Illustration No. 103 I give another example of a survey, and may offer the following remarks with regard to it.   The lines have been numbered in the order in which they have been chained, and the stations likewise, whilst arrows placed against the lines show the directions in which the chaining has been done.

Lines 1, 5 and 6 plot in the form of a triangle at station 8, and line 5 is continued to station 7.   Then lines 7 and 2 will respectively plot from station 9 in line 6 and station 2 in

line 1, in the form of a triangle at station 3 in line 2, and line 2 is continued to station 4.   Then lines 3 and 4 may be plotted from stations 4 and 7 in lines 2 and 5 at station 6, line 3, and if both the work in the field and that on paper has been accurately carried out, the distance between stations 5 and 10 will scale on the plan exactly what line 8 chained in the field, and thus that line will act as a proof line.

In connection with line No. 1 it will be noticed that the hedge breaks back at one point, so that off-sets from the chain line to the boundary would be too long,
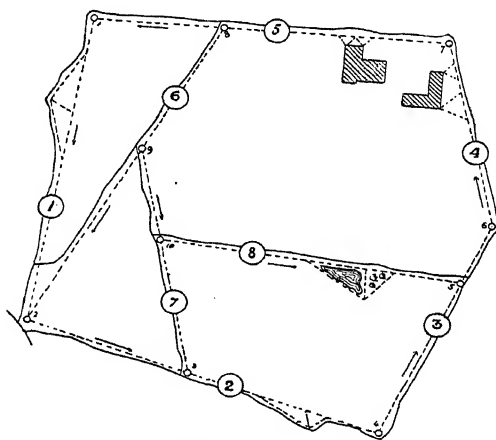
ILLUSTRATION No. 103.

and a small triangle has therefore been constructed.   A similar circumstance exists in the case of line No. 2.

In connection with lines Nos. 4 and 5, it will be observed that the positions of the buildings have been fixed from these lines, and small triangles have been constructed for that purpose.   This is undoubtedly the best means of fixing the positions of important objects.   In the case of line No. 6 it will be seen that the hedge suddenly breaks off at right angles to join the hedge taken up by line No. 1,

and the position of the angle of the hedge has likewise been fixed by a similar triangle.

Once again, on the south side of line No. 8 there is a pond, round which a triangle has been formed, and off-sets have been taken to the water's edge to enable the exact form of the pond to be plotted on the plan. (See Arts. 194 to 211 for correct method of overcoming the numerous little difficulties met with in practice, and the proper method of fixing positions of buildings, ponds, etc., etc.)

FIELD NOTES.

Having now given what seem to be sufficient examples to illustrate the application of the principles by which we should be guided in laying down a system of triangulation for a chain survey, I shall give some field notes of similar surveys, accompanied, as they would be in actual practice, by a key plan with the survey lines and stations numbered thereon.

In the first five examples of field notes which follow, the boundaries have not been sketched in with the object of securing clearness in the necessarily small illustrations, but in the last example this has been done. The plans represented by the field notes, although similar, are not intended to be identical in detail with the illustrations referred to in examples of triangulation.

FIELD NOTES.

Left column:

```
            End of line 2
                568
line 3 - - (562) 5  .   N° 3.
line 5 -    400   5
            300   10
            190   5
            130  .5
            120   10
           0|50   15
           (020)  10    N°2.
           0|00
        From ② to ③
            Line 2


            End of line 1
line 2 - - (620)  25    N°2.
line 6.     600   15
            500   10
            420   15
            310   5
            220   20
            180   12
line 5.     150   15
           (000)  10    N°1.
        From ① to ②
            Line N°1
```

Right column:
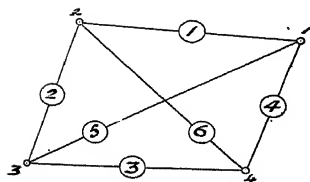
```
            End of Line 6
           (730)     N°4.
           (000)     N°2.
        From ② to ④
            Line 6.


            End of line 5.
           (970)     N° 1.
           (000)     N°3.
        From ③ to ①.
            Line 5.

            End of line 4.
line 1.     545
line 5 - - (535)     N°1.
            500   5
            300   5
line 6.     210   20
            120   30
           (030)     N°4.
           0|00
        From ④ to ①
            Line 4


            End of line 3
line 4 - - (695)  N°4
line 6.     685
            600   10
            500   5
            400   15
            300   15
line 5.     210   8
            120   10
           (005)     N°3.
           0|00
        From ③ to ④
            Line 3.
```

End of line 2
635
(625) No4
600
500
400
300
200
125
(020) No3.
000
line;
line;

From ③ to ④.
Line 2

End of line 1.

Line 2 -- (718) No3
600   5
500   5
430   15
320   5
No2 (290) 5
200   15
100   10
No1. (000)
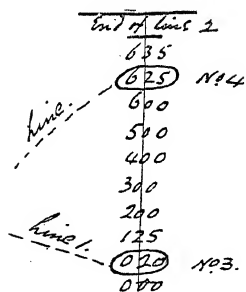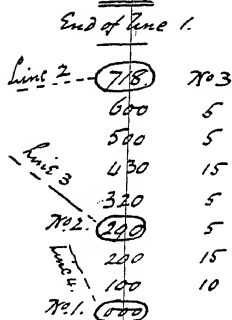
From ① to ③.
Line 1

End of line 4.
(475) No1.
400   5
300   30
200   5
100   30
No5. (070) 10
000   30
line 1;
line 3
line 2

From ⑤ to ①.
Line 4.

End of line 3
675
line (670) No1 line 4.
(425) No5.
400   5
300   10
100   5
No4. (030) 30
000
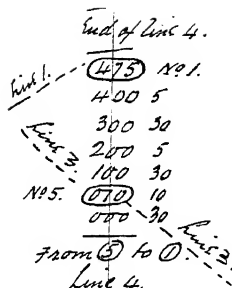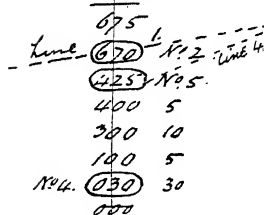
From ④ to ②.
Line 3
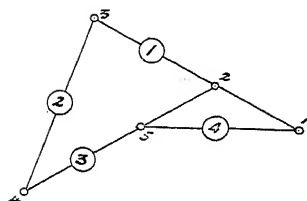
End of line 4. Line 5.
400

No.1..   15   (320)   Line 1.
     3   300
    10   200
     5   100   Line 6.
No.6.   30   (010)   Line 3.
        000

From ⑥ to ①
Line 4.

End of line 3. Line 4.
725

No.6.   (725)   Line 6.
    600
    400
    200
    100   Line 7.
No.5.   (110)   Line 2.
    000

From ⑤ to ⑥.
Line 3

End of line 2.

   300   Line 3
No.5.   (255)   Line 1.
   200
No.4. 10   (025)   Line 8.
    000

From ④ to ③
line 2

End of line 1.

   835
No.4.   (815)   No. 3.
    (570)   No.2.
    (260)
No.1.   (020)
    000

From ① to ④.
line 1.
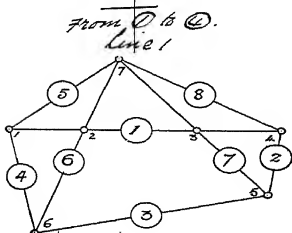
End of line 8.

   555
No.4.   (545)
   15   500
   15   400
    5   300
    5   100   Line 6.
No.7.   30   (005)   Line 5.
      000

From ⑦ to ④
Line 8.

End of line 7.
   680
$-\frac{4}{3}$ (670)   No.7.
$-\frac{8}{}$ (330)   No.3
$-\frac{7}{3}$ (025)   No.5.
    000

From ⑧ to ⑦
line 7.

End of line 6.

   700
$-\frac{4}{3}$ (690)   No.6.
$-\frac{4}{}$ (300)   No.2.
$-\frac{8}{}$ (020) 5   No.7.
    000

From ⑦ to ⑥.
line 6

End of line 5.

   5   438
No.7.   10   (425)   Line 8.
     5   400   Line 6.
    30   300
    10   200
     5   100
No.1.   5.   (010)   Line 4.
     000

From ① to ⑦
Line 5.

End of line 2.

5	630
	(613) — Line 3
5	625
5	250
30	200
5	100
10	(010)
	000

From ③ to ④.
Line 2.

End of line 1

5.	(825) Line 2
30.	700
10	650
15	500
10	400
0	350
	Line 3 Line 3
	(320) 5 — Line 3
	(000)

Line 1

End of line 5.

Line 3   610

No 5.	(580)	Line 1
No 2.	(240)	
5.	250	
20.	100	
5.	(015)	No 6
5.	000	

From ⑥ to ③.
Line 5.

End of line 4.

495

No 6.	15	(475) Line 1
No 1.	5	(025) Line 3
		000 Line 5

From ① to ⑥
Line 4.

End of line 3.

715

20	(710)	No 1. Line 4
5	600	Line 6
20	500	
5	450	Line 6
	(325)	No 5 Line 2
30	200	
25	150	
5	100	
30	050	
	(040)	No 4
	000	

From ④ to ①
Line 3

EXAMPLE No. 5.

End of Line IV 7. Line 8.
No 9. 15 (203)
        10 200   Line 11.
        10 100   Line 1.
No 1. 15 (000)
              Line 4.
From ① to ⑨
Line 7.

End of Tie line 6.
        (390) No 4.
        (080) No 7.
From ⑦ to ④
Line 6. (Tie)

End of Tie line 5
        (125) No 2
        (000) No 8
Line 5. (Tie)

End of line 4. Line 7.
No 1. 10 (125)
        10 100   Line 1.
         5 050   Line 5.
No 8. 30 (000)
              Line 3.
From ⑧ to ①
Line 4.

End of line 3.   Line 4.
No 8. 35 (460)
        35 425   Line 5.
        20 400
        25 350
         5 300
        15 240   Line 6.
        20 120
No 7.   (000)   Line 2.
From ⑦ to ⑧
Line 3.

End of line 2.
        (465) No 7.
                    Line 3
        30 420
        20 400   Line 6.
        20 300
        10 100
         5 050   Line 1.
        20. (000) No 6
From ⑥ to ⑦   Line 9
Line 2.

End of line 1.
        (685)
        (525)
        (395)
        (215)
        (125)
        (000)
From ① to ⑥
Line 1

End of line 11.
No 9. Line 7. (270) Line 8
No 3. Line 1. (000) Line 1.
From ③ to ⑨ (Tie)
Line 11.

End of line 10
Line 8. (285) Line 2. Line 1.
          (000)
From ⑤ to ⑩ (Tie)
Line 10.

End of line 9
No 6. 25 (250) Line 1.
        10 200
        10 050   Line 10.
No 10. 15 (000) Line 8
From ⑩ to ⑥
Line 9.

End of line 8. Line 9.
No 10. 15. (505) Line 10.
         5 300
        15 200
        10 100   Line 11
No 9. 15 (000) Line 7.
From ⑨ to ⑩
Line 8.

EXAMPLE No. 6.

End of line 1.
Line 2   330
N.º 2   720
Line 6.   650   12
fence   580—10
400   15
330·10
16   290
31   240
225
25   215
30   100
090
Line 5.   N.º 1   000   5
From ① to ②
Line 1.

Line 4
End of line 3.
550   2
545   3   N.º 6.
500   5
420   10
300   15
Line 8   290   10   N.º 5.
200   5
100   10
10   050   15
Line 2   010   10   N.º 4.
000   3
From ④ to ⑥
Line 3

End of line 2.
Line 3.   810
800   5
750   3
35   700   5
600
60.
650   35
20   610
500   10
20   400   5
Line 7.   371
300   5
200   10
150   15
Line 6.   100   12
Line 1.   050   9
000   5
From ② to ④
Line 2

End of line 6.

Line 2 :.. (700)   No 2.
     6 50   10
Fence   500   3
     400   8
     340   10
Line 7   335 . 12
No 9.   (330)
     200   5
     100   5   Line 5.
No 8. (000) 10

From ⑧ to ②.
Line 6.

End of line 5.
     759
Line 6 - ½ (748) 2. No 1.
     (475) 3. No 8.
   80   30 - - 230   5
     25 - 200
  80   30
     30 - - 160   5
   80 30
     Ch 3. 100   10
     050   5
Line 4 : (000)   No 7.

From ⑦ to ①.
Line 5

End of line 4.
Line 5   430   3
     (425)   7   No 7.
  20   35 - 400   10
  60   48 - 350   18
     25 - 320   20
  50   60 - 250   15
Line 3 (000)   12   No 6.
From ⑥ to ⑦
Line 4

End of line 8.
No 5.   25 (638)   Line 3.
Ch 3   40   550
Ch 1   20   450
     40   250
     15   150
Ch 3   30   100
Ch 2   25   050
Line 7 : (000)   Line 7   No 10.

From ⑩ to ⑧.
Line 8.

End of line 7
Line 2
No 3. (525)   Line 2   Hedge.
Ch 2   25   420   15
Ch 2   30   350   20
Line 8.   300   10
Hedge.   (215)   Hedge
     200   No 10.
     150   15
     100   10
     050   5
Line 6 : (000)   Line 6   No 9.
From ⑨ to ③.
Line 7

CHAPTER XIII.

THE MEASUREMENT OF LAND.

LAND MEASUREMENT.

(**179**).—The common method of measuring land is by chaining. The chain has already been described in Chapter III. (see Arts. 21 to 25), under the head of " Instruments." The present chapter will deal with the method of chaining in various circumstances.

CHAINING.

Two men are required to use the chain. One drags it from the commencement to the termination of the line ; he is called the leader. The other takes the end of the chain and follows it from point to point throughout the line ; he is called the follower. Their duties are as follows :—

DUTIES OF THE LEADER.

(**180**).—The leader's duties are—

1. To drag the chain in a straight line from station to station.
2. To watch for the directions of the follower as to when and where to put the arrow in.
3. To see that he receives ten arrows on commencing to chain and to put one arrow in at the end of each chain.
4. To put the arrow in the ground in a direct line with the chain outside the brass handle and perfectly plumb.
5. To see that the chain is pulled tight and lies in a perfectly straight line each time before he puts the arrow in.
6. To pull the chain forward one chain length after he has deposited his tenth arrow and then to ask for the ten arrows before continuing to chain.

DUTIES OF THE FOLLOWER.

(**181**).—The follower's duties are—

1. To hold the handle of the chain against the point from which the measurement is being taken, and by waving the hand right or left to assist the leader to place the chain in a direct line between the points from and to which the measurement is to be taken.
2. To pick up each arrow as he comes to it.
3. Finally, when he has picked up the whole of the ten arrows to count them over to the surveyor and hand them back to the leader.

DUTIES OF THE SURVEYOR.

(**182**).—The surveyor's duties are—

1. To see that leader and follower perform their work accurately.
2. To note in the field book lengths of lines measured.
3. To enter the 10 chains in the book each time the leader receives the ten arrows from the follower.
4. To take off-sets from the chain line to hedges, fences and other objects, the positions of which are required to be fixed in that way.
5. Other duties, not necessary to refer to here whilst considering only the subject of how to use the chain.

THE CHAIN.

(**183**). *To Throw the Chain.*—When the chain is required for use, the first thing is to open it, and this is usually done by what is known as " throwing."

To " throw " the chain, remove the strap, take hold of both handles in the left hand, holding the chain in the right hand, shake out free three or four links from the chain and throw the chain forward, at the same moment stepping back a few paces. By this process the chain is stretched out on the ground in a straight line, but folded in half. Then, taking one handle only, walk forward until the chain is lying full length on the ground. If the chain was perfectly folded and is thrown properly, it will open out without kinking.

(**184**). *To Examine the Chain.*—The next step is to examine the chain. Walk along the chain as it lies on the ground, and observe carefully that it is in order and that none of the links are kinked or bent.

(**185**). *To Test the Chain.*—Then, with an assistant take the chain to the " Standard " and test its accuracy, or compare it with the " Standard Chain," where one is employed.

(**186**). *To Fold the Chain.*—After use the chain should be cleaned, when necessary, and carefully folded. As it lies full length on the ground take hold of the " teller " at the 50th link, and w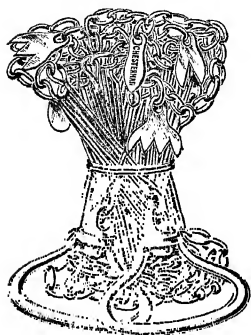alk away until the two handles have come together, as they will naturally do in the process of dragging.   Then take hold of the two links in the centre of the chain in the left hand and, missing the next four (two pair), take the following couple in the right hand and fold the four free links and the two held in the right hand down on to the two held in the left hand, and repeat this process until the folding of the chain is completed.   If the links held in the left hand are caused to revolve as each six links are added, the first two links will form a sort of centre or core, round which the others will rest, crossing each other, as shown in Illustration No. 104.

ILLUSTRATION No. 104.

Finally, place the leather strap round the chain and buckle it tightly.

CHAINING.

(**187**). *Cases.* — The following cases may occur in chaining :—

(1) Where the measurement on a horizontal plane is required and the ground is likewise horizontal, or nearly so ;

    (2) Where the measurement on a horizontal plane is required, but the surface of the land slopes considerably;

    (3) When the area required is that on a plane parallel with the sloping surface of hilly ground;

    (4) When the actual area covered by growing crops on land is required.

TO CHAIN A LINE ON LEVEL GROUND.

(**188**). *Case No. 1.*—The points between which the line is to be run and the distance measured having been definitely established, preferably by driving in " pickets " or " ranging poles," the chain is dragged to the starting point, and the outside of the brass handle is held against the picket by the follower. The leader takes the other end of the chain and one arrow in one hand, and nine arrows in the other (ten arrows in all), and standing so as not to obstruct the view between the pickets, looks back to the follower for direction.

The follower, with his eye behind the picket at the commencement of the line looks away to the picket at the point where it terminates, or, in long lines, some intermediate station in the same line, and by waving his free hand, right or left, signals to the leader who moves his hand accordingly, and when the follower sees that the chain lies perfectly in line between the stations or pickets he signals with a downward motion of the hand to the leader, who then presses the arrow into the ground.

The leader now takes the chain handle, passes an arrow to the chain hand, and drags the chain forward in a direct line towards the station, to which the measurement is to be taken. The follower follows the chain until he reaches the arrow, calls a halt, holds the chain handle to the arrow, as he did in the first

instance to the picket, and again directs the leader, this time sighting from the arrow to the picket in the distance. This process is repeated until the line is completed.

When the tenth arrow has been put down by the leader, he drags the chain forward another chain length and marks the ground where the arrow should be inserted. The follower then coming up draws the tenth arrow, counts them to the surveyor, and hands them to the leader, who also counts them and proceeds as before.

Sometimes an eleventh and distinguished arrow is used to temporarily mark the point when the chain is dragged forward after the insertion of the tenth arrow, and is withdrawn and replaced by an ordinary arrow after the leader has again received the ten. Provided the eleventh arrow is distinguished by a specially coloured piece of cloth, so that no mistake should occur, this is a good plan.

TO CHAIN A LINE OVER A HILL. STEPPING.

(**189**). *Case No. 2.*—In this instance there are three methods of chaining. The first is that known as " stepping." In this case, the horizontal measurement being required, and the ground being on the slope, one end of the chain is held up so that the chain is horizontal ; but the procedure will differ somewhat, according to whether the line is being run up or down hill.

TO CHAIN A LINE UP A HILL.

The follower holds a plummet in his hand with the chain handle, and suspends it exactly over the spot from which the measurement is to be taken, keeping the hand sufficiently high to bring the chain horizontal.

TO CHAIN A LINE DOWN A HILL

The follower holds the end of the chain on the ground in the usual way; but the leader holds his end of the chain and the drop arrow in the same hand sufficiently high to bring the chain horizontal. The arrow is then dropped, and the point to which the horizontal line would reach is thus marked.

Note.—In stepping, if the chain cannot be held horizontal in full chain lengths, it must be done in half-chain or shorter lengths. A spirit level may be used to ascertain if the chain is horizontal, but it is more often left to the judgment of the practised eye. The method of chaining both up and down hill is otherwise similar to that described for level ground.

REDUCTION OF HYPOTENSUAL TO HORIZONTAL MEASUREMENT.

(**190**).—The second method of using the chain under Case No. 2 involves the reduction of hypotensual to horizontal measurement. In this case the chaining on hilly land is precisely similar to that on level land. The method of reducing the hypotensual to horizontal measurement forms no part of chaining, but will be dealt with in its proper place, under the heading of " Correction of Inclines." (See Chapter XXVII, Art. 367.)

SETTING BACK THE ARROW.

(**191**).—The third method of using the chain under Case No. 2 is by what is called "setting back the arrow." Where the gradient of the land is not great, the chaining is performed as if the ground were level; but the arrow is put back a link or so, a distance sufficient to allow for the difference between the length of the line measured on the land and the horizontal distance. It will be seen

that this really has the same effect as "stepping" in the case of lands of greater slope, and the distance to which the arrow is set back must depend on the slope of the land.

SURFACE MEASUREMENTS.

(**192**). *Case No. 3.*—In this case the method of chaining is the same as that already described for level ground.

GROWING CROPS.

(**193**). *Case No. 4.*—In this case the method of chaining is similar to that described for level ground, with this difference that the chain is used slack, instead of tight, so that the actual surface covered by the crops may be ascertained, and the irregularities of the surface of the land may influence the lengths of lines and the superficies calculated therefrom.

OBSTRUCTIONS AND DIFFICULTIES.

TO RANGE A LINE BETWEEN TWO STATIONS THE VIEW BETWEEN WHICH IS OBSTRUCTED BY A HILL.

(**194**).—One of the difficulties often met with in practice is that a line has to be ranged over hilly ground between two stations whose positions are fixed.

Let us suppose that we have been compelled to lay out our base line, *A B* in Illustration No. 105, in such a position that a hill obstructs the view from one station to the other. In order to range out a straight line between the points, it will be necessary to establish two intermediate stations near the top of the hill, each in a line with the stations at the extremities of the line and observable from both of them. It is obvious that for any two stations to be in a line with a third, all three stations

must be in one straight line, and that for any two stations to be in a line with two other stations, one on either side, all the stations must, of course, be in the same straight line : so that what we have to do is to establish two stations near the top of the hill, each in the straight line between the stations on the opposite sides of it. This is done by the following method :—

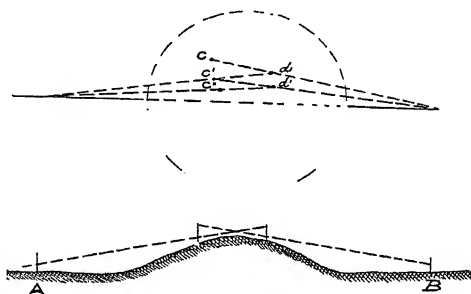Let A and B represent two points between which it is desired to range a line, and the dotted circle in diagram an

ILLUSTRATION No. 105.

intervening hill. Two men, C and D, ascend the hill on opposite sides, C sufficiently high to obtain a view of station B, and D sufficiently high to see station A. Each man carries a picket. C plants his picket as near as he can judge between A and B, say at c, and signals to D with the hand to plant his picket between c and B at d ; D does so, and then looks from his picket and signals to C to move his picket from c to bring it in line between d and A at c'. C moves his picket accordingly, and then looks from his picket c' to B and directs D to move his picket from d to d' in the line c' B. D moves his picket as directed and looks from d' to A, and directs C to move his picket from c' to c'', between d' and A, and this process is repeated until C and D have so placed their pickets that they are each in the line A B.

It will be observed from Illustration No. 105 that each
step brings them nearer into the line, and the process
being continued the desired object is attained.   These
points at the top of the hill in the line between *A* and *B*
having been found and the pickets being placed there,
the line may be chained from either *A* or *B* to them,
and thence continued to the other side.

The number of times the positions of the pickets may
need to be altered before the points in the line are found
will, of course, depend upon the skill of the operators,
but the work may be performed with perfect readiness
after very little practice.   (See also Art. 96.)

TO CHAIN A LINE OBSTRUCTED BY A BUILDING.

(195).—Another difficulty often met with is a building
obstructing a chain line.

In laying down a system of triangulation, as far as
possible all obstructions are avoided, but sometimes a line
has to be run where it will be obstructed by a building.
Let us assume such a case, and see how the difficulty is to
be overcome.

Let *A B*, Illustration No. 106, represent a line which
is being chained from *A* towards *B,* and is obstructed
by a building.   At *a* and *b* on the chain line, about two

ILLUSTRATION No. 106.

chains apart, erect perpendiculars, using the box sextant or
optical square for the purpose, or it may be done with the
chain only.   (See Art. 196.)

Measure off exactly equal lengths on these perpendiculars,
making them sufficiently long to clear the building, and

drive a line from *c* through *d* for a distance of about two chains past the building. When the obstruction has been cleared, drop the perpendiculars, *e g*, *f h*, each exactly the same length as *a b*, *c d*, and the points *g* and *h* will be in the line *A B*. A picket can now be put in at *B* in the line *h g*, and the chaining of the line *A B* may be continued by chaining from *g* through *h*, the distance *d e* being added to the line measured to *b* and continued from *g*. (For Box Sextant see Arts. 105 to 111.)

Note.—When the leader gets past *d* he will be guided .as to the position in which to place the arrows by turning round and looking towards *d* and *c*, instead of being directed by the follower.

TO LAY OUT A RIGHT ANGLE OR ERECT A PERPENDICULAR IN THE FIELD WITH THE CHAIN ONLY.

(**196**).—Let *A B*, Illustration No. 107, be the line on
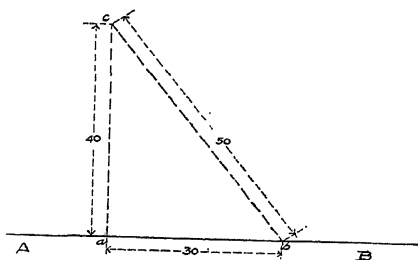

ILLUSTRATION No. 107.

which the perpendicular is required to be erected at *a*. Measure on *A B*, from *a* to *b*, 30 links, and put in an arrow at *b*. Pin one end of the chain at *a* and the 90th link at *b*; pull out the 50th link to *c* until each arm of the chain is equally tight, and put in an arrow at *c*, and the line *a c* will be perpendicular to the line *A B*.

TO CHAIN A LINE OBSTRUCTED BY A POND, AND TO
OBTAIN ITS POSITION AND OUTLINE.

(**197**).—It frequently occurs that a chain line is found to be obstructed by a pond, and, likewise, it is often desired to show an existing pond on the plan.   We will now, therefore, consider how these circumstances are to be met :—

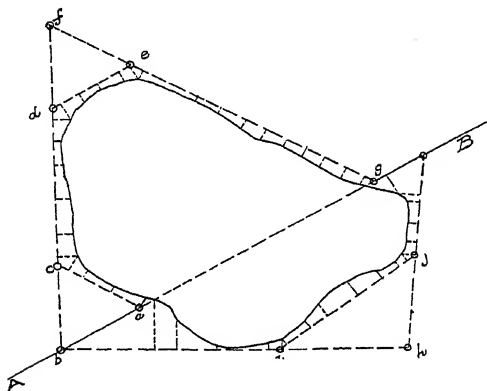Let *A B*, Illustration No. 108, represent a chain line

ILLUSTRATION No. 108.

obstructed by a pond.   In practice it will probably be required—

>   (1) To continue the chain line ;
>   (2) To obtain the distance *b g* to add to the chain line measured to *b* and continued from *g* ;
>   (3) To get sufficient measurements to show the pond accurately on the plan.

The method employed is as follows :—

On arriving near the edge of the pond put in an arrow at *a*, and also one at *b*, the distance *a b* on the chain line being carefully measured 50 links.   Fix the ends of the chain at *a* and *b*, stretch the 50th link to *c* and put in an arrow there, drive a line from *b* through *c* to *f*, sufficiently far to clear the pond, and measure

the line *b f*. Measure back from *f* to *d* 50 links, fix the ends of the chain at *d* and *f*, and pull out the 50th link to *e*. Run the line from *f* through *e* to *g*, measuring *f g* exactly equal to *b f*, which done, *g* will be in the line *A B* and the distance *b f* will equal *b g*. Thus the distance *b g* has been obtained, and the chaining of the line may be continued from *g*.

It will be observed that the line *f g* will form a valuable proof line, as if the work has not been accurately performed a picket put in at *g* will not be in the line *A B*, which may be easily ascertained by sighting across the pond and observing whether *g b A* are in a line.

Another triangle may be formed round the other side of the pond and off-sets may be taken from the sides of the triangles to the edge of the water, to enable the outline of the pond to be shown on plan. This second triangle need not be equilateral, as the distance *b g* has already been ascertained. Its sides should be run near the water's edge, so as to keep the off-sets short; a proof line, *i j* in illustration, should always be taken.

It may be useful here to say a few words as to how the measurements are booked in such cases, although the subject of booking has been dealt with in Chapter XIV.

In the case of ponds or small lakes a sketch would be made in the field book, and the measurements entered, as shown in Illustration No. 109. In other cases the lines would be numbered with the survey lines, and entered in the field book accordingly. The case represented by Illustration No. 109 is that of a pond not obstructing, but lying close to a chain line, which is represented by the column on the right-hand side of the sketch. The figures in that column represent the distances on the chain line from which the lines forming the triangle

round the pond have been run, and the off-sets from
the chain line measured; whilst the lengths of the
lines forming the triangle, the distances on those lines

ILLUSTRATION No. 109.

at which off-sets have been taken, the lengths of the
off-sets, and the tie or proof line, are all given in the
sketch, and may be accurately plotted therefrom.

TO CONTINUE A CHAIN LINE ACROSS A RIVER AND TO
OBTAIN THE WIDTH THEREOF—METHOD NO. I.

(**198**).—Let *A B*, Illustration No. 110, be the chain
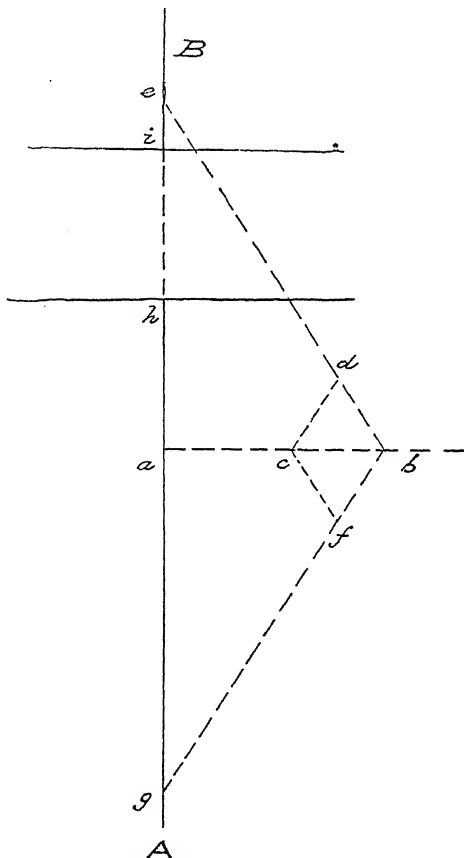line obstructed by a river. On arriving at about one

ILLUSTRATION No. 110.

chain from the bank erect a perpendicular *a b*. On
this perpendicular, a sufficient distance from *a*, measure

b c, 50 links accurately.   Pin the ends of the chain
at *b* and *c,* and pull out the 50th link to *d,* thus
forming the equilateral triangle *c b d.*   Put in pickets
at ·*b* and *d,* and sight from *b* through *d,* and direct a
man to put in a picket on *A B* at *e* on the other side
of the river.   Now turn the 50th link of the chain at
d over to *f,* and sight from *b* through *f,* and have a
pole put in on the line *A B* at *g.*   The distance *a g*
will equal the distance *a e,* which distance must be
added to the line chained to *a* and continued from *e.*

If the width of the river is required it may be found by
deducting the distance *a h* + *i e* from the distance *a e.*

TO CONTINUE A CHAIN LINE ACROSS A RIVER AND TO OBTAIN THE WIDTH THEREOF—METHOD NO. II.

(199).—Let *A B,* Illustration No. 111, be the chain line

ILLUSTRATION No. 111.

obstructed by the
river.   Near the
bank, at *a,* erect
a perpendicular one
chain long on the
line *A B,* putting
in an arrow exactly
at the 50th link,
bisecting the line
at *c.*   At *b* erect the
perpendicular *b d,*
put in a picket at *d*
and move another
picket up and down
on the line *b d* until
the point *e* is found,
where it is in a
straight line with

some object *f* in the line *A B* sighted on the edge of the

bank and a picket at c.   The distance $b\ e$ is equal to the distance $a\ f$ and $b\ e$ minus $a\ g$ will give the width of the river, and the chaining of the line may be continued from f if necessary.

It will be observed that this is a far better method than method No. I. for cases where it is only desired to obtain the width of the river, inasmuch as in this case it will not be necessary for anyone to cross the river, as an object on the bank, in the line $A\ B$, can generally be sighted.

TO CONTINUE A CHAIN LINE OBSTRUCTED BY A RIVER

AND TO OBTAIN THE WIDTH OF THE RIVER—

METHOD NO. III.

(**200**).—Let $A\ B$, Illustration No. 112, be the chain line obstructed by the river, erect the perpendicular $a\ b$ on the chain line, equal 50 links, and put in a picket at b.   At c erect another perpendicular, $c\ d$, equal to $a\ b$, and 50 links distant from it, and on this perpendicular, continued indefinitely, put in a picket at e, where it will be in a line with b and some object h, sighted in the line $A\ B$ on the opposite side of the river.   In this way, it will be observed, two similar triangles $e\ d\ b$ and $b\ a\ h$ are established, and having, therefore, two dimensions of one of those triangles and one corresponding dimension of the other, the remaining corresponding dimension may be easily found by simple proportion thus :—

$$e\ d\ :\ d\ b\ :\ :\ b\ a\ :\ a\ h$$

or assuming $e\ d$ to measure in the field 25 links,

$$25\ :\ 50\ :\ :\ 50\ :\ d\ f$$

$$\text{or} \quad \frac{50\ \times\ 50}{25}\ =\ 100$$

It will be observed that the perpendiculars being made equal to 50 links and erected on the chain line 50 links

apart, the very simple rule—divide 2,500 by $e\,d$ to obtain
the distance $a\,h$— is obtained.

If the width of the river is required, $a\,g$ must, of course,
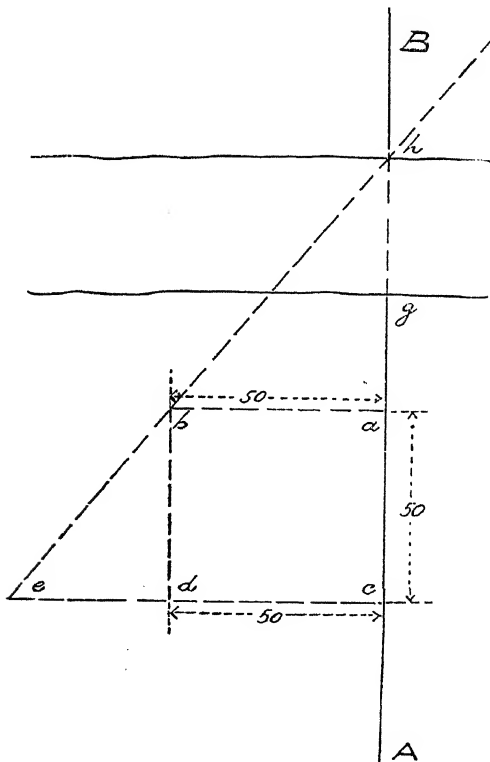

ILLUSTRATION No. 112.

be deducted from the distance $a\,h$.   The chaining of the
line $A\,B$ may be continued from h.

TO CONTINUE A CHAIN LINE OBSTRUCTED BY A RIVER AND
TO OBTAIN THE WIDTH OF THE RIVER—METHOD NO. IV.

(**201**).—Applicable to cases where the chain line
crosses the river obliquely.

Let *A B*, Illustration No. 113, indicate the direction of a chain line crossing the river in an oblique direction. Lay
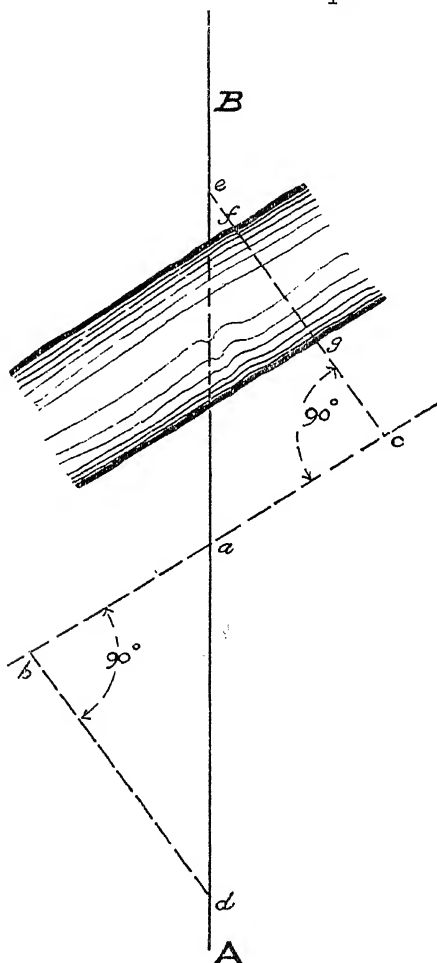
ILLUSTRATION No. 113.

out a line parallel with the river's edge, and measure off on it equal distances on either side of the chain line, say

50 links, from *a* to *b* and *a* to *c*. At *c* and *b* erect the perpendiculars *c g* and *b d*, when *c e* will equal *b d*, which may be measured on the ground, and *b d* minus *c g* + *e f* will give the width of the river.

The chain line may be continued from *e*, the distance *a d* which is equal to *a e*, being added to the line chained to *a* and continued from *e*.

TO CONTINUE A CHAIN LINE OBSTRUCTED BY A RIVER AND TO OBTAIN THE WIDTH OF THE RIVER— METHOD NO. V.

(**202**).—On the obstructed chain line *A B*, Illustration No. 114, near the water's edge, erect the perpendicular

ILLUSTRATION No. 114.

C D, exactly one chain long and read the angle *CDE* with the theodolite.

Then calculate the distance $C\ E$ by trigonometry, which distance must be added to the line chained to a and continued from d.   The distance $a\ b\ +\ c\ d$ must be deducted from the distance $a\ d$, to obtain the width of the river.   (For " Solution of Triangles " see Chapters XXVI. and XXVII.)

TO SURVEY A WOOD, PLANTATION, LAKE, ETC., WITH THE CHAIN—CHAIN ANGLES.

(**203**).—Another difficulty very commonly met with in practice is a wood, plantation, etc., through which lines
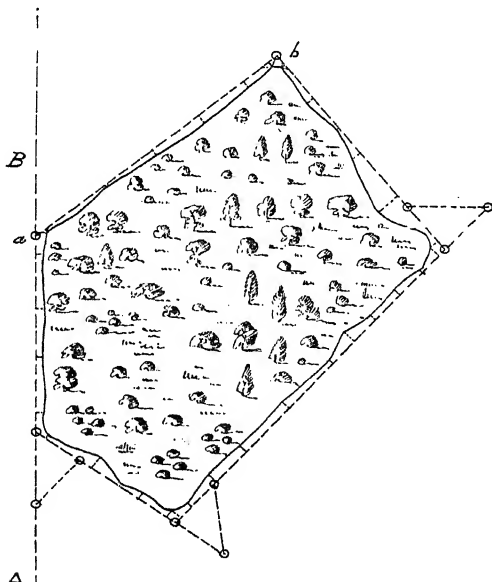
ILLUSTRATION No. 115.

cannot be run, but the boundaries of which must, of course, be accurately shown on the plan.

It may also occur sometimes in cases of litigation that an accurate plan of land has to be prepared, admission

to which cannot be obtained, and the survey has therefore to be conducted from the outside. A survey of this sort is usually best conducted by traversing (see Chapter XXIV.), but it may be made with the chain. The method of procedure is as follows:—

Let *A B*, Illustration No. 115, be a line in the general system of triangulation of a survey of an estate of which the wood forms part, or we may suppose that the wood has to be surveyed apart from other land. Here, as will be seen from the illustration, lines are run round all the boundaries, the direction of the lines being fixed by triangles, except in the case of the last line which, if the work has been perfectly carried out, will, when the plan is plotted on paper, fit accurately according to scale between the stations *a b*, or in other words, the distance *a b* will scale on the plan what it actually measured in the field.

Insets are measured in the usual way, at right angles to the chain lines, to every break or undulation in the boundary.

The theodolite may be set up and each of the angles read as a further guarantee of accuracy; and, remembering that the figure must contain twice as many right angles as the figure has sides, minus four right angles, the accuracy of the theodolite work may in turn be checked by adding to the sum of the angles read four right angles and dividing the result by 90 and again by 2, when the result should equal the number of sides in the figure, and will do so, if the readings have been accurately taken.

TO SURVEY A RIVER, CANAL, ETC., WITH THE CHAIN— CASE I.

(**204**).—A stream or river is often found running through the land to be surveyed, which must, of course, be accurately shown on the plan; or we may have to prepare a plan of a river or canal apart from the surrounding land.

We will first take the case of a stream running through the corner of an estate. Let *A B* and *A C*, Illustration No. 116, be lines in the general system of triangulation of a
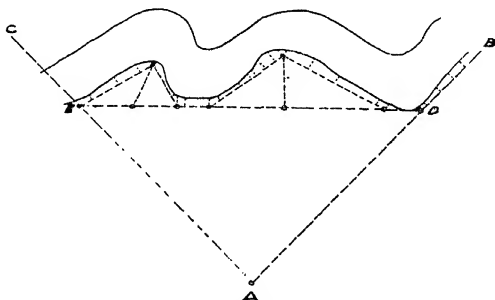

ILLUSTRATION No. 116.

survey, the relative positions of which are fixed by lines not shown. The river passes through the land to be surveyed, and must therefore be accurately shown on the plan.

We run the line *D E* on which we construct the two triangles shown, each of which is properly tied, and from the sides of which off-sets are taken to the water's edge, thus enabling the stream to be accurately plotted.

TO SURVEY A RIVER, CANAL, ETC., WITH THE CHAIN—
CASE II.

(**205**).—Let us now consider the case of a river or canal in which the distance would be too great to admit of being dealt with by the method last referred to.

Let Illustration No. 117 represent the river. From the

ILLUSTRATION No. 117.

point *A*, we run a line *A B* close to the river's edge, leaving a picket at *a*, and another line *B C*, leaving a picket at *b*.

We also measure the tie line *a b*, which enables the relative positions of the lines *A B, B C* to be fixed. As each of the lines are chained, off-sets are taken to the bank so that an accurate plan of the river may be plotted.

The remaining lines are run in a similar manner, and the process is repeated until the survey is completed.

The same is done on the other side of the river to obtain the opposite boundary.

The theodolite may be used to obtain the angles formed by the various lines to check the accuracy of the work. This precaution will be absolutely necessary unless other means of proof are available ; as, for instance, the lines run to survey the river starting from and closing on other properly proved lines.

When the survey is to obtain a plan of a part of a river or canal only, without reference to the surrounding land, and where therefore the lines run to take the undulations of the river are not connected with other survey lines, the distance between and relative positions of the starting and finishing points of the lines on either side of the water must be obtained. This may be done as shown in Illustration No. 118, by putting in stations and forming and measuring the triangles *A B C* and *D E F*. The
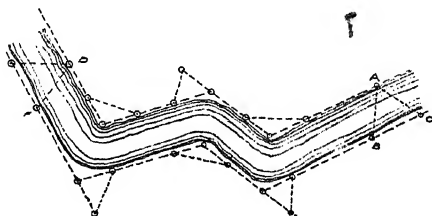
ILLUSTRATION No. 118.

measurements of *A B, A C, D E, D F* may be obtained by methods already given. It would be far preferable, however, in such cases to employ the theodolite.

TO SURVEY A WINDING ROAD WITH THE CHAIN.

(**206**).—The process is similar to that employed in surveying a river already described, with the difference that in this case, as we are able to pass along the road, only one line down the centre, on either side of which off-sets may be taken, is necessary.

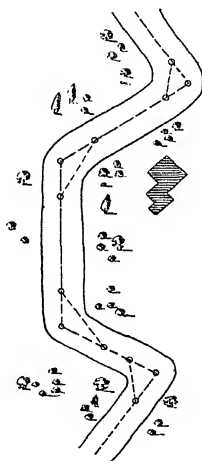In view of the explanation which has been given

ILLUSTRATION No. 119.

of the method of surveying a river or canal, Illustration No. 119 is considered a sufficient explanation.

I may, however, repeat that, unless the lines taking the road start from and close on other properly proved lines, the angles should be taken with the theodolite as a check ; or the survey should be conducted by traversing, which will be explained in its proper place.

TO CHAIN OVER HILLY GROUND.

(**207**).—Another circumstance often occurring in practice which has to be met, is that of sloping ground, in which

case the measurements taken on the surface would not be on a horizontal plane such as is required for a plan.

The horizontal measure is obtained by either of three methods :—

(a) Stepping.

(b) Setting back the arrow.

(c) Reduction of hypotensual to horizontal measurement.

Stepping has already been referred to in explaining the method of chaining, but will now be more fully dealt with.

Stepping.—Let *A* and *B*, Illustration No. 120, be stations on either side of a hill over which a line has to be

ILLUSTRATION No. 120.

chained from *A* to *B*. The leader proceeds up the hill to *a* and the follower holds the end of the chain sufficiently high at *A* to bring it horizontal. For this purpose the follower holds a plumb-line in his hand which he suspends from the chain handle over *A*, the leader puts in his arrow at *a*, and the chain is dragged forward. The follower holds the end of the chain over *a* sufficiently high to bring the chain horizontal, the leader puts in his arrow at *b*, and the process is repeated, arrows being put in by the leader and taken up by the follower, as in the case of chaining on level ground.

When the top of the hill has been reached, the chaining is continued down the slope, the same process being followed, except that the leader now raises the end of the chain, and is enabled to fix the position where the arrow should be put in at the end of the horizontal chain, either by using the plumb-line or a " drop arrow."

When the slope is too steep, to enable the end of the chain to be held sufficiently high to bring it horizontal, the chaining will have to be done in half chains, and, perhaps, even in shorter lengths.

Illustration No. 120 represents a case where the stepping has been done in chain, half-chain, and quarter-chain lengths, but the vertical scale has been much exaggerated in order to show the lines more clearly.

A specially made spirit level may be suspended to the chain as a guide in holding it horizontal, but very little practice will render this unnecessary.

SETTING BACK THE ARROW.

Where ground inclines less than three degrees, it is not generally necessary to make allowance for the slope, and in small slopes the arrow may, after some little practice and experience, be " set back " in the position indicating the end of the horizontal chain with sufficient accuracy without stepping.

REDUCTION OF HYPOTENSUAL TO HORIZONTAL MEASUREMENT.

This is dealt with in Chapter XXVII., under the head of " Correction of Inclines."

TO FIX THE POSITIONS OF BUILDINGS.

(**208**).—It is not sufficiently accurate to fix the positions of buildings or other important objects by taking off-sets from a chain line passing near to them. Lines should be measured from the chain line to the angles of the building, etc., so as to form triangles, the apex of which will fix the positions of the angles of the building, etc., and the distances on the chain line and the lengths of the sides

of the triangles must be entered in the field book, as
shown in Illustration No. 121.

If, when these triangles on the chain line have been
plotted on paper, the dimensions taken round the building
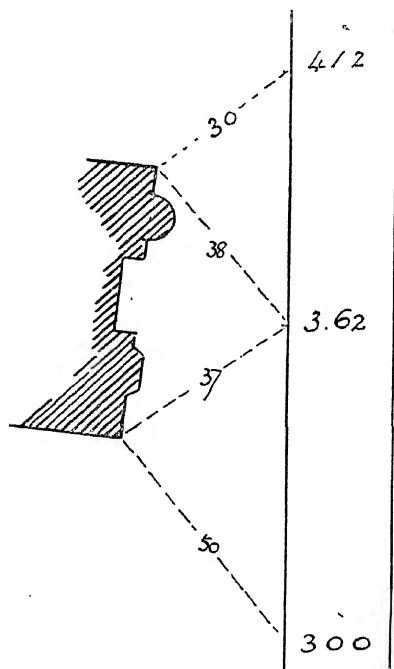

ILLUSTRATION No. 121.

fit in perfectly, there will be proof of the accuracy of that
part of the work.

The double line indicates the column in the field book,
which, as before observed, really represents the chain line,
The lengths of the sides of the triangles formed to fix the
position of the building are noted down on a sketch
made in the margin of the field book, as shown, and dotted
lines are used to indicate clearly to what the measurements
relate.

Generally speaking, the marginal space in the field book being somewhat limited, a larger sketch of the buildings is made on a separate page and the dimensions necessary to plot a block plan taken and shown thereon.   In simple cases the measurements round the building may be shown on the small sketch in the margin of the book.   (See Chapter XIV.)

TO FIX THE POSITIONS OF CORNERS OF FIELDS, JUNCTIONS OF HEDGES, ETC.

(**209**).—As shown in Illustration No. 122, these should be fixed by a small triangle and the measurements are entered in the field book as advised for fixing the positions
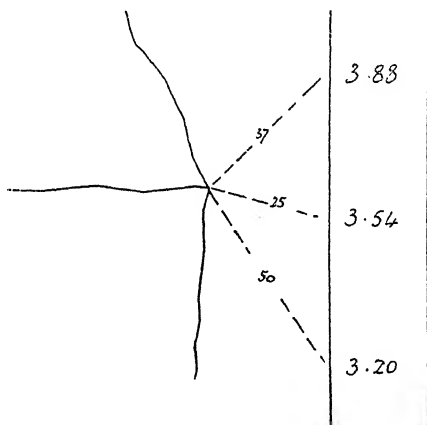
ILLUSTRATION No. 122.

of buildings, but, as in this case there would be nothing to prove that no error had occurred in measuring or plotting the triangle, a proof line should be measured, as shown in illustration.

BREAKS IN HEDGES, ETC., TOO LONG TO BE DEALT WITH AS OFF-SETS.

(**210**).—It often occurs in practice that on arriving at a certain point or points in a chain line from which off-sets or

insets are being taken to a hedge or other boundary, that the
hedge, etc., is found to break out of the general line in a
degree which necessitates some other means of taking the
undulating boundaries than by mere off-sets, which would
be too long.   The best way of dealing with such cases is
to put in pickets and measure lines forming a small
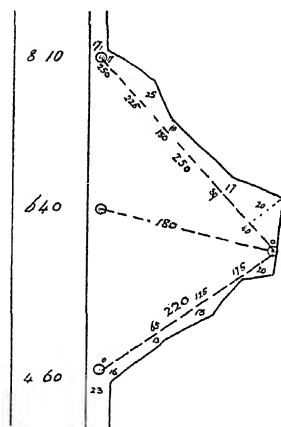triangle, which should be tied or proved, as shown in

ILLUSTRATION No. 123.

Illustration No. 123.   Off-sets or insets, as the case may
be, are then measured from the sides of the triangle, and
the measurements are booked on a sketch made in the
margin of the field book, as indicated.

UNCERTAIN BOUNDARIES.

(**211**).—The common boundaries between fields are—
  (*a*) Hedge;
  (*b*) Ditch;
  (*c*) Hedge and two ditches;
  (*d*) Hedge and ditch;
  (*e*) Fences of various kinds.

In the first three (a), (b) and (c), the centre of hedge or ditch will probably be the boundary. In the fourth case (d), both the hedge and ditch usually belong

ILLUSTRATION No. 124.

to the field in which the ditch is not, as shown in Illustration No. 124.

In the fifth case (e) the fence generally belongs to the land towards which the nails are driven home. Thus in a boarded fence the rails will usually be on the side to the land on which the fence belongs.

In the case of iron fences and hurdles, the feet or spurs will usually be on the land of the owner of the fence.

It is not possible, however, to give absolute rules with regard to boundaries, and where they are uncertain, enquiries should be made from those well acquainted with the land. My advice with regard to obtaining proof of ownership or custom is—listen to any information you can get, but do not necessarily accept it.

There are, too, customs with regard to allowances to be made in fixing boundaries, which differ in various parts.

CHAPTER XIV.

COMPLETE CHAIN SURVEYS.

Surveys illustrating cases necessitating the combination of the rules given separately in previous chapters, and thus instancing their application in actual practice.

In previous chapters I have given separately the rules of procedure to be employed in various cases; the present chapter will deal with complete surveys, involving a combination of those rules, and thus, as far as possible, will make the study as realistic, or as nearly like actual practice in the field, as possible.

Nothing on paper, however, can fully accomplish this; the very fact of the work on paper being brought all under the eye at once renders it far more simple of conception than actual work in the field; but it is hoped that much that is helpful may be found from a careful study of the following examples.

I can probably best give effect to my purpose by assuming a particular case, and going through the procedure in regular order as though engaged in making an actual survey. Before doing this, however, I will give, in tabulated form, and in their natural order, the various steps to be taken in the work. These are as follows :—

    (1) Enter the date, description of the estate, etc., in the first leaf of the field book.

    (2) Make a reconnoitre of the land and prepare a rough sketch plan of it, showing all boundaries, internal fences, hedges, buildings, etc., names

of adjoining owners, cultivation of the various fields, etc.

(3) Pace lines round the boundaries and such tie lines as are necessary to enable the figure to be plotted on paper.

(4) Plot a rough outline plan of the land from the paced measurements at the commencement of your field book, and show thereon all objects required to be shown on the finished plan.

(5) Plan on this sketch the system of triangulation to be employed in the survey and number the lines and stations thereon in the order you decide to chain them. Sometimes it is as well to mark the lines with arrows showing the direction in which they are to be chained, but the numbering of the lines and stations is generally sufficient to show this.

(6) Chain the lines in the order given on the key-plan, numbering the lines and stations in the field book to correspond therewith. If the lengths of the various lines are entered on the key-plan as they are chained, it will be found a great help when the plan has to be plotted.

(7) Where necessary, make separate sketch plans of the buildings, take the dimensions necessary to enable you to plot them and enter same thereon.

(8) With the theodolite read all angles formed by the principal chain lines and enter the readings in the field book.

(9) With the prismatic compass attached to the theodolite take the bearing of the base line.

    (10) Plot the plan from the field notes to a suitable
         scale, checking its accuracy with the proof
         lines and the protractor, and ink same in,
         print, etc.
    (11) Compute the area of the lands from the plan,
         or the field notes.
    (12) The plan completed, except colouring, care-
         fully clean and colour same.

Having now noticed in their proper order the various
steps to be taken, let us assume that we have been
instructed to survey and prepare an accurate plan of a
portion of an estate, which is represented by Illustration
No. 125.

ILLUSTRATION No. 125.

Having arrived on the land with all necessary implements,
instruments and assistants, we proceed to make the
survey, following the order of procedure already laid down.
First we enter in the field book the description and
locality of the land, and the date on which the survey is
made. Then we make a reconnoitre of the land, and
prepare a key-plan on which we enter all necessary
particulars.

If, in making the reconnoitre, it is found there are any uncertain boundaries, doubt as to ownership of certain fields, etc., enquiries should be made, and all difficulties cleared up before the actual survey is commenced, so that there may be nothing to stand in the way of the regular conduct of the work when once it is started.

Our third step is to "pace" lines round the boundaries, and such diagonals as will enable us to plot the figure on paper. This will not, of course, be necessary when any plan of the land is obtainable.

Pacing is performed by simply walking over the lines the lengths of which are required, taking three feet at each step, and either counting the steps or using a passometer to register them. (See Art. 174.)

Referring to Illustration No. 126, if we pace the main constructional lines, we shall be in a position to plot roughly in the first page of our field book an approximately accurate sketch plan of the land, upon which we may lay down a system of triangulation for a survey and from which we may prepare a key-plan as represented by that illustration.

By having a fairly accurate sketch plan of the land, the lines may be run in the best positions, and subsequent departure from the system decided on, which often results in great loss of time and sometimes inaccuracy in the work, is avoided.

A very simple and satisfactory way of making a key-plan, on which we may base our system of triangulation for an actual survey, is to obtain the ordnance sheet containing the particular piece of land, and to make a key-plan from it, amending it as may be necessary for the purpose.

Having prepared an approximately accurate plan at the commencement of the field book, we lay down on it a system of triangulation which will embrace the boundaries and all necessary features of the land.

In laying down the system of triangulation the first question is, Where shall we run our base line ?
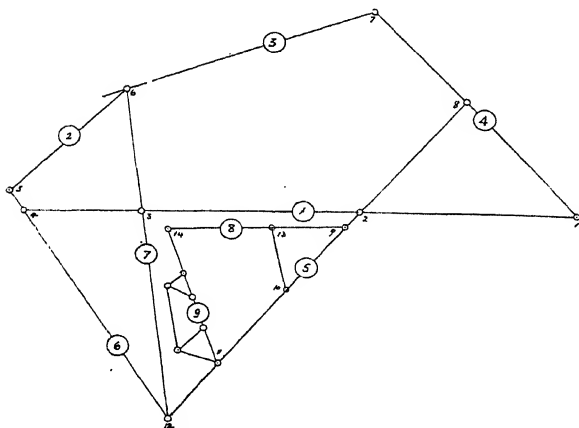

ILLUSTRATION No. 126.

Now there are several considerations to guide us, which I have already pointed out, and will repeat here.

The base line should generally—

      (1) Be on level ground and somewhere about the centre of the land ;

      (2) Run in the direction of its longest dimensions ;

      (3) Be unobstructed and command a view from end to end.

Referring to Illustrations Nos. 125 and 126, it will be seen that the position selected for the base line (line 1) is that which most nearly complies with these requirements. Of course, we cannot always judge from a plan the suitability of positions in which to run lines, hence the reconnoitre.

We see, however, the line is in the centre of the land, runs in the direction of its longest dimensions, is unobstructed, except for the crossing of two fences, and, in addition, has the advantage that the line necessary

to take up the S.E. boundary and the internal hedge will cross it, thus securing the X form, which, it has been pointed out, is a good one.

We have next to consider what lines will be required to take in the boundaries and other features of the land, and will deal with that portion of the estate lying on the south side of the base line first.

We must not forget that—

(1) The lines we decide to measure must be sufficient to enable us to plot an accurate plan of the land on paper ;

(2) There must be additional lines, not needed for plotting the figure, sufficient to tie or prove every part of the work ;

(3) As far as possible the lines should be run so that tie and proof lines may also serve to fix the positions of internal fences, hedges, buildings and other objects, to be shown on the finished plan ;

(4) The lines must be run so as to keep the off-sets or insets as short as possible ;

(5) As far as practicable the lines should be run on level ground ;

(6) All lines should be run so as to avoid, as far as possible, the inconvenience of having to pass through hedges. Where they must cut, they should do so as nearly as possible at right angles;

(7) Very acute or obtuse angles should be avoided.

The positions for the remainder of the lines have been practically dictated to us by the positions of the hedges, fences, river, buildings, etc., but the following remarks may be offered.

The undulating S. boundary, part of the river, and the buildings on the north thereof will be fixed from the base line (line 1).

The N.W. boundary breaks out of the general line
at each end, and it will be necessary to form small
triangles on line 2 there, to prevent long off-sets. (See
Art. 210.)

Line 3 takes the N. boundary on one side, and fixes the
position of the buildings in the N.E. corner, on the other.

The S.E. boundary and the internal hedge in con-
tinuation of it is very irregular, and three small triangles,
as advised in Art. 210, will have to be formed on line 5
to prevent long off-sets.

The off-sets will be on the N.W. side of line 5 as far
as where it crosses the river, and on the S.E. side for the
rest of the distance, and will be entered in the field book
on opposite sides of the centre column accordingly.

Line 6 commences a few links S. of the first station in
that line (station 12, line 5) and proceeds some little
distance past station 4 in line 1.

Line 7 calls for special notice. In the first place,
when lines are made to cross each other, as shown (line 7
crossing line 1), there will be some little extra trouble
needed in ranging them ; and inasmuch as instead of this
line 7, we might have had two lines, one on either side
of the base line joining it at different stations, and that,
by so doing, we should have escaped crossing the pond, in
some respects that course would have been better in actual
practice.

However, the system of lines laid down shows the
application of the rule for continuing a chain line ob-
structed by a pond, and obtaining a plan of the pond.
(See Art. 197.)

Lines 8 and 9, and the other lines connected with them,
need no special notice. Their purpose is to enable the
river on that side to be correctly mapped.

In the foregoing references to the key-plan (Illustration
No. 126) the top of the paper has been assumed to be north.

Although the plotting and finishing of plans, properly belongs to another chapter, I must here refer to the plotting so far as the main lines in the key-plan are concerned, in order that we may examine it as a system of triangulation, and see whether it is both plottable and provable, qualities which, apart from other important features, every system must possess. This we will now do.

First we may lay down the base line (line 1) to scale and mark on it stations 1, 2, 3 and 4. Then we may plot the triangle on the south side of the base line, by taking the lengths representing those parts of lines 5 and 6 which lie on the south of the base line respectively in the compasses, and with stations 2 and 4 as centres, making intersecting arcs.

Now these lines may respectively be continued to stations 5 and 8.

Line 4 may be drawn from station 1 through the extremity of line 5 (station 8) and its length scaled off, giving the position of station 7.

Then lines 2 and 3 may be plotted, stations 5 and 7 being the centres from which the arcs intersecting in station 6 are struck, and thus the outline is completed.

Line 7 has not been needed for plotting the figure and will serve as a proof line. It will be observed that, if any part of the work was inaccurately measured or plotted, line 7 would not scale on the drawing what it measured in the field, the system of lines being such that any inaccuracy in one part would throw the whole figure out. The system planned constitutes one entire figure and not a mere series of figures on either side of a base line; and whilst the system of lines is extremely simple, the slightest inaccuracy will be discovered when the test afforded by the proof line is applied.

Being satisfied, then, that the system of lines decided on will both plot and prove in every part, we now proceed to

the next step and settle the order in which those lines shall be chained, and number the lines and stations accordingly, and thus complete the key-plan as we have it in Illustration No. 126.

Now we proceed to the field work ; put in pickets at the positions on the land indicated by the stations on the key-plan, as they are required, and proceed to chain the lines in the order given and to book them in the manner we shall now describe.

The following diagrammatic plan (Illustration No. 127), with some of the dimensions figured on it as they would be taken in the field, compared with the field book accompanying it, will probably do more to explain the method of chaining and booking than any number of pages of description.

The centre column in the field book represents the chain line measured in the field, shown by dotted lines in the diagram ; the numbers of the lines and stations and the arrows show the order and directions in which the lines have been chained. The measurements across the chain line are some only of the distances thereon at which fences cross, or more important off-sets have been taken. They are, however, sufficient to enable the field notes and diagram to be compared. The off-sets could not be shown, the size of the diagram which it was possible to give, not permitting it.

The off-sets are, of course, entered in the field book on the right or left of the centre column, according as they are on the right or left of the chain line, having regard to the direction in which the chaining is being done. Thus in the case of line 5, chaining as we have in this case from N.E. to S.W., we have off-sets on the right-hand side of the chain line until we cross the river, and then on the left-hand side to the end of the line. If we were chaining in the reverse direction, the respective

off-sets would be booked on the opposite sides of the line. No confusion, however, can arise.

The sketches in the field book are always distorted, inasmuch as the measurements on the chain line are usually entered about equidistant, although the distance on the ground represented by them differs considerably. They are only rough sketches, too, made in the field, and so long as they help in plotting the measurements that is all that is required of them.

We may with profit, perhaps, follow one line, and will select line 1 for that purpose.

We should first notice that in booking we always commence on the last page of the book and write upwards.

First we write across the column representing the chain line, Line 1. From (1) go west to (4) and draw a line. Now we enter 000 in the chain column as representing the commencement of the line, and in a loop to make it more conspicuous (1).

It is as well to make the stations conspicuous by drawing a circle round them in this way, because in plotting it is very handy to be able to find the stations readily.

At 150 links along the chain line the fence crosses, so we put 150 in the chain column and draw the lines on either side of it representing the fence. This fence joins another which runs parallel with the chain line, and the point at which an off-set let fall from the point of junction would cut the chain line at right angles is at 195 links along the latter. Hence we enter 195 in the chain column, and measuring the off-set 10 links, we enter it as shown.

Off-sets are similarly taken and booked to every break, etc., in the hedge or bank of the river so that an exact plan may be plotted.

At 675 on the chain line we put in our station for line 5 and book it accordingly, as shown.

At 695 another fence crosses and is booked, and a line drawn to represent it as before.

Again at 1260 another fence crosses and is booked likewise.

At 1356 we establish station 3.

From 1530, 1570 and 1650 in the chain line, we measure to the angles of the building with the object of fixing its position, and put down these measurements against a sketch, as shown. We also measure round the building, and note the dimensions against the sketch.

We now continue to the end of the line, which we reach at 1730 links. This is entered, station 4 and line 6 crossing also being noted.

Finally, we draw a line across the column, write " End of line 1," and then a double line to show that the line is completed.

NOTE.—It is usual to book the ten chains each time the arrows are exchanged to prevent the possibility of mistake.

All the other lines in the survey are chained and booked in a precisely similar manner. There is no necessity to refer further to the matter, a comparison of the field notes with the illustration may be simply made.

Having completed the chaining, we now make a larger sketch of the building at the N.E. corner of the land, the position of which has been fixed by triangles from the chain line.

It is not always necessary to make separate sketches of the buildings. Where they are very simple in outline the measurements may be put down against the sketch shown in the field notes, as has been done in the case of the building on the west side of the land, but where there is any amount of detail, it is better to make a separate sketch rather than attempt to crowd all the dimensions on to the necessarily very small one in the margin of the field book.

Line No. 7 (upper left diagram)

End of line 7

	11.51
20	9.50
35	8.70
	7.40
15	7.00
5	6.47
35	6.35
20	4.80
47	4.45
	4.39
	3.12
	2.92

Line 5 · · ○No12 · Line 6

Fence

River

Fence ○No3 · Line No11

straight fence

Pond Tie from 4.070 ○ 090

1.50
1.05
0.00 ○No6

Line 3 · · Line 2

From ○ No6 line 2
go S to ○12 line 5

Line No 7.

Line No 6 (lower left diagram)

End of Line No6 · Line No 9.

	9.75
30	9.65	○No5
20	9.00
10	8.80	○834 on Line No1.
5	8.30
10	6.50
20	5.50
0	4.50	Line No1
8	4.10	Fence
5	0.48	○No10

Fence

Line No 5.

Line No 9 (right upper diagram)

End of Line 9

○No11 · 4.80

3.56

2.08

1.60

0.30
000 ○No10

From ○ to Line No8
go S.5 ○ 4 line No.

Line No 9

Line No 8 (right lower diagram)

End of Line 8

○No4	5.68	10
	5.55	5
	4.80	5
	3.50	15
	2.90	8
	2.50	10
○No3	2.25	5
No852	1.80	5
Line 3	1.20	10
210	0.50	5
○No2	000	

Line No9

From ○ No4 Line 5

Illustration No. 128 represents a sketch of the building in the N.E. corner of the land, made on a separate page in
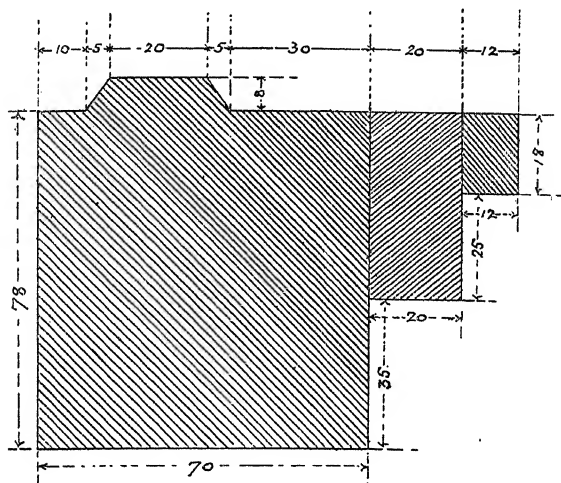
ILLUSTRATION No. 128.

the field book, with all the dimensions shown on it, as they have been taken from the building.

The walls in this case are all at right angles to each other and, therefore, no ties have been necessary, but where such is not the case, as many ties must be taken as are requisite.

Where the land is measured with a Gunter's chain, as would be the case where the plan was required to be plotted to a chain scale, the dimensions round the buildings should be taken in links and parts. The measurements can generally best be taken with the tape, one side of which is usually divided into links and the other into feet and inches.   (See Art. 30.)

The survey referred to in the foregoing remarks has been given merely as an example of chain work and triangulation.

In all but quite unimportant cases, however, in practice, the further guarantee of accuracy obtained by the use of the theodolite should be taken advantage of.

Applying it to the present case, the instrument would be set up at stations 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12 and 14, and the angles read and entered in the field book as indicated by the example given in Illustration No. 129.
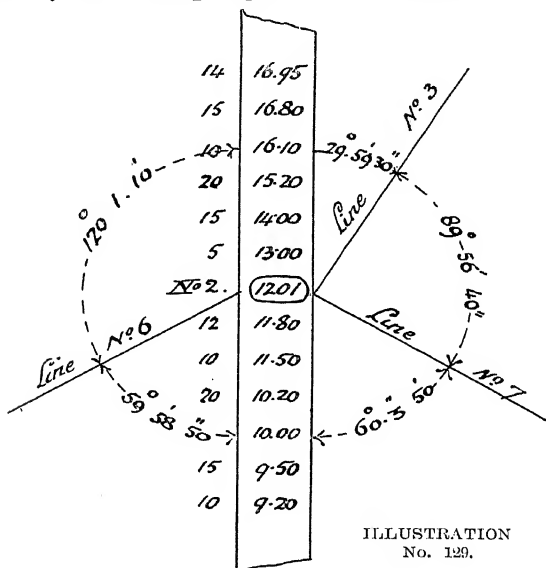
ILLUSTRATION
No. 129.

It will be seen that the entries are simply made in their natural place between the lines to which the angles relate.

While the theodolite is adjusted over station 1 the bearing of the base line is taken.   This enables the plan to be so laid down on the paper that the top of the sheet may represent north, or the north-point to correctly shown thereon.

It must not be forgotten, however, that the needle does not point true north, and that the declination varies from time to time.   The method of finding the true meridian has been dealt with in Arts. 325 and 326, Chapter XXIV.

CHAPTER XV.

PLOTTING SURVEYS.

Preparation of Drawings—Plotting—Order—Method—Printing—Border Lines—To Remove Drawing Ink from Plans — Scale — North-Point — Colouring of Plans — Coloured Lines on Coloured Drawings — Enlarging, Reducing and Copying Drawings.

———

The present chapter deals with the order and method of plotting surveys from field notes, printing, colouring and generally the preparation of drawings.

As necessity is the mother of invention, so method may be said to possess a similar relationship to accuracy. Having this in mind, I shall now give:—

    (1) The order in which surveys should be plotted ;

    (2) The method of plotting from field notes ;

    (3) Printing on plans ;

    (4) Colouring.

PLOTTING.

(**212**). *The order in which plotting should be done and the preparation of drawings.*—The following is the order in which the work of plotting from field notes and the preparation of the plan should be done :—

    (1) Plot all the constructional lines, without reference to off-sets or other detail, in pencil.

    (2) Draw in the lines and stations plotted in pencil, in red liquid colour.

    (3) Plot all the off-sets and other details in pencil.

    (4) Ink in neatly all the boundaries, fences, ditches, streams, buildings, trees and other detail.

(5) Transfer this "rough" plan to a clean sheet of paper (omitting the lines used in survey) for a finished plan. The best method of doing this will be found described at the end of this chapter.

(6) Print, put on north-point, scale, border line, etc., and other necessary finishing.

(7) Carefully calculate the areas of the various fields. For the method of computing areas from plans see Chapters XVI., XVII. and XVIII., Arts. 213 to 236.

(8) Remove all pencil lines from the drawing and thoroughly clean it by rubbing with bread crumb.

(9) Where the plan is a large one and is to be coloured, it is sometimes damped and fastened perfectly flat on the board to ensure evenness of colour, but this is not usually necessary.

(10) Colour the plan.

The method of plotting surveys from field notes is as follows :—

Lay down the base line to the scale required, and, if possible, in such a position that the top of the paper may represent north. Now look to the key-plan and the field notes, and plot any two lines which are connected with the base line and each other, in other words, any lines which are plottable, and proceed in like manner with all the remaining lines until the whole have been plotted.

Where the angles have been taken with the theodolite, check the accuracy of the plotting with the vernier protractor as the work proceeds. It will be noticed that the survey could easily be plotted without the key-plan, but reference to it simplifies the work and saves much time.

It will likewise be noticed that it does not follow that

the lines are measured and booked in the order in which they can be plotted, and it will often be found necessary to pass over some of the lines in the first instance, and return and plot them as may be possible afterwards.

A brief glance through the field notes will give an idea of the size and form of the plan, and enable the first line to be laid down so that the drawing may be central on the paper when finished.

The lengths of lines which have to be taken in the com-passes for the purpose of ascertaining the points of inter-section for plotting should be first laid down on the paper (they should not be taken direct from the scale with the compasses, which spoils the scales).

Where the plan is being plotted to a large scale, beam-compasses may be necessary for describing the arcs to fix the positions of the intersection of lines.

When all the constructional lines have been laid down in pencil, they should be drawn in permanently in red fluid colour.

As soon as the constructional lines are quite dry, lay the scale parallel with the first line on which off-sets or insets occur, with the zero of the scale at the end from which the measurements are calculated, the scale being steadied by paper weights and placed at such a distance from the chain line that the centre division of the off-set piece, when slided on the edge of the scale, may coincide with it. Slide the off-set piece along the edge of the scale to the several points at which off-sets or insets occur, marking lightly, with a finely pointed hard lead pencil, the lengths thereof, and proceed in this way until the line is completed. This done, carefully draw in the boundaries and other details through the various points thus determined.

In the case of buildings, the small triangles fixing their relative positions to the chain lines are plotted first, and

then the buildings are added from the dimensions which, where necessary, are put down to separate sketches in the field book as suggested in Chapter XIV.

Where buildings are cross-ruled as shown in Illustration No. 128, the section lines should be perfectly parallel and equidistant. A " sectioner " will be found useful for this purpose.

When all the lines and details have been plotted in pencil, they should be very neatly drawn in India ink, which should be freshly rubbed up on each occasion, unless, as is usually the case in this time-saving age, bottled liquid ink is used. Higgin's American liquid waterproof India ink will be found very convenient where the colouring has to be done directly the drawing is finished, as it will not wash up.

TO REMOVE DRAWING INK FROM PLANS.

Wet the portion to be erased with a brush charged with clean water, blot up with perfectly clean blotting paper after allowing it to stand for a few seconds, and rub briskly, but lightly, with good white india-rubber. Repeat the process if necessary. This will be found better than scratching with a knife or ink-eraser.

When the rough plan, viz., the correct though unfinished plan, has been plotted, the plan (omitting the constructional lines) should be transferred to a clean sheet of paper by one of the methods described at the end of this chapter, inked in, printed, etc., etc., and finally cleaned and coloured.

PRINTING AND FINISHING DRAWINGS.

Printing on Plans.—No one who is unable to print neatly can satisfactorily finish a drawing ; the best plan will be quite spoilt if indifferently printed. For printing to be satisfactory, the letters must be accurately formed,

perfectly spaced, and of a size and character suitable to the drawing on which they appear.

There is no royal road to becoming a good printer, it is a matter of practice properly guided and helped by good copy.

There are a few suggestions which may be made : (1) Do not bear on the pen as you would in writing, but draw in the lines without causing the nibs to deflect at all ; (2) dip the pen in the ink so as to wet the back of the nib only ; (3) never go on printing when the pen is running dry ; (4) hold the pen so that the nibs bear perfectly evenly on the paper, to obtain a clear, sharp line ; (5) in spacing out the headlines, sketch them out roughly first on a slip of paper, then divide the space occupied into two equal parts and set same off on either side of a central line drawn on the plan. In this way the different headlines are set down centrally under each other, which is particularly necessary ; (6) in headings a combination of the styles of printing will often give a good effect, but, generally speaking, no printing is to be preferred to plain upright block ; and the man who can execute this style well will find little difficulty with the remainder ; (7) avoid stencil plates in the case of all except unimportant work.

BORDER LINE.

Another matter of finish to a drawing is a neat border line.

SCALE AND NORTH-POINT.

The scale on a plan is likewise a matter requiring notice. It should be absolutely accurate, neat and effective. Every plan should have a north-point on it to show the aspect of the land, a most important feature.

THE COLOURING OF PLANS.

Colouring.—The skilful colouring of drawings is not to be acquired except by practice, but the following directions may help the student.

(1) The board on which the drawing is pinned should be slightly tilted at the top, about 15 or 20 degrees, so that the colour will tend to run down.

(2) Amply sufficient colour to finish the plan should be mixed before the colouring is commenced ;

(3) The colours should be well mixed and used very thin. Beginners almost always err by using the colours so thick that it is impossible to spread them with a perfectly flat even surface, which is what should be aimed at.

(4) Use the largest brush that may conveniently be employed, having regard to the size of the plan to be coloured.

(5) Always commence to colour at the top left side of the drawing, working from left to right, and from top to bottom.

(6) Never let the brush get dry, keep it well charged, and likewise prevent the lower edge of the colour which you are gradually working downward from becoming a fixed dry line. The slight tilt on the board will help this by tending to keep the colour running in that direction, thus there will be a small volume of colour along this lower edge keeping it moist.

(7) Where the plan is very large and broken up into separate distinct fields, take advantage of this and colour one of the enclosures at a time up to the boundary lines to decrease, as far as possible, the size of the washes you have to

deal with. Care must be taken, however, not to go over the lines, in which case, when the adjoining field is coloured, you get two coats in places, forming hard disfiguring patches on the plan.

(8) Never go over the same ground twice, unless, after a plan is quite dry, it is found necessary to give some part another coat. With care, however, this never should occur.

(9) Always have another dry brush at hand to take up any superfluous colour when finishing off a wash. Never use blotting paper for that purpose.

(10) In colouring water, the edge is coloured only, the colour gradually dying away towards the centre. To do this, pass a brush charged with clean water down the centre of the stream or river where it is not wide on plan (or along the inner edge of what will be the coloured part of the water) then draw a brush charged with blue colour along each edge of the water, when the colour amalgamating with the water on one side will give the desired effect. Some practice is necessary to do this effectively.

(11) When two different colours will meet at any point, one should be quite dry before the other is put on.

(12) Colouring must be done boldly and expeditiously. Good brushes are essential.

COLOURED LINES ON COLOURED DRAWINGS.

Lines in Colour.—In any case where on a coloured drawing the constructional lines, or the lines on which

the levels have been taken, or the arcs indicating the theodolite readings, etc., etc., have to be shown in coloured lines, they should be put on after the colouring has been done and when the plan is quite dry. If put on before, they lose their sharpness and are apt to run when the colouring is done.

In the case of the plan required to be sent in by candidates for the Surveyors' Institution examinations, the constructional lines, the arcs indicating the theodolite readings, and the lines over which the levels have been taken, as shown in the sections, have to be indicated on the plan in coloured lines. See "Instructions to Candidates," issued by the Surveyors' Institution to examinees.

TO ENLARGE OR REDUCE DRAWINGS.

Method No. 1.—Prepare a sheet of very transparent tracing paper divided with finely and very accurately ruled lines into any number of equal squares the sides of which represent one division of the scale to which the plan required to be enlarged or reduced is plotted. The sizes of the squares must depend upon the amount of detail in the drawing to be copied, the scale to which it is drawn, etc.

Similarly prepare a sheet of drawing paper, but let the squares equal a corresponding division of the scale to which the copy is to be drawn.

Pin the plan to be copied on a drawing board with the ruled sheet of tracing paper over it. It should be well pinned down all round to prevent the slightest movement.

Now note carefully the positions of the fences, etc., etc., in the various squares, marking off the exact points at which the vertical and horizontal lines are cut by scaling from the drawing, and transferring to the copy, according to scale.

In this way the whole of the detail as well as the main outline is delineated.
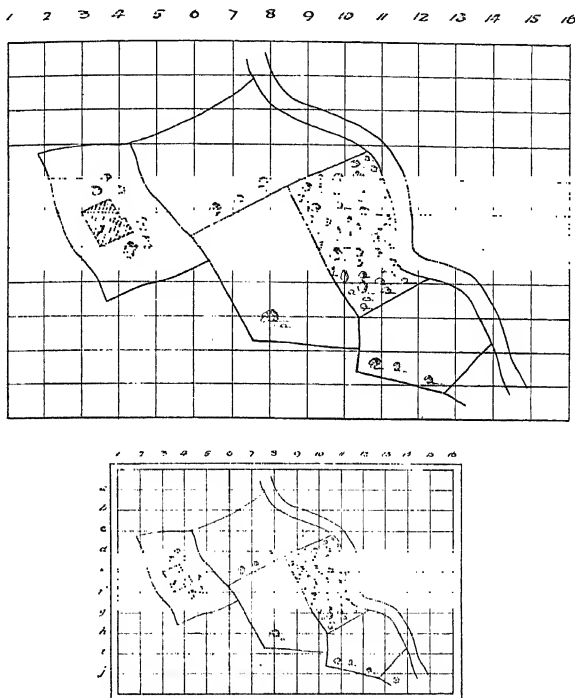
ILLUSTRATION No. 130.

Illustration No. 130, representing a plan and copy to a reduced scale prepared by this method, is all that need be given in further explanation.

Method No. 2.—Select some convenient point about the centre of the drawing to be copied and draw radiating lines from it through all the angles formed by fences, etc., etc., and to the angle of buildings and other objects to be delineated.

Scale these radiating lines from the drawing and mark off on them corresponding distances according to

the scale to which the copy is to be prepared, thus fixing the positions of the points to which they relate. Trace the outline of the boundaries, etc., through these points, and proceed in like manner until the drawing to enlarged or reduced scale is complete.
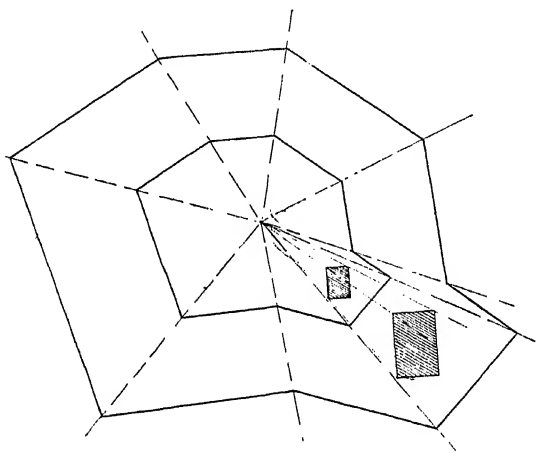
ILLUSTRATION No. 131.

Illustration No. 131 will be sufficient to explain the process.

In the case of curved boundaries, they may be divided up into a number of segments, radiating lines drawn through the points of division, and these points transferred on the radiating lines, which will enable the exact contour of the curve to be delineated on the reduced or enlarged scale.

I generally work on a sheet of good transparent tracing paper pinned over the drawing, as in that case the original is protected from injury, and the copy is produced either in the centre of the drawing being copied or surrounding it, according to whether the copy is larger or smaller scale than the original. In this way the whole is kept under the eye at once, and turning from drawing to copy

is avoided. The drawback, however, is that the copy has to be transferred to a clean sheet of paper afterwards.

Method No. 3.—Employ the pantagraph (see Chapter X., Arts. 119 to 123).

Method No. 4.—Employ the eidograph (see Chapter X., Arts. 124 to 127).

Note.—In all cases greater accuracy is procured in reducing than in enlarging.

TO COPY DRAWINGS.

Method No. 1.—Use the glass tracing table (see Chapter X., Art. 118).

Method No. 2.—Prepare a sheet of transfer paper by rubbing one side of a sheet of very thin tracing or tissue paper with black lead. It is better prepared with the black lead from good drawing pencils finely ground up than with common lead. Pin the sheet of clean drawing paper on a drawing board with the transfer paper over it, black face downwards, and the drawing to be copied on the top, face upwards, with a sheet of thin tracing paper over it for protection. The whole must be well pinned to prevent the slightest movement.

Very accurately trace the outline of the drawing with a hard pencil held slightly sloping from left to right in the direction of the line you are tracing, but perpendicular in other directions.

Ink in the pencil imprint which will be produced on the clean sheet of paper.

Method No. 3.—Make a tracing, viz., a copy on tracing paper or cloth, where a copy produced in that way is sufficient to meet the requirements of the case, as much time is thereby saved.

In making a cloth tracing, draw on the dull, and colour on the glazed side of the tracing linen. Rub the dull side of the linen with powdered chalk and dust off with a clean cloth before commencing to trace.

CHAPTER XVI.

COMPUTATION OF AREAS FROM PLANS WITH THE COMMON SCALE.

Simple Cases of Straight Boundaries—Cases to be Dealt with—Linear and Square Measure—Method of Reduction—Links to A.R.P—Yards or Feet to A.R.P.—Examples—Square — Rectilineal Figure — Right Angled Triangle — Any Triangle — Trapezoid — Rhombus — Rhomboid — Trapezium — Figure of any Number of Sides—Examples—Method—Important Observations.

The subject of the present and following chapters is the computation of the area of lands—

(*a*) From plans ;

(*b*) From the field book ;

and will deal with the different methods to be employed under varying circumstances.

It will no doubt clear the ground somewhat if I first give in tabulated form the several cases which will be dealt with. They are as follows :—

(1) Computation of areas from plans with the common scale.

(*a*) Simple cases of straight boundaries.

(*b*) Irregular boundaries.—Casting.

(*c*) Irregular boundaries. — Calculation of off-sets.

(*d*) Curved boundaries. — Simpson's Rule.

(*e*) Complicated outlines.—Reduction of complicated to simple forms.

(2) Computation of areas with the computing scale.

(3) Computation of areas with the planimeter.

(4) Method of ascertaining the true area from a given erroneous area computed from a plan by a different scale to that which the plan has been drawn.

(5) Land measure. Reduction of customary to statute measure, and *vice versa*.

(6) The computation of areas from field notes—

        (*a*) Chain surveys;

        (*b*) Trigonometrical surveys; and

        (*c*) The traverse.

It will be seen that, with regard to the computation of areas from plans with the common scale, there are five, and, in the case of computations from field notes, three distinct cases to which I must direct your attention. Before doing so, however, I will give the following tables of linear and square measure.

TABLES.

Linear Measure.

Inches.	Feet.	Yards.	Poles.	Furlongs.	Miles.
12	1	—	—	—	—
—	3	1	—	—	—
—	—	5½	1	—	—
—	—	—	40	1	—
—	—	—	—	8	1

Inches.	Links.	Poles.	Chains.	Furlong.
7·92	1	—	—	—
—	100	4	1	—
—	—	—	10	1

SQUARE MEASURE.

Mile.	Acres.	Roods.	Chains.	Perches. (Poles)	Yards.	Feet.	Links.	Inches.
1 =	640	—	—	—	—	—	—	—
—	1 =	4	10	160	4840	43560	100000	6272640
—	—	1 =	2½	40	1210	10890	25000	1568160
—	—	—	1 =	16	484	4356	10000	627264
—	—	—	—	1 =	30¼	272¼	625	39204
—	—	—	—	—	1 =	9	—	1296
—	—	—	—	—	—	1 =	—	144

METHOD OF REDUCTION.

(**213**). *Links to Acres, Roods and Poles.*—Inasmuch as an acre contains 100,000 links, we may convert links into acres by simply placing the decimal point five places to the left, thus 100000 links becomes 1·00000, or 1 acre, and all the figures to the right of the point become decimals of an acre, and therefore by successively multiplying these decimals by 4 (the number of roods in an acre) and 40 (the number of poles in a rood), each time marking off 5 places, we may reduce square links to acres, roods and poles.

Example.—Reduce 24987654 links to acres, roods and poles.

A.   249·87654
                    4
            —————
R.   3·50616
                  40

P.   20·24640

Answer: 249 acres, 3 roods, 20·24640 poles.

(**214**). *Yards to Acres, Roods and Poles.*—In reducing inches, feet, etc., to acres, the above does not, of course, apply, and we must first reduce to yards, and then divide by 4840 (the number of square yards in an acre) to bring the yards to acres. The remainder multiplied successively by 4 and by 40, and divided successively by 4840 will give roods and poles.

Example.—Convert 25498710ft. to acres, roods and perches.

$$9)25498710$$

$$484,0)283319,0(585$$
$$2420$$

$$4131$$
$$3872$$

$$2599$$
$$2420$$

$$179$$
$$4$$

$$484)716(1$$
$$484$$

$$232$$
$$40$$

$$484)9280(19$$
$$484$$

$$4440$$
$$4356$$

$$84$$

Answer: 585 acres, 1 rood, 19¼ perches.

THE COMPUTATION OF AREAS FROM PLANS WITH THE COMMON SCALE.

(**215**). *To Compute the Area of a Square. Rule.*—Multiply the side by itself (or square the side) and the product will be the area.
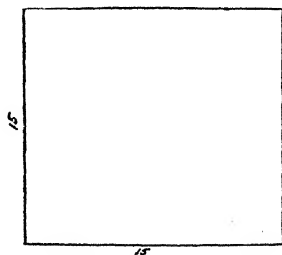
ILLUSTRATION No. 132.

Example: $15 \times 15 = 225$ft. area.

(**216**). *To Compute the Area of a Rectilineal Figure.*
Rule.—Multiply the length by the breadth and the
product will be the area.
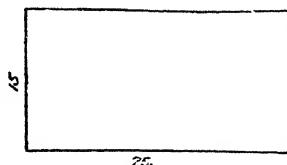
ILLUSTRATION No. 133.

Example: 25 × 15 = 375ft. area.

(**217**). *To Compute the Area of a Right Angled
Triangle.*   *Rule.*—Multiply the base by half the perpen-
dicular for the area, or multiply the base by the perpen-
dicular and take half the product for the area.
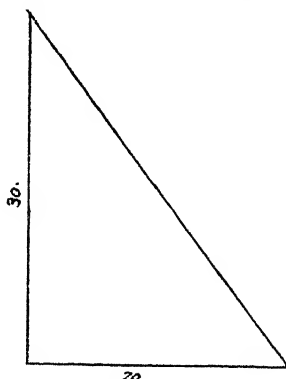
ILLUSTRATION No. 134.

Examples: (1) $\dfrac{20 \times 30}{2} = \dfrac{600}{2} = 300$ area.

(2) $30 \times 10 = 300$ area.

The latter rule is found most convenient in many
cases in practice, as will be seen hereafter.

(**218**). *To Compute the Area of any Triangle. Rule
No. 1.*—Multiply the longest side or base by half the

perpendicular height from it to the apex of the triangle, and the product will be the area; or multiply the base by the perpendicular height, and take half the product for the area.
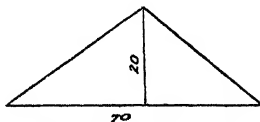
ILLUSTRATION No. 135.

Examples: (1) $70 \times 10 = 700$ area.

(2) $\dfrac{70 \times 20}{2} = 700$ area.

The latter rule is found more convenient in practice in many cases, as will be seen later on.

(**219**).—*To Compute the Area of a Trapezoid*, viz., a four-sided figure with two of its sides parallel and the other two not.

Rule.—Add the parallel sides, and multiply half their sum by the perpendicular height between them, and the product will be the area; or multiply the sum of the parallel sides by the perpendicular height between them, and half the product will be the area.
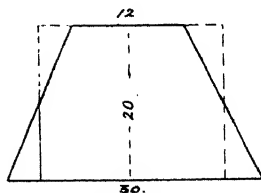
ILLUSTRATION No. 136.

Examples: (1) $\dfrac{30 + 12}{2} \times 20 = 21 \times 20 = 420$ area

(2) $\dfrac{(30 + 12) \times 20}{2} = \dfrac{840}{2} = 420$ area.

It will be seen that in taking the mean between the parallel sides and multiplying the result by the perpen-

dicular height, we really convert the trapezoid into a rectilineal figure of equal area, as indicated by the dotted lines, and then compute the area of that figure.

(**220**).—*To Compute the Area of a Rhombus*, viz., a four-sided figure having all its sides and each pair of its opposite angles equal.

Rule.—Multiply the side by the perpendicular height between the sides, and the product will be the area.
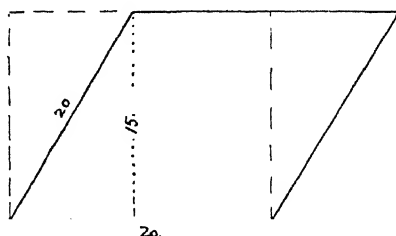
ILLUSTRATION No. 137.

Example: 20 × 15 = 300 area.

Here again it will be seen that in effect, by multiplying the base by the perpendicular height, we convert the rhombus into a rectilineal figure, as indicated by the dotted lines, and then compute the area of that rectilineal figure.

(**221**).—*To Compute the Area of a Rhomboid*, viz., a four-sided figure with its opposite sides and angles equal.

Rule.—Multiply the base by the perpendicular height between the sides, and the product will be the area.
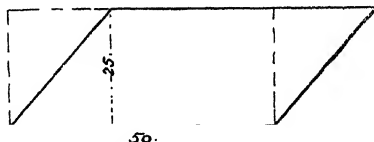
ILLUSTRATION No. 138.

Example: 50 × 25 = 1250 area.

The rule in this case is the same as that for the rhombus.

(**222**).—*To Compute the Area of a Trapezium,* viz., a four-sided figure, whose sides are not parallel.

Rule.—Measure the diagonal *BD* and the perpendiculars *AE, CF,* and multiply the diagonal by half the sum of the two perpendiculars for the area; or multiply the diagonal by the sum of the perpendiculars, and take half the product for the area.
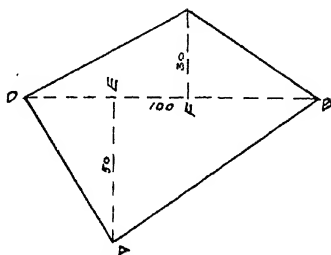
ILLUSTRATION No. 139.

Examples: (1) $100 \times \dfrac{30 + 50}{} = 100 \times 40 = 400$ area.

(2) $\dfrac{100 \times (30 + 50)}{2} = \dfrac{100 \times 80}{2} = 400$ area.

(**223**).—*To Compute the Area of a Figure bounded by any Number of straight Sides.*

The rules given in the foregoing simple examples are those by which the areas of the most complicated figures bounded by straight lines may be computed, as will be seen from the following rule and examples.

Rule.—Divide the figure up into triangles, trapezoids, trapeziums, etc., compute the areas of these figures, and add the results for the area of the whole figure.

Example No. 1.—The figure shown in Illustration No. 140 is divided up into three triangles, and their areas are computed and added for the area of the whole figure.

The base line has been multiplied by the whole perpendicular in each case, and the double area obtained divided by 2 to get the true area. This will be found the more convenient method when there are a number of

triangles, the areas of which have to be added to obtain
the area of a figure of which they form part.
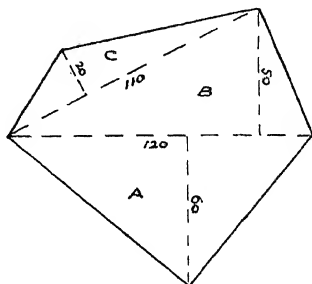
ILLUSTRATION No. 140.

Triangles A and $B = 120 \times (60 + 50) = 120 \times 110 = 13200$
Triangle $C = 110 \times 20$ $= $ 2200

Double areas   2)15400

Area of the whole figure   7700

The same base line has been made to serve for the two
triangles A and B, which, of course, reduces the calculations.

Example No. 2.—Illustrations Nos. 141 and 142 represent
respectively the correct and incorrect method of dividing
the figure for computation.

It will be seen that it might be divided up into five
separate triangles, each having a different base as shown in
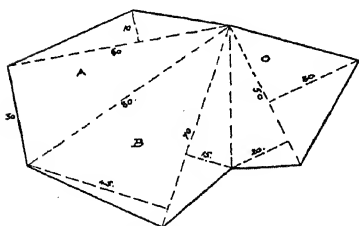
ILLUSTRATION No. 141.
Correct Method.
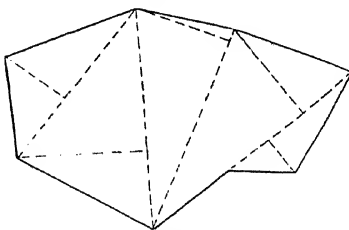
ILLUSTRATION No. 142.
Incorrect Method.

Illustration No. 142. This would involve five separate cal-
culations in obtaining the area, which is obviously undesir-
able, multiplying the labour and increasing the risk of error.

The same figure might be divided up, as shown in Illustration No. 141, into three trapeziums, or six triangles, each base serving for two, thus reducing the necessary number of calculations in computing the area to three instead of five as in the previous case.

The calculation will be as follows :

Double areas.

Trapezium *A* 60 × (10 + 30) = 2400
   ,,   *B* 70 × (15 + 45) = 4200
   ,,   *C* 50 × (30 + 20) = 2500
                  2)9100
Area of the whole figure = 4550

Example No. 3.—It not infrequently occurs that it is more expeditious to construct a figure which exceeds the area of the figure to be computed, and then to deduct the excess. Illustration No. 143 represents this case. Here it is more simple to form a trapezium round the ten-sided figure, and then deduct the two small triangles, than it
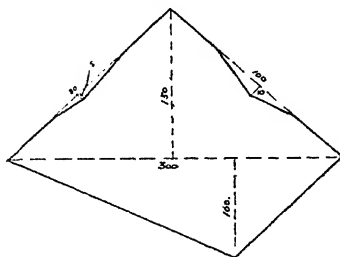
ILLUSTRATION No. 143.

would be to divide it up into the numerous small triangles, etc., which would otherwise be necessary to compute the contents.

The working is as follows :—

Double

Trapezium 300 × (150 + 100) = 75000
Deduct 100 × 10 = 1000
         80 × 15 = 1200
                ——   = 2200
                     2)72800
                      36400

Example No. 4.—Illustration No. 144 gives a form which is most simply divided into a trapezoid and three triangles, two of which have to be added to, and one
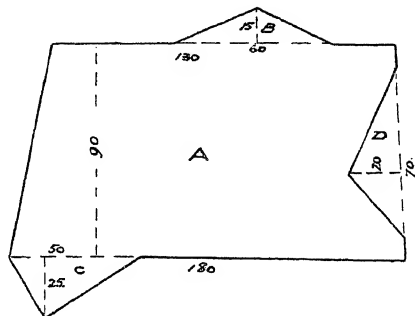
ILLUSTRATION No. 144.

deducted from the area of the trapezoid, to obtain the area of the whole figure.   The working is as follows :—

```
                                         Double
                                         areas.
        Trapezoid  A  (130 + 180)  × 90 = 27900
        Triangle   B  60 × 15 =   900
            ,,     C  50 × 25 = 1250
                                 ────
                                 2150
  Deduct Triangle D  70 × 20 = 1400
                                      =    750
                                         ──────
                                       2)28650
                                         ──────
                                         14325
```

The variety of forms to be dealt with is practically without limit, but it is believed that what has been said will be sufficient to explain the rules by which we should be guided in all cases.

METHOD TO BE FOLLOWED IN COMPUTING AREAS.

(**224**).   *Important Observations.*—What we have to aim at is (1) simplicity, and (2) accuracy, and these may best be obtained by (*a*) dividing the figure up in the most simple way, (*b*) reducing the number of calculations as far

as possible, (c) proceeding in a perfectly methodical manner, and (d) carefully checking the accuracy of all calculations.

When computing areas by scaling from maps, the best method of procedure is very accurately to trace off the outline of the piece of land the area of which is required, and to divide up the figure on that tracing, lettering or numbering the triangles, trapeziums, etc., as done in the foregoing examples, by which means mistakes as to the exact boundaries of the piece of land intended, and also the omission of some of the figures into which it has been divided for computation purposes, are avoided.

As a guarantee of accuracy in the " scaling off" and calculations, the area may be taken out twice independently of each other, and by a different method, and the results compared.

To guard against serious mistakes, it is a very good plan to take the figures in round numbers, so rendering the calculations very simple, and then to compare the result with that from more detailed calculations. Another safeguard is to compare the results of calculations and the relative sizes of figures as they may be judged by the eye on glancing at the plan.

THE COMPUTATION OF AREAS FROM PLANS—*Continued.*

Casting — Off-sets — Simpson's Rule — Reduction of Complicated to Simple Forms— Computing Scale— Planimeter.

IRREGULAR BOUNDARIES.

There are several methods of computing the areas of figures, the boundaries of which are not straight lines. The first of these, that which I am about to describe, is known as "casting."

(**225**).—*Casting* consists in laying down give and take lines, thus substituting straight for crooked boundaries, when the areas of the figures they enclose may be computed by the simple rules given in Chapter XVI.

Let us suppose that it is required to compute the area of the land represented by Illustration No. 145.
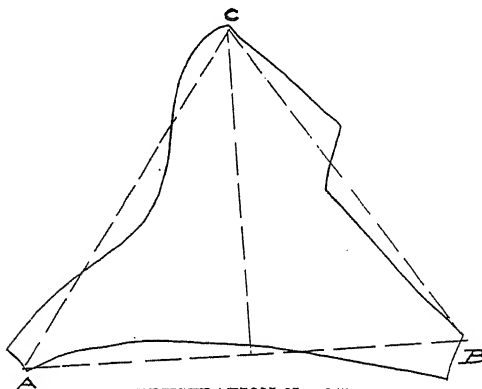
ILLUSTRATION No. 145

First reduce the irregular figure to a single triangle, as shown ·by the dotted give and take or "casting" lines,

which, it will be observed, exclude parts of the figure on one side and add an equal quantity on the other, so that the area of the triangle is equal to the area of the more complicated figure.

It would be found very difficult with the ordinary ruler to draw these equalising lines so as to exclude at one part a quantity exactly equal to that included at another, as to do this properly you need to see on both sides of the line at once, hence the following method.

Stretch a piece of thin silk or, what has been suggested to me is better, a hair from *A* to *B*, moving one end until the exact position for the equalising line is found ; make a pencil mark under the silk or hair, to denote its position, and then draw in the line with an ordinary ruler.

Casting is more properly applied to the straightening of lines which are nearly straight, that is to fences the undulations in which are not sufficiently large on the plan from which the figure is to be computed to require dealing with as " off-sets," or to figures the forms of which, as in the case of Illustration No. 145, specially lend themselves to being dealt with in that way.

With a little practice a remarkable amount of precision in the use of casting lines may be obtained.

(**226**). *Off-sets.*—In by far the greater number of cases to be dealt with in practice, the boundaries of land are not perfectly straight lines, but undulating, such as would necessitate the taking of off-sets or insets from the chain line in surveying the land with the object of preparing a plan thereof. In such cases the method of computing areas is the same as where the boundaries are straight, with this difference, that the areas of the off-sets or insets have to be calculated separately, and added to or subtracted from the area of the main figure, as the case may be.

Example No. 1.—Suppose it is required to compute the area of the land represented by Illustration No. 146.

Straight lines are drawn from point to point round the boundaries, constituting in this case a triangle, and the area of this triangle is computed as shown in Art. 218.

To the area thus found must be added the areas of all the off-sets on lines AB and AC and the area of the insets on line BC must be deducted.

When computing areas from a plan of land which you have surveyed, and of which you have therefore field notes, the lines of the figures into which the land is divided for
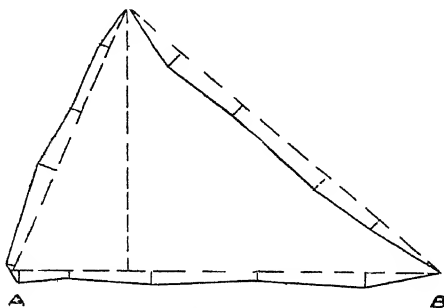
ILLUSTRATION No. 146.

computation purposes should be made coincident with the chain lines taken along the boundaries, so that the off-sets on each of the lines may be calculated from the dimensions given in the field notes.

In cases where the computation is being made from a plan, without reference to the field notes, lines may be drawn round the boundaries, and the off-sets at every break in the boundary, and their distances apart, scaled, so as to obtain practically the same information as would be given in the field notes, but this, of course, can only be done where the plan is drawn to a large scale. In other cases " casting " must be resorted to, or the area may be computed with the planimeter.

Example No. 2.—Let Illustration No. 147 represent a chain line with off-sets taken from it to the irregular boundary on the left at every point at which it changes direction, as would be done in an ordinary survey for a

0	900
60	850
30	700
70	580
60	430
90	320
50	250
70	150
0	000
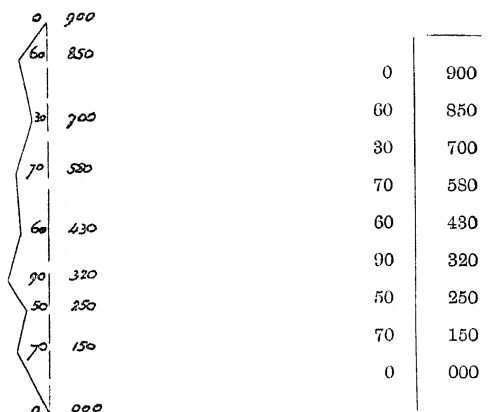
ILLUSTRATION No. 147.

plan. The figures given in the illustration represent, respectively, the distances along the chain line, and the lengths of the off-sets in links. The corresponding field notes are also given for the sake of clearness. Here we have two triangles and six trapezoids, and the calculation would be as follows :-

Double areas.

$$150 \times 70 = 10500$$
$$100 \times (50 + 70) = 12000$$
$$70 \times (50 + 90) = 9800$$
$$110 \times (60 + 90) = 16500$$
$$150 \times (60 + 70) = 19500$$
$$120 \times (30 + 70) = 12000$$
$$150 \times (60 + 30) = 13500$$
$$50 \times 60 = 3000$$

$$2)96800$$

Area = 48400 links.

It will be seen that the area of the off-sets may be calculated direct from the field notes, without reference to the

plan, and, therefore, provided the chain lines round the
boundaries are utilised for the figures into which the land
is divided for computation purposes, as they should be, the
trouble of scaling the numerous off-sets will be avoided,
and, what is more important, greater accuracy obtained.

Example No. 3.—Required the area of the land repre-
sented by Illustration No. 148. The outer lines into which
the figure is divided for computation purposes and the chain
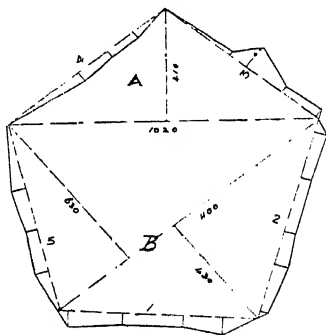
ILLUSTRATION No. 148.

lines round the boundaries have been made coincident, so
that the off-sets may be accurately calculated from actual
measurements.

It will be noticed that line 4 is outside the figure, and
the insets thereon must therefore be deducted.

The triangle A and the trapezium B are first dealt with,
their areas calculated and entered in the second column,
where they are added, their sum being carried forward to
the extreme right-hand column.

The off-sets on lines 1, 2, 3 and 5 are then calculated
and entered in the first column, the result of each line
being carried to the second column, where they are added
together.

The insets on line 4 are then computed, the results

being entered in the first column and their total carried to the second column, where it is deducted from the total off-sets. The total area of off-sets, minus insets, is then carried to the right-hand column and added to the area of the main figures. The resulting sum, which represents double areas, is then divided by 2 and reduced to acres, roods and poles as explained in Art. 213.

Skeleton field notes relating to lines 1, 2, 3, 4 and 5 are given below, and the workings on the following page :—

End of line 2.			End of line 4.
750	0		
700	45		
580	35	0	680
330	40	40	370
210	50	25	250
030	30	30	140
000	0	0	000
Line No. 2.			Line No. 4.

End of line 1.		End of line 3.		End of line 5.	
650	0	640	0		
535	65	370	15	700	
410	40	300	90	550	50
210	40	160	25	390	40
025	60	025	30	240	50
000	0	000	0	000	0
Line No. 1.		Line No. 3.		Line No. 5.	

Calculations Relating to Example 3, Illustration 148.

<table>
<tr><td></td><td></td><td></td><td>Double
areas.</td></tr>
<tr><td colspan="3">Triangle A 1020 × 410</td><td>418200</td></tr>
<tr><td colspan="3">Trapezium B 1100 × (630 + 430)</td><td>1166000</td></tr>
<tr><td></td><td colspan="2" align="center">Main Figure</td><td>1584200</td></tr>
</table>

	Off-sets on line 1.	Double Areas.	
Add	25 × 60	= 1500	
	185 × 60 + 40	= 18500	
	200 × 40 + 40	= 16000	
	125 × 40 + 65	= 13125	
	115 × 65	= 7475	
			56600

	Off-sets on line 2.	Double Areas.	
	30 × 30	= 900	
	180 × (50 + 30)	= 14400	
	120 × (50 + 40)	= 10800	
	250 × (40 + 35)	= 18750	
	120 × (35 + 45)	= 9600	
	50 × 45	= 2250	
			56700

	Off-sets on line 3.		
	25 × 30	= 750	
	135 × (25 + 30)	= 7425	
	140 × (25 + 90)	= 16100	
	70 × (90 + 15)	= 7350	
	270 × 15	= 4050	
			35675

	Off-sets on line 5.		
	240 × 50	= 12000	
	150 × (50 + 40)	= 13500	
	160 × (40 + 50)	= 14400	
	150 × 50	= 7500	
			47400

			196375
Deduct	In-sets on line 4.		
	140 × 30	= 4200	
	110 × (30 + 25)	= 6050	
	120 × (25 + 40)	= 7800	
	310 × 40	= 12400	
		30450	
			165925

2)1750125
8·75062
4
3·00248
40
0·09920

Area = 8 acres 3 roods

Example No. 4.—Let Illustration No. 149 represent a wood which. has been surveyed by chain angles, and the
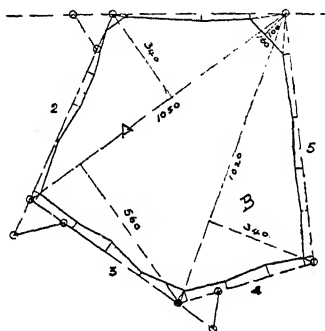
ILLUSTRATION No. 149.

area of which is required. Skeleton field notes of lines 1, 2, 3, 4 and 5 are given on the following page, from which the off-sets may be calculated. Here we first compute the area of the five-sided figure formed by the lines circumscribing the wood, and then deduct the area represented by the insets and the small triangles at the commencement and end of line 1.

Field Notes and Workings Relating to Example 4, Illustration 149.

Line No. 2 — End of line 2.

Offset	Distance	Station
	800	station 6
	670	station 5
25	640	
10	455	
28	325	
28	210	
	125	station 4
	050	
	000	station 2

Line No. 4 — End of line 4.

Offset	Distance	Station
	480	station 11
25	450	
40	325	
30	175	
	140	station 10
35	030	
	000	station 8

Line No. 3 — End of line 3.

Offset	Distance	Station
	740	station 9
	600	station 8
50	590	
15	440	
40	300	
25	170	
	140	station 7
35	020	
	000	station 5

Line No. 1 — End of line 1.

Offset	Distance	Station
	710	station 3
	595	station 2
10	540	
25	290	
20	120	
	000	station 1

Line No. 5 — End of line 5.

Distance	Offset	Station
820		station 1
690	20	
480	18	
315	25	
165	20	
020	30	
000		station 11

Double Areas.

Trapezium A 1050 × (340 + 560)   945000
Triangle B 1020 × 340   346800

Main figure   ..   1291800

Insets on line 1.   Double Areas.
120 × 20   =   2400
170 × (20 + 25)   =   7650
250 × (25 + 10)   =   8750
55 × 10   =   550
———— 19350

Insets on line 2.
160 × 28   =   4480
115 × (28 + 28)   =   6440
130 × (28 + 10)   =   4940
185 × (10 + 25)   =   6475
30 × 25   =   750
———— 23085

Insets on line 3.
20 × 35   =   700
150 × (35 + 25)   =   9000
130 × (25 + 40)   =   8450
140 × (40 + 15)   =   7700
150 × (15 + 50)   =   9750
10 × 50   =   500
———— 36100

Insets on line 4.
30 × 35   =   1050
145 × (35 + 30)   =   9425
150 × (30 + 40)   =   10500
125 × (40 + 25)   =   8125
30 × 25   =   750
———— 29850

Insets on line 5.
20 × 30   =   600
145 × (30 + 20)   =   7250
150 × (20 + 25)   =   6750
165 × (25 + 18)   =   7095
210 × (18 + 20)   =   7980
130 × 20   =   2600
———— 32275

Triangles.
80 × 30   =   2400
100 × 100   =   10000
———— 18400
——— 159060

2)1132740
5·66370
4
————
2·65480
40

Area $=$ 5 acres 2 roods 26 poles.   26·19200

(**227**). *Simpson's Rule.*—Simpson's Rule gives a method
of finding the area of a figure bounded by a curved line by
means of off-sets taken at regular distances.

Rule.—Along the chain line measure off any *even* number
of *equal* distances (say one chain) and take the length of
the off-set from the chain line to the boundary at each
point.   Then to the sum of the first and last off-sets add
four times all the even off-sets, and twice all the odd off-
sets (not including the first and last), multiply the result
by the common distance between the off-sets, and take one
third of the product for the area.
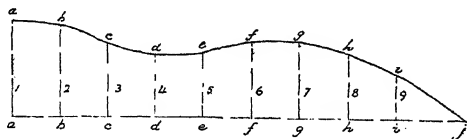
ILLUSTRATION No. 150.

Example No. 1.—Suppose it is required to compute the
area of the piece of land represented by Illustration No. 150.
Along the line *a j* measure off an even number of equal
distances, and at each of these points measure the perpen-
diculars *aa, bb, cc, dd, ee, ff, gg, hh, ii.*

To the sum of the first and last off-sets, Nos. 1 and 9,
add four times the sum of all the even off-sets, viz., Nos. 2,
4, 6 and 8, and twice the sum of all the odd off-sets, not in-
cluding the first and last, viz , Nos. 3, 5 and 7 ; multiply
the result by the common distance between the off-sets,
and take one third of the product for the area.

Referring again to Illustration No. 150, it will be seen
that eight equal parts have been set off along the line from
a to *i*.   The ninth part, a triangle, must be dealt with
separately, its area being computed and added to the area
found by the rule.

Of course, the line might have been divided up into some

even number of equal parts, when there would have been no piece over to calculate separately; but it is more simple in practice to mark off a number of equal distances, and deal with any remainder separately, than equally to divide up the line.

Example No. 2.—Let Illustration No. 151 represent a piece of land the area of which is required to be computed by Simpson's rule. First we measure off a common distance on the line AB from A, in this case 100 links. At each of the points, we then measure the perpendicular from
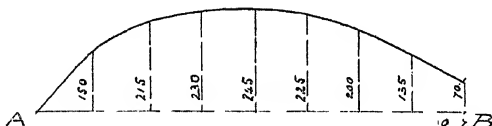
ILLUSTRATION No. 151.

the line AB to the curved fence. In this instance the line is divided into an even number of equal parts, and therefore there will not be any piece over at the end to calculate separately, as would otherwise be the case.

The calculation of the area by Simpson's rule will be as follows :—

Sum of the first and last off-sets	=	70
Four times all the even off-sets = 4 (150 + 230 + 225 + 185)	=	2960
Twice all the odd off-sets (not including the first and last) = 2 (215 + 245 + 200) ..	=	1320
		4350
Common distance		100
One third		3)435000
		1·45000
		4
		1·80000
		40
Area = 1 acre 1 rood 32 pole		32·00000

Example No. 3.—Illustration No. 152 represents a case

ILLUSTRATION No. 152.

involving a combination of the rules already given. The
workings are as follows :

Triangle *A* and *B* 1300 × (460 + 580)   2)1352000

Add (by Simpson's rule)   676000

      4 (60 + 90 + 50)   =   800

      2 (80 + 80)   =   320

                                   1120

      Common distance   100

      One third   112000 ÷ 3 =   37333

                                    713333

        Deduct.   Double Areas.

      70 × 65   =   4550

      105 × (80 + 65)   =   15225

      100 × (80 + 70)   =   15000

      103 × (70 + 35)   =   10815

      80 × 35   =   2800

                    48390 ÷ 2 =   24195

                               6·89138

                                     4

                               3·56552

                                     40

Area = 6 acres 3 roods 22·6 poles.   22·62080

(**228**). *Reduction of Complicated to Simple Forms of Equal Area.*—It is shown by a well-known proposition of Euclid that triangles of equal base and perpendicular height are equal in area. This furnishes us with a means whereby we may change the form of a triangle without altering its superfices. For example, the triangles *ABC*, *ABD*, *ABE*, Illustration No. 153, differ in form but are equal in area, they being on the same base, and also of equal perpendicular height, or between the same parallel lines.
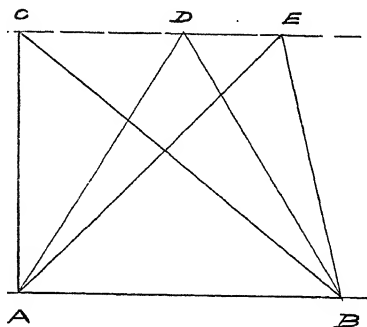
ILLUSTRATION No. 153.

If, therefore, we have a figure of several sides, we may reduce it to a single triangle of equal area by first dividing it successively into a series of triangles, and then so changing their forms as to cause one of their sides to coincide with a given line, so reducing the figure by one side at each step until it is ultimately reduced to a single triangle. The method of doing this, a description of which follows, is applicable to cases in which "casting," dealt with in Chapter XVII. (Art. 225), might be employed, viz., the reduction of irregular boundaries to straight lines; but properly speaking, casting should be confined to the straightening of merely undulating boundaries, whilst the method under consideration is more particularly applicable

to the reduction of a many-sided figure to one of more simple form.

We can easily conceive a case where both methods might be applied with advantage.

Example No. 1.—In a case such as that indicated by Illustration 154, we might first straighten the undulating

ILLUSTRATION No 154.

boundaries by casting, as indicated by the fine lines, and then reduce the five-sided figure to a simple right-angled triangle of equal area.

The method of reducing is as follows :

Rule.—Draw a line of indefinite length perpendicular to the longest side of the figure, *A B* in Illustration 154, and number the angles as therein shown. Now using the parallel ruler or set-squares, make a mark on *A B* at the point *a*, at which it would be cut by a line parallel with 1—3 passing through angle 2 ; similarly, make a mark on *A B* at *b*, the point at which it would be cut by a line parallel with *a* 4 passing through angle 3 ; again, make a third mark on *A B* at the point *c*, at which the perpendicu-

lar would be cut by a line parallel with b 5 passing through angle 4; finally, draw a line from c to 5, and the right-angled triangle, c A 5, obtained will be equal in area to the original five-sided figure.

The process is the same for a figure of any number of sides.

The following gives in a simple form the steps to be taken.

(1)   Erect the perpendicular.

(2)   Number the angles.

(3)   Lay the parallel ruler from 1 to 3, slip the free arm to 2 and make mark a on perpendicular; lay the parallel ruler from a to 4, slip the free arm to 3 and make mark b on perpendicular; lay the parallel ruler from b to 5, slip the free arm to 4 and make mark c on perpendicular; lay the parallel ruler from c to 6, slip the free arm to 5 and make mark e on perpendicular—and so on for any number of angles.

Curves may be dealt with in the same way by dividing them into a number of parts and numbering the points as angles.
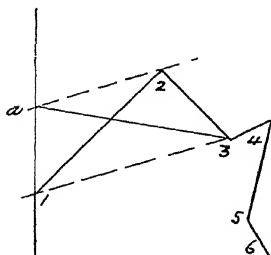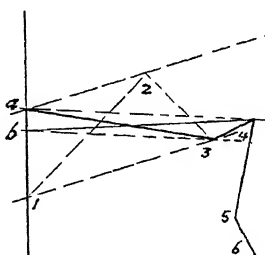
ILLUSTRATION No. 155.   ILLUSTRATION No. 156.

Example No. 2.—Required to reduce the seven-sided figure shown in Illustration No. 155 to a right-angled triangle.

In this case one of the sides chances to fall within the perpendicular, and the angles are therefore numbered as shown in illustration. We lay the ruler from 1 to 3, slip the free arm to 2 and mark a on the perpendicular, and are thus able to substitute triangle a 1 3 for triangle 1 2 3, and the figure is reduced by one side and becomes of the form shown in thick lines in Illustration No. 156. Similarly, we
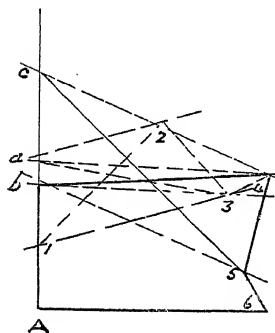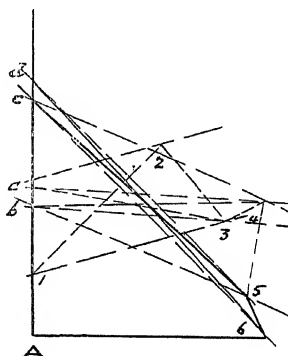
ILLUSTRATION No. 157.   ILLUSTRATION No. 158.

lay the ruler from a to 4, slip to 3 and mark b, and are thus enabled to substitute triangle b 3 4 for a 3 4, and the figure is further reduced and becomes of the form shown by thick lines in Illustration No. 157. Again we lay the ruler from b to 5, slip to 4 and mark c, and substitute triangle c b 5 for b 4 5, and the figure becomes of the form shown by thick lines in Illustration No. 158. Once more, we lay the ruler from c to 6, slip to 5 and mark d, and substitute triangle d 6 c for c 5 6. Finally, the line d 6 is drawn in and the right-angled triangle A 6 d is equal in area to the original seven-sided figure.

In the illustrations, for the sake of clearness, I have drawn in the lines, which is not desirable in practice.

Example No. 3.—This example shows the application of the rule to a figure bounded on one side by a curve, but it

does not seem necessary to do more than just give the steps and Illustration No. 159, the process being the same a in
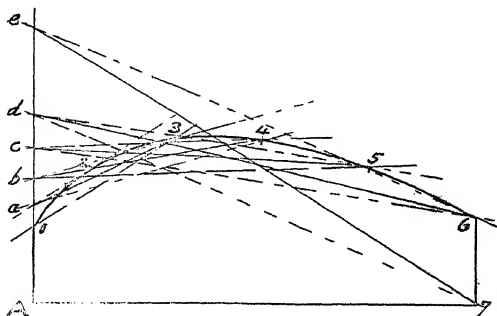
ILLUSTRATION No. 159.

the case of figures bounded by straight lines, with the exception that the curve has to be divided, the points of division taking the place of angles in a figure bounded by straight lines.   The steps are :—

```
From 1 to 3 slip to 2 mark a
  ,,    a ,, 4   ,,    3   ,,  b
  ,,    b ,, 5   ,,    4   ,,  c
  ,,    c ,, 6   ,,    5   ,,  d
  ,,    d ,, 7   ,,    6   ,,  e
```

Join e and 7, and the right-angled triangle A 7 e is equal in area to the original figure bounded on one side by a curved line.

COMPUTATION OF AREAS WITH THE COMPUTING SCALE.

(**229**).—This instrument and the method of using it has already been explained in Chapter X., under the head of "Instruments," and no further reference to it need here be made (see Arts. 128 and 129).

The computation of areas from plans by several different methods usually form the subject of a compulsory question in the Surveyors' Institution Professional Associateship examination, and examinees are offered the loan of a computing scale for the purpose : it is all the better to be in a position to accept the invitation.

COMPUTATION OF AREAS WITH THE PLANIMETER.

(**230**).—The planimeter and the method of using it have likewise already been described in Chapter X. (see Arts. 130, 131 and 132).

CHAPTER XVIII.

THE COMPUTATION OF AREAS—*Continued*.

Method of ascertaining the True Area from a Given Erroneous Area, Computed from a Plan by a Different Scale to that to which the Plan has been drawn, etc., etc.—Reduction of Customary to Statute and Statute to Customary Measure.

It sometimes happens that the area of a piece of land is computed from a plan by a scale other than that to which the plan has been plotted, consequently the area obtained is erroneous, and either in excess or defect of the true area.

There is in all cases, however, a ratio existing between the true area, the erroneous area, the square of the scale used in plotting the plan, and the square of the scale used in computing the erroneous area, and therefore having given any three of these terms we may discover the fourth.

The four cases which may present themselves are :—

	GIVEN.	REQUIRED.
(*a*)	1. The true area ; 2. Any erroneous area computed ; 3. The scale used in computing the given erroneous area.	The true scale, viz.: the scale to which the plan has been plotted.
(*b*)	1. The scale to which the plan has been plotted ; 2. The scale used in computing the area ascertained ; 3. The area ascertained.	The true area.
(*c*)	1. The scale to which the plan has been plotted ; 2. The true area ; 3. Any other given scale.	The erroneous area which would be found if the given scale were used.
(*d*)	1. The scale to which the plan has been plotted ; 2. The true area ; 3. Any given erroneous area.	The scale used in computing the given erroneous area.

Therefore, if we have the true area, the erroneous area, and the scale used in computing it, we can find the scale

to which the plan has been plotted; and, conversely, if we have given the scale to which the plan was drawn, the scale used in computing the area, and the erroneous area obtained, we may find the true area. These are the two cases most likely to occur in practice.

Likewise, having given the scale to which a plan has been plotted, and the area of the land, we may ascertain what area would be found from dimensions taken off by any given scale; or, again, having given the scale to which the plan has been plotted, and the true area, ascertain what scale must have been made use of in taking off the dimensions which give a certain erroneous area.

Questions coming under one or other of the heads above named are very common in the examinations in land surveying, and candidates should be prepared to answer them.

The rules applicable to each of the foregoing cases are as follows :—

RULES.

(**231**). *Case* (*a*).—(1) When the true area is greater than the erroneous area.—The erroneous area is to the true area as the square of the scale used is to the square of the scale to which the plan has been plotted.

(2) When the erroneous area is greater than the true area.—The erroneous area is to the true area as the square of the scale used is to the square of the scale to which the plan has been plotted.

(**232**). *Case* (*b*).—(1) When the scale used in computing the area is smaller than the scale to which the plan has been plotted.—The square of the scale used in computing the area is to the square of the scale to which the plan is plotted as the area ascertained is to the true area.

(2) When the scale used in computing the area is larger than the scale to which the plan

has been plotted.—The square of the scale used in computing the area is to the square of the scale to which the plan is plotted as the area ascertained is to the true area.

(**233**). *Case* (*c*).—(1) When the given scale is smaller than the scale to which the plan has been plotted.—The square of the scale to which the plan has been plotted is to the square of the given scale as the correct area is to the area which would be found if the given scale were used.

(2) When the given scale is larger than the scale to which the plan has been plotted.—The square of the scale to which the plan has been plotted is to the square of the given scale as the correct area is to the area which would be obtained if the given scale were used.

(**234**). *Case* (*d*).—(1) When the true area is greater than the given erroneous area.—The true area is to the given erroneous area as the square of the true scale is to the square of the erroneous scale assumed to have been used.

(2) When the true area is smaller than the given erroneous area.—The true area is to the given erroneous area as the square of the true scale is to the square of the erroneous scale assumed to have been used.

It will be noticed that the rules in 1 and 2 of each case are identical, but they are repeated for the sake of clearness.

A little confusion is sometimes experienced by students owing to the fact that the smaller scales are represented by the higher numbers, and that the smaller the scale used the greater will be the area found.

I will now give an example in each of the eight cases, supposing the same scales and areas in each case, so that the truth of the rules may be evident.

Example No. 1.—The area of a certain piece of land is

ten acres.   From the plan of it I computed the area with
a scale of one chain to an inch, and the result was $2\frac{1}{2}$ acres.
To what scale is the plan plotted ?

$$2 \cdot 5 \; : \; 10 \; :: \; 1^2 \; : \; \text{square of scale sought.}$$
$$2 \cdot 5 \; : \; 10 \; :: \; 1 \; :$$
$$= \quad \frac{10 \times 1}{2 \cdot 5} \quad = \quad 4$$

and the square root of 4 equals 2.   The scale to which the
plan is drawn is two chains to an inch.

Example No. 2.—The area of a certain piece of land is
ten acres.   From the plan of it I computed the area, using
a four chain scale, and the result was 40 acres.   What is
the scale of the plan ?

$$40 \; : \; 10 \; :: \; 4^2 \; : \; \text{square of scale sought.}$$
$$40 \; : \; 10 \; :: \; 16 \; :$$
$$\frac{10 \times 16}{40} \quad = \; 4$$

and the square root of 4 equals 2.   The scale to which the
plan is plotted is two chains to an inch.

Example No. 3.—From a plan which was plotted to a
scale of two chains to an inch I computed the area with a
scale of four chains to an inch, and the area resulting was
40 acres.   What is the true area ?

$$4^2 \; : \; 2^2 \; :: \; 40 \; : \; \text{true area}$$
$$16 \; : \; 4 \; :: \; 40 \; :$$
$$\frac{4 \times 40}{16} \quad = \quad 10$$

The true area is ten acres.

Example No. 4.—From a plan which was plotted to a
scale of two chains to an inch I computed the area, using a
scale of one chain to an inch, and the resulting area was
2a. 2r.   What is the true area ?

$$1^2 \; : \; 2^2 \; :: \; 2 \cdot 5 \; : \; \text{true area}$$
$$1 \; : \; 4 \; :: \; 2 \cdot 5 \; :$$
$$= \quad \frac{4 \times 2 \cdot 5}{1} \quad = \quad 10$$

The true area is ten acres.

Example No. 5.—I have a plan drawn to a scale of two
chains to an inch, and the area of the land it represents is

ten acres. What area should I obtain if I used a scale of four chains to an inch?

$$2^2 \ : \ 4^2 \ :: \ 10 \ : \ \text{area which would be obtained}$$
$$4 \ : \ 16 :: \ 10 \ :$$
$$= \ \frac{16 \ \times \ 10}{4} \ = \ 40$$

The area which would be obtained is 40 acres.

Example No. 6.—I have a plan drawn to a scale of two chains to an inch, and the area of the land it represents is ten acres. What area should I obtain if I used a scale of one chain to an inch?

$$2^2 \ : \ 1^2 \ :: \ 10 \ : \ \text{area which would be obtained with 1 chain scale}$$
$$4 \ : \ 1 \ :: \ 10 \ :$$
$$= \ \frac{1 \ \times \ 10}{4} \ = \ 2\tfrac{1}{2}$$

Therefore $2\tfrac{1}{2}$ acres is the area which would be obtained if the scale of one chain to an inch were used.

Example No. 7.—I have a plan plotted to a scale of two chains to an inch, the true area it represents is ten acres. I have computed the area and make it $2\tfrac{1}{2}$ acres. What scale must I have used?

$$10 \ : \ 2\tfrac{1}{2} \ :: \ 2^2 \ : \ \text{scale used}$$
$$10 \ : \ 2{\cdot}5 :: \ 4 \ :$$
$$\therefore \ \frac{2{\cdot}5 \ \times \ 4}{10} \ = \ \frac{10}{10} \ = \ 1$$

and therefore the scale used must have been a scale of one chain to an inch.

Example No. 8.—I have a plan plotted to a scale of two chains to one inch. The true area of the land it represents is ten acres. I have computed the area and make it 40 acres. What scale must I have used?

$$10 \ : \ 40 \ :: \ 2^2 \quad \text{scale used}$$
$$10 \ : \ 40 \ :: \ 4$$
$$\frac{40 \ \times \ 4}{10} \ = \ 16 \text{ and } \sqrt{16} \ = \ 4$$

and therefore the scale used must have been a scale of four chains to one inch.

Customary and Statute Measure.

REDUCTION OF CUSTOMARY TO STATUTE AND STATUTE TO CUSTOMARY MEASURE.

Originally land measure differed in various parts of England, and was governed by the custom of the particular locality in which the land was situate, hence the term " Customary Measure."

Now, however, land measure for the whole of England is governed by Act of Parliament; hence the term " Statute Measure " (see Appendix).

Consequently, in old documents, we occasionally come across plans, etc., giving the area of lands in customary acres, roods and perches, and it is therefore necessary to reduce customary to statute, and sometimes statute to customary measure.

The same applies also in some cases when dealing with land in Scotland or Ireland, or other parts of the world where the land measure differs from our statute measure.

I shall now give the rules for converting customary to statute, and statute to customary measure; having given the number of square feet in the customary perch, or the number of square yards in the customary acre.

WHEN THE NUMBER OF FEET IN A CUSTOMARY PERCH IS GIVEN.

(**235**). *Rule No. 1.*—Reduce the customary quantity to square feet, and divide by 272·25, the number of square feet in a statute perch, and again divide the quotient by 160, the square perches in an acre, for acres, and the remainder multiplied successively by 4 and by 40, and divided successively by 160, will give the roods and perches statute measure.

(**236**). *Rule No. 2.*—Reduce the area (customary measure) to acres and decimals of an acre, multiply by the number of square yards in a customary acre, and divide by 4840, the square yards in a statute acre, and reduce the decimals to roods and perches by multiplying successively by 4 and by 40.

I shall now give worked examples showing the application of each of the foregoing rules.

It will be seen that in reducing customary to statute measure, the quantity given is first reduced to some unit which is identical in both, and that the result is divided by the number of those units contained in the statute acre, rood, or perch.

Thus, referring to Example No. 1 on the following page, the customary measure has first been reduced to square feet, which are alike in both the customary and statute measure, and then divided by 272·25, the square feet in a statute perch, and again by 160 to reduce the statute perches to statute acres.

The example has been worked in this way as it seemed to set forth the rule more clearly in that form, but the workings would have been shorter if the square feet had been divided by 9 to reduce them to square yards, and again by 4840 to bring the square yards to acres, the remainder being reduced to roods and perches by multiplying successively by 4 and by 40, and dividing by 4840.

In other examples throughout the work, brevity has sometimes been sacrificed for the sake of clearness, and the advantage of following a possibly shorter course in individual instances has not been taken when the same would not be applicable to cases in general.

Example.—By Rule No. 1 :—

Reduce 16a. 1r. 10p. customary measure, when the customary perch is 15ft., to statute measure.

```
                                    A.  R.  P.
Area, customary  measure   =   16   1   10
            Roods in an acre        4
                                   ———
         Customary roods           65
           Perches in a rood       40
                                   ———
        Customary perches        2610
Sq. ft. in customary perch        225
         (15 × 15)
                             ·  13050
                                5220
                                5220
                                ————
Sq. ft. in statute perch 272·25)587250·00(2157 statute perches
                                54450
                                ————
                                42750
                                27225
                                ————
                               155250
                               136125
                               ————
                               191250
                               190575
                               ————
                                  675
```

```
Square perches in an acre 160)2157(13 acres
                              160
                              ———
                              557
                              480
                              ———
                               77
                                4
                              ———
                          160)308(1  rood
                              160
                              ———
                              148
                               40
                              ———
                          160)5920(37 perches
                              480
                              ———
                             1120
                             1120
```

Area, statute measure, 13 acres, 1 rood, 37 poles.

Example.—By Rule No. 2 :—

Reduce to statute measure 16a. 1r. 10p., when the customary acre contains 36,000 square feet.

                          A. R. P.
  Area, customary measure 16 1 10

                        $= 16$ and $\dfrac{50}{160}$ acres

                        $=$ 16·3125 acres
  Yards in customary acre   4000

  Yards in statute acre   484,0)6525,00000(13·4814
                               484
                               ———
                               1685
                               1452

                                2330
                                1936
                                ———
                                 3940
                                 3872

                                  680
                                  484
                                  ———
                                  1960
                                  1936
                                  ———
                                   240

                     13·4814
                           4
                     ———————
                     1·9256
                        40

                     37·0240

Area, statute measure, 13a. 1r. 37p.

To reduce statute to customary measure the process is, of course, precisely similar, only the multipliers and divisors are transposed.

CHAPTER XIX.

THE METHOD OF LAYING OUT LAND OF GIVEN AREA IN ANY REQUIRED FORM.

General Observations—To find the Side of a Square of Given Area—To find the Side of a Parallelogram, the Area and Other Side being given—To find the Perpendicular of a Triangle, the Area and Base being given—To find the Base of a Triangle when the Area and Perpendicular Height are given—To find the Diameter of a Circle when the Area is given—To find One of the Diameters of an Ellipse, the Area and Other Diameter being given—To find the Foci of an Ellipse, the Transverse and Conjugate Diameters being given—To find the Sides of a Rectilineal Figure of Given Area, when the Sides are to bear a Given Proportion to each other—Work in the Field—To lay out Land in the Form of a Square—Parallelogram—Triangle—Circle—Ellipse.

It would seem convenient in some respects to divide this portion of our subject into two parts as follows :—

> (1) The rules for calculating the required parts from the particulars given ;
> (2) The method of constructing the figures in the field ;

and I will therefore deal with the subject in that order.

A knowledge of euclid, geometry and mensuration is what we need in the first place, and an acquaintance with the methods of procedure in the field, in the second.

The subject, however, is very simple, and will offer no difficulty to anyone who has mastered the chapters on "The Computation of Areas."

I have said there are two steps to be taken; first to calculate the dimensions, and secondly to lay out those dimensions in the field. Let me illustrate this :—

Suppose it is required to lay out a given quantity of land in the form of a square. We must ascertain the length of the side of a square which will contain the required area and then lay out the sides at right angles to each other.

Similarly, if we wish to lay out a circle of given area, we must first find the radius of a circle of that area; a triangle, its base and perpendicular; a parallelogram, its two sides; a polygon, its radius and side; or a figure having its sides in a given proportion to each other, then the lengths of those sides, and so on.

Most of the rules and methods are extremely simple, but for the sake of completeness I will give them here.

RULES FOR ASCERTAINING THE REQUIRED PARTS FROM THE GIVEN AREA.

TO FIND THE SIDE OF A SQUARE OF GIVEN AREA.

(**237**). *Rule.*—Extract the square root of the given area in square links for the side in links.

Rectangle

TO FIND ONE SIDE OF A PARALLELOGRAM, THE AREA AND THE OTHER SIDE BEING GIVEN.

(**238**). *Rule.*—Divide the given area in square links by the given side in links, and the quotient will be the side sought, in links.

TO FIND THE PERPENDICULAR OF A TRIANGLE, THE AREA AND BASE BEING GIVEN.

(**239**). *Rule.*—Divide the area in square links by half the base in links, and the quotient will be the perpendicular in links.

TO FIND THE BASE OF A TRIANGLE WHEN THE AREA AND THE PERPENDICULAR HEIGHT ARE GIVEN.

(**240**). *Rule.*—Divide the area in square links by half the perpendicular height in links, and the quotient will be the base in links.

TO FIND THE DIAMETER OF A CIRCLE WHEN THE AREA IS GIVEN.

(**241**). *Rule.*—Divide the area in square links by ·7854, and extract the square root of the quotient for the diameter in links.

TO FIND ONE OF THE DIAMETERS OF AN ELLIPSE, THE AREA AND THE OTHER DIAMETER BEING GIVEN.

(**242**). *Rule.*—Divide the area in square links by the given diameter in links, multiply by ·7854, and the quotient will be the diameter sought, in links.

TO FIND THE FOCI OF AN ELLIPSE, THE TRANSVERSE AND CONJUGATE DIAMETERS BEING GIVEN.

(**243**). *Rule.*—From the square of the semi-transverse diameter subtract the square of the semi-conjugate diameter, and the square root of the quotient will be the distance of the foci from the centre or intersection of the diameters.

TO FIND THE SIDE OF A RECTILINEAL FIGURE OF GIVEN
AREA WHEN THE SIDES ARE TO BEAR A GIVEN PROPOR-
TION TO EACH OTHER.

(**244**). *Rule.*—Divide the area in links by the product
of the terms of proportion, and multiply the square root
of the quotient separately by the terms of proportion
for the length and breadth respectively.

An illustration here may be useful to elucidate the
rule. Let *A B C D*, Illustration No. 160, represent a
piece of land containing twenty acres, which has been
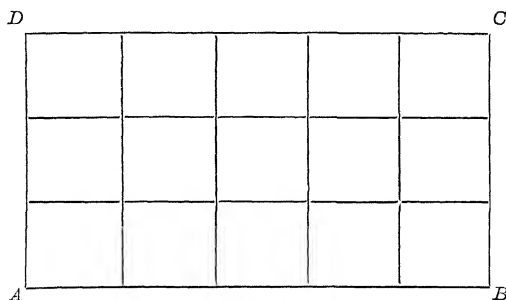
ILLUSTRATION No. 160.

laid out in a rectangle having its sides in the proportion
as 3 is to 5, by the foregoing rule.

Multiplying together the terms of proportion, 3 and 5,
we obtain 15, and dividing the area by 15 we obtain
the area of one of the fifteen parts into which the whole
figure is divided, which are, of course, squares.

Now, if we extract the square root of one of these
squares, we get the length of its sides, which being
multiplied by 3 gives the shorter side *A D* or *B C*, and
multiplied by 5 gives the longer side *A B* or *C D*.

WORK IN THE FIELD.

TO LAY OUT LAND IN THE FORM OF A SQUARE.

(**245**).—Carefully chain a straight line in the required position for one of the sides, and drive down pegs at each end. By aid of the optical square, box sextant, or theodolite, have pickets put down in the direction the other two corners of the field are to occupy, so that lines run from them to the opposite corners already fixed will be at right angles with the first line. Chain the two sides, the directions of which you have thus established, each exactly equal to the first, and drive down pegs at each of the corners, which will give the positions for the corner posts of the fences.

TO LAY OUT LAND IN THE FORM OF A ~~PARALLELOGRAM~~ Rectangle.

(**246**).—The method of procedure here is obviously similar to that described in the case of a square, the only difference being that the sides at right angles to each other are not of equal length.

TO LAY OUT LAND IN THE FORM OF A TRIANGLE.

(**247**).—Carefully chain the base line and put down pegs at each of its extremities. With the aid of the optical square or other instrument, erect the perpendicular at the desired position on the base line, carefully chain off the proper length on this perpendicular, and drive down a peg to fix the position for the third corner post of the fences.

Where the perpendicular is required to cut a given point in an existing fence, erect it at any point on the base line laid down, and another perpendicular a chain or two closer to the existing fence. Chain off on each of these perpendiculars the required height, and put in pickets at their extremities. Now sight by means of these pickets to the fence, and so transfer this perpendicular height to

THE APPORTIONMENT OF LANDS.

*To separate a Given Quantity of Land from an Estate—
Where the Estate is Rectangular in Form and the
Dividing Fence is to be Parallel with one of its Sides—
Ditto when the Piece of Land separated is to be in the
Form of a Trapezoid and the Position of One End
of the Dividing Fence is fixed—Ditto when the Piece
of Land Separated is to be in the Form of a Triangle
—When the Estate is Triangular in Form and the
Dividing Fence is to be Parallel with One of the
Boundaries—Ditto when the Dividing Fence is to be
Run from One of the Angles of the Triangular Field
to the Side opposite—Ditto when the Fence is to be
run from a Given Point in One Side to the Side
opposite—When the Estate is Irregular in Shape and
the Portion to be cut off is to be in a Given Position
so that as far as possible it will be bounded by
Existing Fences—Examples in Each Case.*

This subject is one which divides itself into three
distinct cases.

Case 1.—When it is desired to part off a given quantity
of land from an estate.

Case 2.—The division of an estate into any number of
apportionments of equal or unequal *quantity.*

Case 3.—The apportionment of lands of variable value
between any given number of owners in shares of equal or
unequal *value.*

The present chapter will deal with the first of these.

Although the subject might be sub-divided into a number
of cases according to the form of the piece of land to be
dealt with, the methods to be employed in the different

CHAPTER XX.

THE APPORTIONMENT OF LANDS.

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Dividing Fence is to be Parallel with one of its Sides—
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so that as far as possible it will be bounded by
Existing Fences—Examples in Each Case.*

This subject is one which divides itself into three
distinct cases.

Case 1.—When it is desired to part off a given quantity
of land from an estate.

Case 2.—The division of an estate into any number of
apportionments of equal or unequal *quantity.*

Case 3.—The apportionment of lands of variable value
between any given number of owners in shares of equal or
unequal *value.*

The present chapter will deal with the first of these.

Although the subject might be sub-divided into a number
of cases according to the form of the piece of land to be
dealt with, the methods to be employed in the different

instances are practically the same, and anyone who has thoroughly mastered Chapter XVI., on "The Computation of Areas," will readily follow the application of the rules in the present case.

I am persuaded that brief statements of the rules, followed by simple examples, will be found more helpful than a long treatise on that which needs little explaining.

To Separate a Given Quantity of Land from an Estate.

CASE I.—(*a*) WHERE THE ESTATE IS RECTANGULAR IN FORM AND THE DIVIDING FENCE IS TO BE PARALLEL WITH ONE OF ITS SIDES.

(**250**). *Rule.*—Divide the area to be parted off in square links by the length of the side to which the dividing fence is to be parallel in links, and the quotient will be the depth.

Example.—Let *A B C D*, Illustration No. 161, represent the land from which a portion is to be parted off; the area
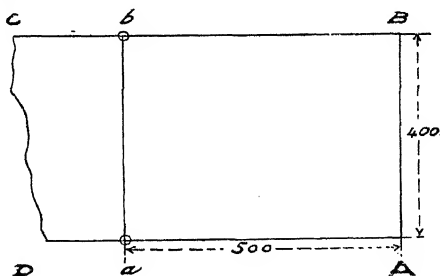
ILLUSTRATION No. 161.

to be separated equal two acres; and the length of the side, *A B*, to which the dividing fence is to be parallel, equal 400 links.

$$2 \text{ acres} = 200000 \text{ links.}$$
$$\frac{200000}{400} = 500 \text{ links.}$$

Then 500 links equals the depth of the land to be parted

off.   Measure 500 links from *A* to *a* and from *B* to *b*, and
drive down pegs at these points, to fix the position of the
fence, which will part off two acres from the estate.

(b) WHEN THE PIECE OF LAND SEPARATED IS TO BE IN
THE FORM OF A TRAPEZOID AND THE POSITION OF ONE
END OF THE DIVIDING FENCE IS FIXED.

(**251**).   *Rule.*—Divide twice the area to be parted
off in square links by the side of the field which is to
become for its entire length one of the boundaries of the
separated area, in links, and the quotient will be the
perpendicular of a triangle containing that area.   Subtract
from this perpendicular the length proposed for the
opposite parallel boundary of the trapezoid, and the
difference will equal the other parallel side.

Example.—Let *A B C D*, Illustration No. 162, represent
the estate from which the trapezoid is to be separated;
two acres the area to be parted off; 400 links the length of
the side *A B*, which is to become one of the boundaries of
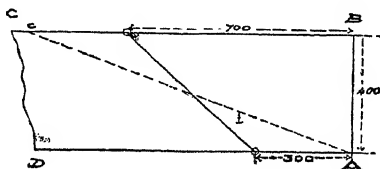
ILLUSTRATION No. 162.

the separated figure, and 300 links the distance from *A* to
the point on *A D*, from which the dividing fence is to be
run.

2 acres = 200000 square links.
Twice the area = 400000.
$$\frac{400000}{400} = 1000$$

Then 1,000 links equals the perpendicular of a triangle
on *A B* containing the given area, and 1,000—300, the

length fixed for the boundary on *A D*, gives 700, the length of the boundary on *B C*. Measure off these distances, drive in pegs, and you have fixed the position of the dividing fence which will part off the required area in the form of a trapezoid.

(c) WHEN THE PIECE OF LAND TO BE SEPARATED IS TO BE IN THE FORM OF A TRIANGLE.

(**252**). *Rule.*—The rule in this case has been given in the last example, which see.

There are other instances which might arise out of the foregoing case, but they will present no difficulty if the simple examples given are understood.

CASE II.—WHEN THE ESTATE IS TRIANGULAR IN FORM.

(a) WHEN THE DIVIDING FENCE IS TO BE PARALLEL WITH ONE OF THE BOUNDARIES.

(**253**). *Rule.*—Subtract the area to be parted off from the area of the whole figure, to find the area of the triangle remaining. Then, as the area of the original
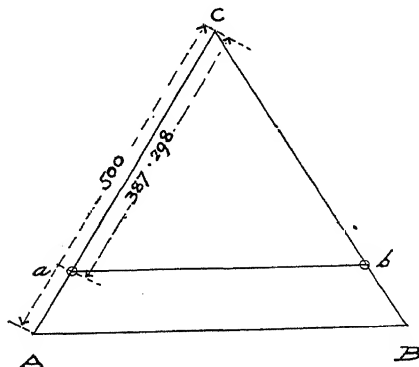
ILLUSTRATION No. 163.

triangle is to the area of the triangle which will remain, so

is the square of the side of the whole triangle to the square
of the side of the remaining triangle.

Example.—Let *A B C*, Illustration No. 163, represent the
triangular field from which the land is to be parted; five
roods the area of the whole field; two roods the area to be
parted off; and 500 links the length of the side *A C*,
adjacent to the side to which the dividing fence is to be
parallel.

$$5 \text{ roods} - 2 \text{ roods} = 3 \text{ roods}.$$

$$5 \quad : \quad 3 \quad : : \quad 500^2 \quad : \quad a\,C^2$$

$$\frac{250000 \times 3}{5} \quad a\,C^2$$

$$= \; 750000 \quad = \; a\,C^2$$

$$= \quad \sqrt{\;150000} \; = \; a\,C$$

and $\sqrt{\;150000}$ (= 387·298

```
              9
       68   600
            544
      767   5600
            5369
     7742   23100
            15484
    77449    761600
             697041
   774588    6455900
             6196704
```

Then 387·298 links is the distance *C a*, and *a* is the
point from which the fence must be run parallel to *A B* in
order that the figure *A B b a* may contain two roods.

(*b*) WHEN THE DIVIDING FENCE IS TO BE RUN FROM ONE
OF THE ANGLES OF THE TRIANGULAR FIELD TO THE SIDE
OPPOSITE.

(**254**). *Rule.*—Divide twice the area to be parted off
by the length of the side which is to become the base of
the separated triangle, and the quotient will be the
perpendicular.

This perpendicular being erected anywhere on its base, and a line ranged through its extremity at right angles to it (which will, of course, be parallel with the base) until it cuts the boundary fence, will give the point therein from which the dividing fence must be run.

Example.—Let *A B C*, Illustration No. 164, represent the triangular field ; two acres the quantity to be parted off ;
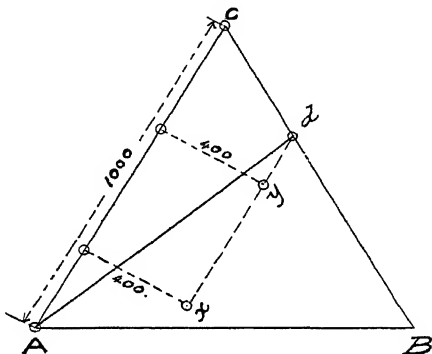
ILLUSTRATION No. 164.

A the angle from which the dividing fence is to be run to the side *B C* ; and 1,000 links the length of side *A C*.

$$2 \text{ acres} = 200000$$
$$\text{Twice the area} = 400000$$
$$\text{Twice area,} \quad \frac{400000}{1000} = 400$$
$$\text{Base,}$$

Then 400 links is the length of the perpendicular which, being set up at any two convenient points on *A B* (the farther apart the better) and transferred to *d* by sighting from *x* through *y*, gives the point *d* to which the fence must be run from *A*, in order that exactly two acres may be parted off.

(*c*) WHEN THE FENCE IS TO RUN FROM A GIVEN POINT IN ONE SIDE TO THE OPPOSITE SIDE.

(**255**). *Rule.*—Subtract the area of the trapezoid to be parted off from the area of the whole triangle, to obtain the

area of the remaining triangle. Divide twice the area of the remaining triangle by the distance measured from the angle opposite the dividing fence, to the point in the boundary from which it is to start, and the quotient will be the perpendicular, which being set up and ranged to cut the opposite boundary will give the point therein to which the dividing fence must be run to part off the trapezoid of given area.

Example.—Let *A B C*, Illustration No. 165, represent the triangular field from which it is desired to part off a given quantity by a fence from a given point in one side to the side opposite, and let *d* be the given point ; 15 acres the
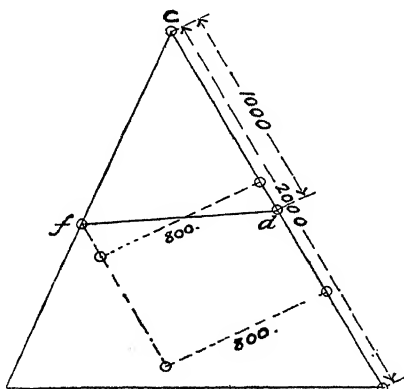
ILLUSTRATION No. 165.

area of the whole estate ; 11 acres the quantity to be parted off ; and 1,000 links the distance *C d*. The process is as follows :—

Area of whole triangle	..	15 acres.
Portion to be parted off	..	11 ,,
Triangle remaining ..	..	

$$\frac{4 \text{ acres} = 400000}{\text{Twice area in links} = 800000} = 800 \text{ links}$$
$$\text{Distance } C d = 1000$$

Then the perpendicular of a triangle on *C d* containing four acres equals 800 links, which being set up and transferred to cut the boundary *A C*, as indicated in the last example, gives the point *f*, to which the fence must be run in order that the trapezoid containing 11 acres may be parted off.

(*d*) WHERE THE ESTATE IS IRREGULAR IN SHAPE AND THE PORTION TO BE CUT OFF IS TO BE IN A GIVEN POSITION SO THAT AS FAR AS POSSIBLE IT WILL BE BOUNDED BY EXISTING FENCES.

(**256**).—In this case the surveyor first speculates as to where the fence should come, computes the area of the piece cut by his guess line and then adds or deducts a triangular piece representing the quantity by which his guess is in excess or defect of the required area.

Rule.—Range out a line called a guess line, which will cut off about the required area of land, chain the necessary lines, and compute the area of this speculative figure and subtract same from the total area to be severed ; or subtract the area to be severed from the computed area, according as the one is greater or less than the other. The difference will be the area which must be added to or deducted from the speculative figure. Divide twice this difference by the guess line, and the quotient will be the perpendicular of a triangle which will contain the area to be added or deducted to give the required area.

This perpendicular being set up on the guess line, and transferred by running a line at right angles thereto until it cuts the required boundary, will give the position for the dividing fence.

Example.—Let *A B C D E*, Illustration No. 166, represent a piece of land from which it is desired to sever say five acres, the severed portion to be bounded by the fences, *B C*, *C D* and *D E*. We range our guess line, and compute the

area of the figure by chaining the base line and the per-
pendiculars from E and C for that purpose.
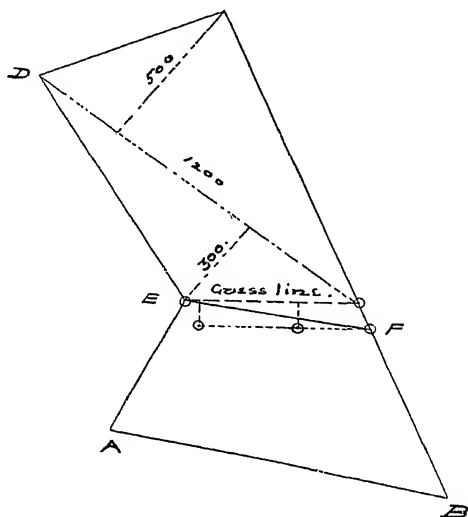
ILLUSTRATION No. 166.

The area, we find, is four acres three roods eight poles,
which is 32 poles short of the area required to be parted
off.   This equals 20,000 links, or the double area equals
40,000 links, which divided by 500 links, the length of the
guess line, gives us the perpendicular of a triangle thereon
containing 32 poles.

This distance is set up perpendicular to the guess line
at any two convenient points therein (the farther apart
the better) ; pickets are put down at the extremities of
these perpendiculars and a line sighted thereby to cut
the fence at F, the point to which the new fence from
E must be run in order that exactly five acres may be
parted off.

The workings are as follows :—

$$
\begin{array}{rl}
\text{Base} & = & 1200 \text{ links} \\
\text{Perps.} & = & 500 \\
\text{,,} & = & 300 \\
& & \overline{} \quad 800 \\
\end{array}
$$

$$
\begin{array}{rl}
2 \mid & 960000 \\
\text{A} & 4\cdot80000 \text{ double area} \\
& 4 \\
& \overline{} \\
\text{R} & 3\cdot20000 \\
& 40 \\
& \overline{} \\
\text{P} & 8\cdot00000 \\
\end{array}
$$

Area to be parted off = 5a. 0r. 0p.
Area parted off by guess line = 4   3   8
Difference = 0   0   32

32 poles = 20000 links
2

Guess line = 500) 40000 double area
Perp. of triangle on base
500 links to contain 32 poles = } 80 links

Note.—In any case where boundaries are not straight lines, off-sets or insets must, of course, be taken, and their areas computed and added to or subtracted from the area of the main figure, as the case may be, in computing the guessed area.

CHAPTER XXI.

THE APPORTIONMENT OF LANDS.—*Continued.*

THE DIVISION OF AN ESTATE INTO ANY NUMBER OF
APPORTIONMENTS OF EQUAL OR UNEQUAL *quantity or
value.*

*When the Field is Rectangular, the Shares are Equal, and
the Fences are to be Parallel with a given Side—Ditto
when the Shares are Unequal—When the Field is
Triangular in Form, the Shares are Equal or Unequal,
and the Fences are to be Parallel with a Given
Boundary—Ditto when the Shares are Equal, and the
Fences are to be run from one of the Angles to the
Side opposite—Ditto, but the Shares are Unequal—
Ditto, but the Fences are to Run from a given Point
in one of the Sides to the Side opposite—When the
Field to be divided up is in any Form whatever,
the Shares are to bear a given Proportion to each
other, and the Value of the Land is Uniform in all
parts—Ditto when the Value of the Land is not
Uniform.*

––––––––

The land surveyor is often concerned with :—

    (1) The division of an estate into any number of
apportionments of equal or unequal quantity;
and

    (2) The division of an estate into any number of
apportionments of equal or unequal value.

I shall now give the rules and methods applicable to
cases coming under one or other of these heads, and first
with regard to :—

CASE I.

THE DIVISION OF AN ESTATE INTO ANY NUMBER OF APPOR-
TIONMENTS OF EQUAL OR UNEQUAL *quantity*.

(*a*) WHEN THE FIELD IS RECTANGULAR, THE SHARES ARE
EQUAL AND THE FENCES ARE TO BE PARALLEL WITH A
GIVEN SIDE.

(**257**). *Rule.*—Divide the side from which the fences
are to be run into as many equal parts as there are shares,
and range out the fences parallel with the given boundary.

Example.—Let *A B C D*, Illustration No. 167, represent
a rectangular field, which is to be divided into six equal
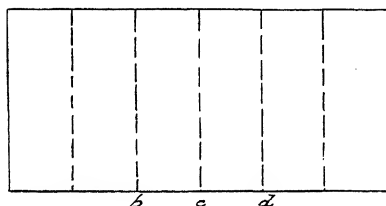
ILLUSTRATION No. 167.

parts by fences parallel with the boundary *C D*, and let *C B*
equal 1,200 links.

Then the length on *C B* attributable to each apportion-
ment will equal

$$\frac{1200}{6} = 200 \text{ links.}$$

Measure off from *A* to *a* and *B* to *a*, 200 links, and
the same from *a* to *b*, *b* to *c*, *c* to *d*, *d* to *e*, *e* to *C*,
and *e* to *D* on the boundaries *A D* and *B C*, drive in
pegs at these points, and you have fixed the positions of
the fences.

(b) WHEN THE FIELD IS RECTANGULAR, THE SHARES ARE
UNEQUAL, AND THE FENCES ARE TO BE PARALLEL WITH A
GIVEN SIDE.

(**258**). *Rule.*—Divide the area of each person's share
by the length of the given side, and the quotient will
be the length of the other side.

The dimension sought for laying out each person's
share must be found separately.

Example.—Let *A B C D*, Illustration No. 168, re-
present a rectangular field which it is required to divide
between three persons, E, F and G, in unequal shares,
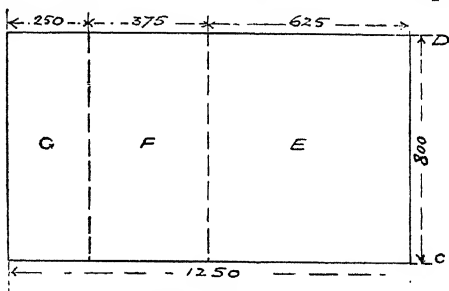
ILLUSTRATION No. 168.

by fences parallel with the boundary *D C* ; the whole
land comprises ten acres ; E's share five acres ; F's
share three acres; and G's share two acres ; the longer
dimension of the entire land 1,250 links, and the shorter
dimension 800 links.

E's share equals five acres, equals 500,000 links.

$$\frac{500000}{800} = 625 = \text{the frontage on } AD \text{ of E's share.}$$

F's share equals three acres, equals 300,000 links.

$$\frac{300000}{800} = 375 \text{ links} = \text{the frontage on } AD \text{ of F's share.}$$

G's share equals two acres, equals 200,000 links.

$$\frac{200000}{800} = 250 \text{ links} = \text{the frontage on } AD \text{ of G's share.}$$

$$
\begin{aligned}
\text{E's frontage} &= 625 \text{ links.} \\
\text{F's \quad,,} &= 375 \quad,, \\
\text{G's \quad,,} &= \underline{250} \quad,, \\
\text{Total} &= 1,250 \quad,,
\end{aligned}
$$

The figures assumed in the above example are so simple that it would appear better to divide the boundary, giving one-half to E, three-tenths to F, and the remainder, one-fifth to G, but where the total acreage and the shares are represented by more complicated figures, the proportion would not be so apparent. The rule given is better as a general one.

It may likewise seem unnecessary to calculate G's share, as he would receive the remainder of the 10 acres, after deducting E's and F's shares; but, in the first place, the land represented might be part only of an estate, and, in the second place, it would in any case be better to calculate all the shares for the test of the accuracy of the workings which is afforded by so doing. Thus we find, in the example given, the frontages of the several shares, as calculated, sum to 1,250 links, which is, we know, the total frontage.

These distances being accurately measured, and pegs driven down on each of the opposite boundaries, the positions for the fences will be fixed.

(*c*) WHEN THE FIELD IS TRIANGULAR IN FORM THE SHARES ARE EQUAL OR UNEQUAL, AND THE FENCES ARE TO BE PARALLEL WITH A GIVEN BOUNDARY.

(**259**). *Rule.*—As the area of the whole triangle to be divided up is to the square of any one of its sides so is the area of the triangle to be parted off to the square of its similar side.

Note.—As the calculation depends on the proportion which exists between similar triangles and the squares of corresponding sides, each apportionment, whether the shares are equal or unequal, must be calculated separately.

The first and second shares are added together to form the second term in the proportion in calculating the length on the given boundary attributable to the second share; and the first, second and third shares are added

together to form the second term in the proportion, in calculating the length on the given boundary attributable to the third share, and so on.

The side chosen should be one of those from which the fences are to be run (not that to which the bases of the parted figures will be parallel), then the ascertained measurements can be set off on the opposite boundaries, and pegs driven down to fix the positions of the ends of the fences.

Example.—Let *A B C*, Illustration No. 169, represent a triangular field which it is desired to divide up between
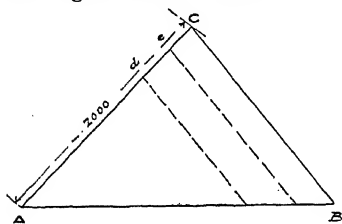
ILLUSTRATION No. 169.

three persons, D, E, F; 18 acres equal the whole estate, 20 chains the length of the boundary *A C*, from which the division fences are to be run ; and the proportion of the several shares to be as follows : D, eight acres; E, six acres; F, four acres.

D's share :—

$$18 \text{ acres} : 8 \text{ acres} \quad : 20^2 : A\ d^2$$

$$\therefore \frac{8 \times 400}{18} \quad = \quad A\ d^2$$

$$\sqrt{} \quad \frac{8 \times 400}{18} \quad = \quad A\ d$$

$$14 \cdot 53 \text{ links} \quad = \quad A\ d$$

E's share :—

E's share is six acres, which added to D's share, eight acres, equal 14 acres.

$$18 : 14 : : 20^2 : A\ e^2$$

$$\frac{14 \times 400}{18} \quad = \quad A\ e^2$$

$$\sqrt{} \quad \frac{14 \times 400}{18} \quad A\ e$$

$$17 \cdot 63 \text{ links} \quad = \quad A\ e$$

and 1763—1453 = 310 links = *d e*, or the distance on *A C* attributable to E's share.

The distance on the boundary *A C*, attributable to F's share is, of course, the remainder, after deducting that attributable to D's and E's shares; therefore we subtract 1,763 links from 2,000 links, which gives 237 links, or the distance on the boundary *A C* attributable to F's share.

These measurements, being carefully set out on the boundary *A C* and marked by pegs driven down, the points from which the division fences must be run to boundary *A B*, parallel with *B C*, have been fixed.

(*d*) WHEN THE FIELD IS TRIANGULAR IN FORM, THE SHARES ARE EQUAL, AND THE FENCES ARE TO RUN FROM ONE OF THE ANGLES TO THE SIDE OPPOSITE.

(**260**). *Rule.*—Divide the side opposite the angle from which the fences are to run into as many equal parts as there are shares ; carefully measure off the distances along the boundary attributable to each share, and drive in pegs to fix the positions for the ends of the fences.

Example.—Let *A B C*, Illustration No. 170, be the triangular field to be divided up amongst six persons in equal shares ; *A* the angle from which the division fences are to be run to the side *B C* opposite; and 1,410 links the length of the boundary *B C*.

The length of the boundary *B C*, 1,410 links, divided by six gives 235 links, which is the distance on boundary *B C* attributable to each share. It will be seen that, although the triangular fields
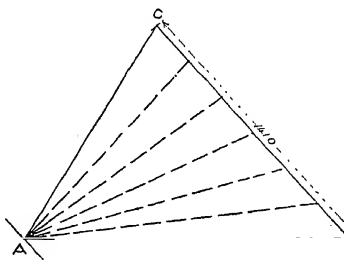
ILLUSTRATION No. 170.

representing each share vary in form, they are each on an equal base and between the same parallels, or of equal perpendicular height, hence their areas are equal.

(*e*) WHEN   THE   FIELD   IS   TRIANGULAR   IN   FORM,   THE
SHARES   ARE   UNEQUAL,   AND   THE   FENCES   ARE   TO   RUN
    FROM   ONE   OF   THE   ANGLES   TO   THE   SIDE   OPPOSITE.

(**261**). *Rule.*—As the whole area is to the area of a
share, so is the whole base to the part of that base
attributable to the share.

Example.—Let *A B C*, Illustration No. 171, represent
a triangular field which it is desired to divide between
three persons, D, E and F, in unequal shares by fences
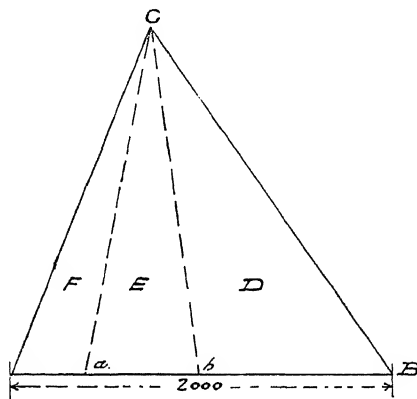
ILLUSTRATION No. 171.

from *C* to *A B* ; and let the whole area be twenty
acres ; D's share ten acres ; E's share six acres ; and
F's share four acres, and the side *A B* equal 2,000
links.

To find D's share.

$$20 \quad 10 :: 20 : \; B \, b$$
$$\therefore \; \frac{10 \times 20}{20} = B \, b$$
$$10 = B \, b$$

Thus we find that D's share of the boundary *A B* equals
10 chains, or 1,000 links, or half the whole base.

Note.—Inasmuch as the area of D's share is half the whole figure, we might have done this by the last rule, simply dividing the base in half to find the point to which the fence must be run, but I have purposely chosen this proportion in order that the truth of the rule may be evident.

To find E's share.

$$20 : 6 :: 20 : a \, b$$
$$. \frac{6 \times 20}{20} \quad a \, b$$
$$\frac{120}{20} : a \, b$$
$$6 = a \, b$$

or 600 links equal the distance on the base *A B*, attributable to E's share.

To find F's share.

$$20 : 4 :: 20 : A \, a$$
$$\frac{4 \times 20}{20} = A \, a$$
$$4 = A \, a$$

or 400 links equal the distance *A a* on the base line *A B*, attributable to F's share.

(f) WHEN THE FIELD IS TRIANGULAR IN FORM, THE SHARES ARE EQUAL OR UNEQUAL, AND THE FENCES ARE TO BE RUN FROM A GIVEN POINT IN ONE OF THE SIDES.

(262). *Rule.*—Find the position in the side containing the given point, which the end post of the fence would have to occupy if run to the angle *C* (Illustration No. 172), by rule given in the last case, and then convert the triangle thus formed into one of equal area having one of its angles at the given point, by the rules given in Chapter XVII., Art. 228.

Example.—Let *A B C*, Illustration No. 172, represent a triangular field which it is desired to divide between three persons, D, E and F, in unequal shares ; the whole area equal 20 acres, D's share 10 acres, E's share 6 acres, and
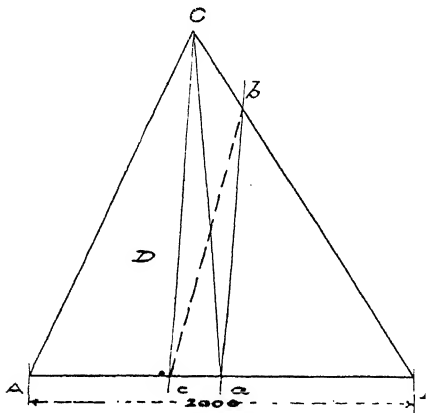
ILLUSTRATION No. 172.

F's share 4 acres, the side *A B* equal 2,000 links, and *c* the point in the boundary *A B* from which it is required to run the fences.

Find the point *a* in the base from which the fence parting off D's share would have to be run if it met the boundaries *A C*, *B C* at the angle *C*, as shown in the last example, draw in the line *a C* in pencil, connect the point *c* in base and the angle *C*, by doing which you will cut off from the triangle *A a C* the triangle *c a C*, which add by substituting another triangle of equal area, but with one side lying in the direction *c b*, which it is required the fence shall occupy. This is done by laying out the line *a b* parallel with *c C*, and so transferring the perpendicular height of triangle *C c a* to *b*, and obtaining the equal triangle *C c b* in the required position.

The remaining triangle *c* B *b* may be similarly divided between E and F, but this can be more simply done by the rule given in the previous example, as follows :—

As the whole area of the triangle *c* B *b* (equals 10 acres) is to E's share (equals 6 acres), so is the distance *b* B (which would, of course, have to be ascertained) to E's share of that boundary.

(g) WHEN THE FIELD TO BE DIVIDED UP IS IN ANY FORM WHATEVER, THE SHARES ARE TO BEAR A GIVEN PROPORTION TO EACH OTHER, AND THE VALUE OF THE LAND IS UNIFORM IN ALL PARTS.

(**263**). *Rule.*—Ascertain the area of the land to be divided up, and the acreage due to each party according to the proportion his share bears to the whole, and lay out each share in the most convenient form for all parties concerned, by the rules given in Chapter XIX., Arts. 237 to 249 ; also Chapter XX., Arts. 250 to 256.

CASE II.

THE DIVISION OF AN ESTATE INTO ANY NUMBER OF APPORTIONMENTS, WHEN THE ESTATE TO BE DIVIDED UP IS IN ANY FORM, THE SHARES ARE OF EQUAL OR UNEQUAL VALUE, AND THE VALUE OF THE LAND TO BE DIVIDED IS NOT UNIFORM.

(**264**). *Rule.*—(1) Prepare an accurate plan of the whole land to be divided up and show thereon "quality" lines separating the lands of varying value into parcels ;

(2) Fix the value per acre and value each part separately ; also calculate the total value of the whole of the lands.

(3) Settle as to in which of the parcels the several shares shall respectively fall.

(4) Then the total *value* of any of the parcels is to the value of any one share as the acreage of that parcel is to the acreage of the share in it

ILLUSTRATION No. 173.

Example.—Let Illustration No. 173 represent a plan of the land to be divided up between eight persons, A, B, C, D, E, F, G, H, in equal shares.

The land is shown in illustration divided up into five parcels by quality lines, and the values per acre and total values are also given thereon.

From this we see the total value of the lands to be divided up is as follows :—

Parcel 1.—40	Acres	at	£3	=	£120
2.—30		at	5	=	150
3.—25		at	6	=	150
4.—50		at	10	=	500
5.—60		at	12	=	720
		Total value		=	£1640
$\frac{1}{8}$ = value of each share				=	£205

Now, suppose it has been decided that the lands shall be apportioned so that A's share falls in parcels 1 and 2; B's share in parcels 2 and 3, and so on, commencing with A's share and parcel 1, and finishing with H's share and parcel 5.

Each share then is to be worth	...	...	£205
The total value of parcel 1 is only	...	...	120
So that A must have the whole of parcel 1, and			———
in parcel 2 an acreage of the value of		...	£85

Then to find his acreage in parcel 2 by the rule given :—

$$
\begin{pmatrix} \text{Total} \\ \text{value of} \\ \text{parcel 2} \end{pmatrix} \text{ is to } \begin{pmatrix} \text{value of that} \\ \text{portion of A's} \\ \text{share which} \\ \text{will fall in} \\ \text{parcel 2} \end{pmatrix} \text{ as } \begin{pmatrix} \text{the} \\ \text{acreage} \\ \text{of} \\ \text{parcel 2} \end{pmatrix} \text{ is to } \begin{pmatrix} \text{A's} \\ \text{acreage} \\ \text{in} \\ \text{parcel 2} \end{pmatrix}
$$

$$150 \quad : \quad 85 \quad :: \quad 30 \quad :$$

$$
\begin{array}{r}
30 \\
\hline
15{,}0\,)\ 255{,}0\ (\ 17\ \text{acres.} \\
15 \\
\hline
105 \\
105 \\
\end{array}
$$

Therefore A's share comprises parcel 1 = £120
And 17 acres in parcel 2 = 85

Total   £205

Then B's share will take the remainder of
    parcel 2 = £150 — £85 =   £65
And an acreage in parcel 3, which shall be of
    the value of £205 — £65 or £140.
  Then, again, following the rule :—

$$150 \quad : \quad 140 \quad :: \quad 25 \quad :$$

$$
\begin{array}{r}
25 \\
\hline
700 \\
280 \\
\hline
15{,}0\,)\ 350{,}0\ (\ 23\tfrac{1}{3}\ \text{acres.} \\
30 \\
\hline
50 \\
45 \\
\hline
5 \\
\end{array}
$$

And $23\tfrac{1}{3}$ acres at £6 per acre =   140

Total   £205

Very simple figures have been taken in the example for the sake of clearness. The results might almost have been arrived at mentally, but the examples given illustrate the rule as a general one, and its application to more complicated cases.

In the same manner all the other shares may be calculated, but the process is too simple to need further example.

With regard to the work in connection with staking out the shares in the field, see Chapter XIX., Arts. 237 to 249.

CHAPTER XXII.

Principles and Practice of Levelling.

The present chapter will deal with the art and practice of levelling, a distinct and very important branch of land surveying.

The various instruments used in levelling operations, their character, use, and adjustment, have already been fully dealt with in Chapter V., Arts. 43 to 73. Some reference has also been made in defining the technical terms applicable to surveying, to those concerning levelling operations, but it will considerably clear the ground if I commence by giving full definitions of all the various technical terms applicable to this branch of our work.

DEFINITIONS.

(**265**). *Levelling.* — The art of ascertaining the relative positions, vertically, of any number of given points in the surface of lands. It is concerned with natural slopes rather than mere irregularities of surface.

(**266**). *Sections.* — Drawings showing the result of levelling operations.

The nature of a section may, perhaps, be better explained by illustration.

Suppose a deep cutting be made through the land along the line on which the levels have been taken, then the section, with modifications, to which I shall refer directly, is, as it were, a scale drawing of the side of the cutting. It is the necessary measurements to enable us to prepare such a drawing that we obtain in the process of levelling.

However, owing to the smallness of the vertical variations representing the difference in level at different points, compared with the horizontal distance between them, the rise or fall from one to the other would scarcely be observable, in many cases, in a drawing representing a true picture of such a section as that I have described, and therefore it is usual to draw the section to two different scales, the vertical scale, usually being very much larger than the horizontal scale, by which means the variation in level between different points in the surface is made apparent.

Another point to be noted is that it is not every irregularity of the surface that is obtained by levelling, but only the relative vertical positions of those points at which the levels are taken, and therefore the upper surface is represented, not by a waved or broken line, as it would be in fact, but by a series of straight lines connecting the points representing the relative heights on the line of section.

The method of plotting a section from the level book will, of course, be fully dealt with in its proper place, and examples of sections will then be given. The above description is sufficient for our present purpose.

(**267**). *Longitudinal Section.—Cross Section.*—A longitudinal section is a section in the longer direction of the land, as compared with one taken in the shorter direction, called a cross section.

(**268**). *Trial Levels.*—Levels taken at considerable distances apart, generally for ascertaining the feasibility of some scheme, such as a projected line of railway, a canal, etc., etc.

In taking trial levels, the horizontal distances are not, as a rule, measured, but the proposed line is laid down on the best map obtainable, and is followed as nearly as possible. The levels are taken at points which can be identified on the map, particularly at important points, such as roads, streams, etc., which would be crossed by the line.

By this means the surveyor is able to see what part, if any, of the line would have to be in tunnel, cutting or embankment, etc., the bridges required to cross roads, streams, etc., and the various difficulties to be encountered.

Without this information it would be impossible to intelligently consider the advisability of following one or other of two or more possible routes, or to form any reliable estimate of the cost of the proposed undertaking.

(**269**). *Complete Levels.*—Levels taken at exactly ascertained horizontal distances on lines accurately defined by a plan, such as would be required for the preparation of complete working sections.

(**270**). *Check, Running or Flying Levels.* — Levels taken from the end of a line of section back to the starting point, for the purpose of checking more detailed operations ; or, where the line is very long, between bench marks thereon.

The readings are taken as far apart as possible, to shorten and simplify the work. The original course

is not necessarily followed; any convenient route back to the commencement of the line may be taken, it only being required to find the total rise or fall from point to point.

(**271**).   *Simple Levelling.*—Levelling in which backsights and foresights only are taken.

(**272**).   *Compound Levelling.*—Levelling in which backsights, foresights, and intermediates are taken.

(**273**).   *Backsight.*—The first reading taken each time after setting up the level.

(**274**).   *Foresight.*—The last reading taken each time before the level is moved to another station.

(**275**).   *Intermediates.*—The readings taken between backsights and foresights.

(**276**).   *Rise.*—The greater height of one point in a line of section than that immediately preceding it, indicated by a *lower* reading on the staff.   Thus, if the first reading is 4·40 and the second is 3·20, there is a rise of 1·20.

(**277**).   *Fall.*—The extent to which one point in a line of section is lower than the one immediately preceding it, indicated by a *higher* reading on the staff.   Thus, if the first reading is 4·40, and the second is 5·20, there is a fall of 0·80.

(**278**).   *Reduced Levels.*—The heights of the various points at which the levels have been taken above the assumed base, arrived at by adding the rises and subtracting the falls from the reduced level of the point immediately preceding.   At the commencement that reduced level is the assumed height above base.   We may speak of the base line as the datum, and in that case it would be so many feet below the first point in the section.

(**279**).   *Datum.*—Some standard of measure to which all other measurements are referred.

(**280**). *Base Line.*—The datum assumed, and from which the reduced levels are plotted.

(**281**). *Bench Mark.*—Some permanent immovable object, easily identified, on which the levelling staff may be held. Bench marks are required for enabling any line of section on which they occur to be checked at some future time, if necessary, and also as a point from which levelling operations may be continued.

Bench marks are indicated by B.M. on sections and maps.

Permanent bench marks, as shown in Illustration No. 174, have been carved in the masonry of buildings, etc., in different parts of the country by the Royal Engineers. These B.M.'s and their level values are shown on the ordnance maps. The centre line of the horizontal cut

ILLUSTRATION No. 174.

represents the position which the heel of the staff should occupy when referring levels to an ordnance bench mark.
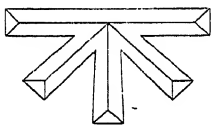
It is mostly in extensive operations, such as levelling for railways, canals, etc., etc., that B.M.'s are required. In such cases the levelling operations, extending as they do over a course many miles in extent, it is usual to take the first reading on some such permanent, immovable and clearly-defined point, and to carefully describe same in the level book for the purpose of future identification. The levels are also taken on suitable objects as bench marks at frequent intervals along the entire line ; and so is the last reading at the end of each day's work, so that operations may be resumed therefrom.

(**282**). *Apparent Level.*—The level which is obtained from the readings taken with the instrument, before the proper allowances for curvature and refraction have been made.

(**283**). *True Level.*—A truly level line is one which is at every point equidistant from the earth's centre, or a curved line, such as still water naturally assumes. It is obtained by making the proper allowance for the error arising from curvature and refraction, from the actual readings.

(**284**). *Curvature.*—The difference between true and apparent level, attributable to the line of sight (except as affected by refraction which has a counteracting influence), being a horizontal line, whereas the earth's surface, and consequently true level, is curvilineal.

(**285**). *Refraction.*—That effect caused by the density of the atmosphere by which objects viewed from a great distance appear higher than they really are. It is usually taken at one-sixth or one-seventh the correction for curvature, and is deducted therefrom.

(**286**). *Vertical Scale.* — The scale to which the vertical measurements in a section are plotted, generally much larger than the scale used in plotting the horizontal measurements in the same section. Different scales are used with the object of exaggerating the difference between the relative vertical positions of the different points at which the levels are taken, and so rendering the rise or fall from one point to another the more readily observable.

(**287**). *Horizontal Scale.*—The scale to which the horizontal measurements in a section are plotted, usually much smaller than the vertical scale used for the same section.

(**288**). *Reading.*—The measurement registered by the intersection of the cross wires of the telescope on the staff.

(**289**). *Contouring.*—Levelling for the purpose of ascertaining a number of points of equal level value, with the object of showing on a plan or map contour lines repre-

senting the variations in level in different parts of any
tract of land.

(**290**). *Contour Lines*.—The lines by which the result
of contouring is shown, practically representing the
outlines which would be formed by water rising to a
number of levels round a hill or series of hills, and
thus showing the difference in altitude between different
localities.

PRINCIPLES AND PRACTICE.

(**291**). *General Observations*.—The nature and object
of levelling has already been explained in connection with
the definitions; and in the description of the level and
staff already given under the head of " Instruments," the
method by which levels are obtained has been briefly
dealt with.   (See Arts. 48 to 73.)

From what has been there said, the following points will
have been gleaned :—

> (1) That the measurements we obtain in levelling
> operations are merely comparative quantities
> from which may be gathered by how much
> one point is relatively higher or lower than
> another ;

> (2) That the measurements obtained are the
> vertical distances from the horizontal line
> of sight given by the level, to the point in
> the surface of the earth on which the staff is
> held ; and

> (3) That so long, and so long only, as the level
> remains in the same position, that is with the
> level value of the horizontal line of sight
> unchanged, a comparison of any number of
> readings taken with it may be made, and
> the rise or fall from one point to another
> obtained.

However, speaking of any number of readings taken with the position of the level unchanged, as a series of readings, it is seldom indeed that a line of section is so short as to admit of all the readings being taken in one series.

Usually the level has to be reset many times, and consequently as this cannot be done so as to give the same level value to the telescope and line of sight by which the readings are registered, some method of forming a connecting link between the readings taken with the instruments in different positions must be resorted to.

No difficulty will be experienced in following the very simple means by which this object is achieved, if we remember that the measurements obtained are only for the purpose of enabling the level value of one point to be compared with the level value of another.

From the staff still held at the last point in one series of readings, a second reading is taken, as the first reading in the second series, directly the level is reset. Thus the level value of the first point in the second series is fixed by its being also the last point in the first series ; and the second point in the second series is fixed by its being found to be so much lower or higher than the first reading in that series.

It has been pointed out that the measurements obtained in levelling operations are merely comparative quantities showing one point as so much higher or lower than that preceding it, but if we assign a level value, above a horizontal base, to the first point, say 100ft. above base, the true *comparative* level value may be assigned to each of the other points by adding the rise or deducting the fall from one point to the level value of the point immediately preceding it.

This practice of assigning a level value to the first

point is always resorted to, because by doing so the actual
heights of each of the points above a given base are
obtained, which much simplifies the work of plotting
sections, as well as giving the information in a tangible
form.

To illustrate this, suppose we have taken the levels
at every chain length, and have calculated the height
above base or reduced levels at each of these points.
With this information it will be a simple matter to
prepare a section.

We shall only need to lay down a horizontal line,
marking off on it to scale the various points at which
levels were taken ; to erect indefinite perpendiculars at
each of these points, and to scale off on them the heights
above base, which will give the points through which
the line of section must be drawn.

The foregoing general remarks are only offered, how-
ever, to clear the way for a more detailed explanation
of simple and compound levelling.

SIMPLE AND COMPOUND LEVELLING.

(**292**). *Simple Levelling.* — In simple levelling the
process is as follows : The level is set up as far from
the first point in the line of section as convenient for
reading, the staff is held at the first point and the
reading, a backsight, is taken, and entered in the level
book. The staff is then carried forward about an equal
distance on the other side of the instrument and the
second reading, a foresight, taken and entered. The level
is then moved forward as far as possible to allow of a
reading being conveniently taken to the staff, which is
still held at the second point. This reading, a backsight,
is entered, and the staff is again carried forward and
a fourth reading, a foresight, taken and entered, and so
on to the end of the line.

For example, let $C\,D\,E\,F\,G\,H$, Illustration No. 175, represent the positions occupied by the instrument, and
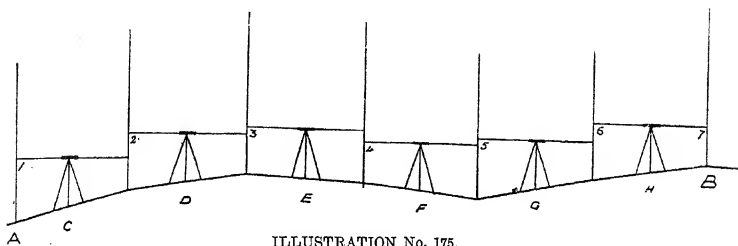
ILLUSTRATION No. 175.

1 2 3 4 5 6 7 the positions of the staff, in the course of simple levelling from A to B.

The first reading, a backsight, is 6·90, and the second reading, a foresight, is 4·60, which gives a rise of 2·30 from points 1 to 2. The third reading, which is taken with the level in a new position, and the staff still at point 2, is a backsight, 7·80; the fourth reading, a foresight, is 6·35, which shows a rise of 1·45 from point 2 to 3; the fifth reading, a backsight to the staff held at point 3, is 8·75, and the sixth, to the staff held at 4, a foresight, 9·30, which gives a fall of 0·55 from point 3 to point 4. This process is continued for each of the other points. (See Arts. 273 and 274.)

COMPOUND LEVELLING.

(**293**).—Simple levelling, as we have seen, is useful for finding the rise or fall between two or more points, but, for the purpose of preparing proper sections, it is necessary to ascertain the levels at every point at which there is a change in the natural slope of the ground; hence compound levelling.

Frequently the levels are taken at regular horizontal distances, as, for instance, every chain length, so that the

readings are comparatively close together; hence intermediate readings between the backsights and foresights.

I will now give an example of compound levelling.

Suppose it is required to take the levels on the line AB from A, Illustration No. 176, at every chain length.
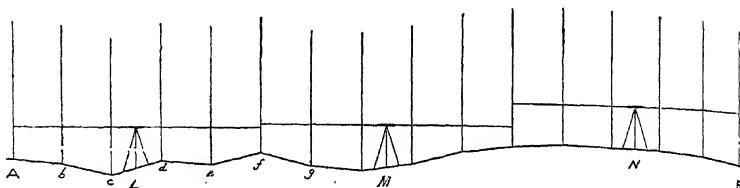
ILLUSTRATION No. 176.

The instrument is set up at L, as far from the first point at which the level is to be taken as possible, so long as the reading can be conveniently taken. The distance will vary with the power of the telescope, etc.

The first reading is a backsight, 4·70, the second an intermediate, 5·20, thus there is a fall from A to b of 0·50. The third point is also an intermediate, 6·30, thus there is a fall from b to c of 1·10. The fourth reading, also an intermediate, 4·90, shows a rise of 1·40 from c to d. The fifth reading, likewise an intermediate, is 5·30, which gives a fall from d to e of 0·40. The sixth reading, being the last which can be taken without moving the level, is a foresight, 4·30, which gives a rise of 1·10 from e to f.

The level having been moved to M, another reading is taken to the staff still held at f, and this being the first reading after resetting the level, is a backsight, 5·20. It has nothing to do with the first series of readings, it is the first reading in the second series which serves to show the rise or fall between this point and the following one. The staff is next held at g and the seventh reading, an intermediate, is taken, 6·30, which shows a fall from f to g

of 1·10.   From *e* to *f* there was a rise of 1·10, therefore points *e* and *g* are level.

The remaining readings are taken in a similar manner to the end of the line of section.

From the above explanation it will be seen :—

(1) That the height of the telescope of the level above the ground, and the level value of the ground on which it stands, in no way affects the operations ;

(2) That the backsight, the first reading in the following series, has no connection with the foresight, the last reading in the preceding series, the instrument having been in a different position when they were respectively taken, but that the backsight, intermediates and foresight in each series are comparable quantities, enabling the rise or fall from one point to another to be ascertained by calculation ; and

(3) That the connecting link between the different series of readings is obtained by the foresight in one series and the backsight in that which follows being taken from the staff in the same position.

Some further examples of levelling will be given when we come to consider the level book and method of reducing levels.

METHOD OF CHECKING THE ACCURACY OF LEVELLING OPERATIONS.

(**294**).   *Check Levels.*—Owing to the extreme importance of accuracy in levelling operations, it is necessary to have some means by which the quality of the work may be tested.   This is done by what is known as check levelling.

On the completion of a line of section, flying or check

levels are taken back to the commencing point, and if the difference between the total rises and total falls in the flying levels equals the difference between the total rises and total falls in the complete or trial levels, the work is proved.

It is seldom that the two results will be identical, but in ordinary cases the error should not exceed 0·03. In cases of large operations, where the line is several miles long, the allowable error is greater, say ·2, or even ·3.

In cases where two or more lines of section cross each other, a reading should always be taken in each line at the point where the lines cross.

Let Illustration No. 177 represent a case where two lines of section, AB, CD, are required to be taken for roads which will cross each other obliquely at X, and let L
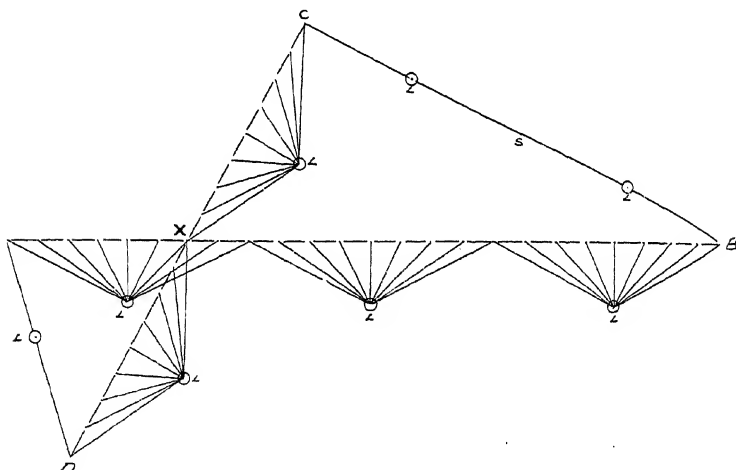
ILLUSTRATION No. 177.

represent the positions occupied by the instrument; the radiating lines the lines of sight therefrom to the staff held at each of the points in lines AB and CD cut thereby, and

s the positions of the staff in the check levels from B to C, and D to A.

The levels are taken as already indicated, at each of the points in line AB from A to B. Flying levels are then taken from B, the end of the first line, to C, the commencement of the second line. The levels are then taken at each of the points in CD from C to D, and finally flying levels are taken from D, the end of the second line, to A, the beginning of the first line.

The check or flying levels are taken over the shortest and most convenient course from B to C and D to A.

When the levels are reduced, about which more will be said, the level value of A will occur twice, as the first and the last entries, and if these are equal the accuracy of the work will be proved.

In the survey which candidates for the Professional Associateship Examinations of the Surveyors' Institution are required to make, the levels on two lines of section, one in the longer and the other in the shorter direction of the land, have to be taken, and check levels, as indicated above, must be given. See " Instructions to Candidates," issued by the Surveyors' Institution.

CURVATURE AND REFRACTION.

(**295**). *General Observations.*—There are two errors which may have to be taken into account in levelling operations. They are :—

  (1) Curvature ;
  (2) Refraction.

But, as a matter of fact, they are not either usually considered in ordinary levelling practice, for two reasons : first the correction, except where the distance between the points is very considerable, would be too small to need notice ; and, secondly, because in ordinary levelling operations the level is placed equidistant between the forestaff

and backstaff, or thereabouts, which entirely obviates the
necessity for any correction being made, the error in that
case on the one side balancing that on the other.

Thus, if the reading on the backstaff is one inch too
low owing to curvature, the reading on the forestaff at the
same distance will also be an equal amount too low, and as
the process of levelling.only gives the height of one point
in its relation to another the error is balanced.

(**296**). *Curvature.*—Curvature arises from the line of
sight through the telescope of the level being horizontal
(except as affected by refraction), whereas true level follows
the curve of the earth, or is a curved line equidistant at
every point from its centre, and consequently the greater
the distance of the object sighted from the instrument, the
greater becomes the difference between apparent and true
level.
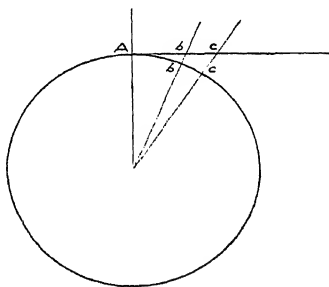
ILLUSTRATION No. 178.

Thus, referring to Illustration No. 178, when the distance
is *A b* the error is *b b*, when it is *A c* the error is *c c*, the
error increasing as the square of the distance, from which
is deduced the following :—

(**297**). *General Rule for Calculating the Correction
for Curvature.*—Divide the square of the distance by twice
the earth's radius, and the result will be the correction for
curvature.

The earth's diameter may be taken for our purpose at 7,920 miles, which gives the deduction for curvature in inches for one mile $\frac{12}{7920}$, equal to $\frac{1}{7920}$ of a mile, or 8 inches, or two-thirds of a foot.

RULE FOR CALCULATING THE CORRECTION FOR CURVATURE WHEN THE DISTANCE IS IN MILES.

(**298**).   *Rule 1.*—Take two-thirds the square of the distance in miles for the correction for curvature in feet.

Calculating the curvature for a distance of one mile, we get $\frac{2}{3} \times 1^2 = \frac{2}{3}$ of a foot $= 8$in.

Similarly, when the distance is two miles, we have $\frac{2}{3} \times 2^2 = \frac{8}{3}$ feet, or $2\frac{2}{3}$ feet, or 2ft. 8in.

Again, when the distance is three miles, we have $\frac{2}{3} \times 3^2 = \frac{18}{3} = 6$ft., and so on.

By an examination of the same figures, we obtain the rule applicable when the distance is given in chains, thus : Two-thirds the square of the distance in miles equals the correction for curvature in feet. Therefore, if the distance is in chains and we divide by 80, we convert same into miles, thus 1 chain equals $\frac{1}{80}$ mile, and, therefore, taking the basis of one chain and following the rule applicable where the distance is given in miles, we get $\frac{2}{3} \times (\frac{1}{80})^2$ equals correction in feet, or $\frac{2}{3} \times (\frac{1}{80})^2 \times \frac{12}{1}$ equals the correction in inches, which is more convenient, since the correction for 20 chains is only 6 inches.

$$\frac{2}{3} \times \left(\frac{1}{80}\right)^2 \times \frac{12}{1}$$

$$= \frac{2}{3} \times \frac{1}{6400} \times \frac{12}{1}$$

$$= \frac{8}{6400}$$

$$= \frac{1}{800}$$

Hence the following rule :—

RULE FOR FINDING THE CORRECTION FOR CURVATURE WHEN THE DISTANCE IS IN CHAINS.

(**299**). *Rule 2.*—Divide the square of the distance in chains by 800 for the correction in inches.

Again from the same figures we may obtain the rule applicable when the distance is given in yards. Thus: Two-thirds the square of the distance in miles equals the correction for curvature in feet, therefore if the distance is in yards and we divide by 1760, the yards in a mile, we convert same into miles. Thus 1 yard equals $\frac{1}{1760}$ mile, and therefore taking the basis of 1 yard, and following the rule applicable where the distance is given in miles, we get $\frac{2}{3} \times (\frac{1}{1760})^2 \times \frac{12}{1}$ equals the correction in inches.

$$\frac{2}{3} \times (\frac{1}{1760})^2 \times \frac{12}{1}$$
$$= \frac{2}{3} \times \frac{1}{3097600} \times \frac{12}{1}$$
$$\overline{387200}$$
$$= \cdot 00000257 \text{ inches.}$$

Hence the following rule :—

RULE FOR FINDING THE CORRECTION FOR CURVATURE WHEN THE DISTANCE IS GIVEN IN YARDS.

(**300**). *Rule 3.*—Square the distance, multiply the result by 257, cut off 5 places of decimals from the product, and the whole numbers will equal the correction in inches.

REFRACTION.

(**301**). *General Observations.*—Refraction has already been defined as the effect caused by the density of the atmosphere by which objects viewed from a great distance appear higher than they really are.

It will have been observed that curvature makes the reading on the staff greater than it should be, or in other words, it makes the level of the point to which the

observation is taken appear lower, and consequently as refraction makes objects appear higher, it has a counter-acting effect, and the correction for refraction must be deducted from the correction for curvature.

(**302**).   *Rule.*—Refraction varies with the condition of the air, but it is usual to correct for it at the uniform rate of one-sixth or one-seventh that for curvature.

I will now give one or two examples of finding the correction for curvature and refraction.

Examples.—(1) The level was adjusted and the horizontal cross web of the telescope observed to cut the top of a wall 5·30ft. high at a distance of two miles.   The height of the telescope above the ground was 4ft. 6in. Required the rise in the ground from the point at which the level stood to the wall sighted.

$$\tfrac{2}{3} \times 2^2 = \text{cor. for curvature} = \quad 2\cdot666$$
$$\tfrac{1}{7} = \text{cor. for refraction} = \quad \cdot381$$
$$\overline{\qquad\quad 2\cdot285}$$
$$\text{Apparent level } 5\cdot30 - 4\cdot50 = \text{fall} \qquad \cdot800$$
$$\text{Rise} = \overline{1\cdot485\text{ft.}}$$

Which shows that, although, according to the reading, there was an apparent fall of ·80ft. from the one point to the other, as a matter of fact there is a rise of 1·485ft.

(2) It is desired to know the fall from one point to another three miles distant.   The level is adjusted and the horizontal web of the telescope is found to cut the parapet of a house the height of which above the ground is known to be exactly 70ft.   The height of the telescope of the level above the ground on which it was set was 5ft.

$$\text{Apparent fall } 70 - 5\text{ft.} = \qquad\qquad 65.$$
$$\tfrac{2}{3} \times 3^2 = \text{cor. for curvature} \quad 6\cdot$$
$$\tfrac{1}{7} = \text{cor. for refraction} \qquad 0\cdot857$$
$$\overline{\qquad\qquad 5\cdot143}$$
$$\text{Fall} = 59\cdot857 \text{ ft.}$$

(3) It is desired to know the fall between two points, 20 chains distant from each other, in order that the

feasibility of connecting a drain, which is 4ft. below the surface at point A, to a sewer, the invert of which is 13·66ft. below the surface at point B, may be ascertained. Required the gradient the drain would have if laid.

The level was set up near point A and the staff held there read 4·03; the staff held at point B read 16·80.

$$16·80 - 4·08 = \text{apparent fall of} \qquad\qquad 12·7700$$

$$\frac{20^2}{800} = \frac{400}{800} = \text{cor. for curvature} \qquad ·5$$

$$\frac{·5}{7} = \text{cor. for refraction} \qquad\qquad ·0715$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \overline{} \qquad ·4285$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \overline{12·3415}$$

Depth of sewer below surface
at point B .. .. 13·66
Depth of drain at point A 4·00
                                              ———   9·6600
                                    Fall   22·0015ft.

The drain, therefore, would have a fall of 22ft. in its length of 20 chains.

$$\left.\begin{array}{l} 22 \text{ feet} = \dfrac{264 \text{ inches}}{20 \text{ chains} = 1320 \text{ feet}} \end{array}\right\} = ·2 \text{ or } \tfrac{1}{5}\text{in. in a foot.}$$

or 2in. in 10ft., which would be sufficient.

I pointed out in Art. 293, in explaining the principles of levelling, that the height of the telescope above the ground did not affect the operations; but it will be noticed in Examples 1 and 2 above, that the height of the telescope above the ground is a factor in the calculations. This is so because in those cases (unlike the ordinary case of levelling) the height of the telescope really takes the place of the backsight.

THE LEVEL BOOK.

(**303**). *Level Book in Simple Levelling.*—The level book and the method of reducing the levels will now be illustrated.

I first give a case of simple levelling, and would ask

the reader's attention to the accompanying level book and diagram, and the sketch section, Illustration No. 179.

LEVEL BOOK.—SIMPLE LEVELLING.

Back-Sight.	Fore-Sight.	Rise.	Fall.	Reduced Levels.	Distance.	Remarks.
2·50				100·00	0·00	Centre of Alexander Road.
4·00	4·90		2·40	97·60	9·00	
4·20	2·25	1·75		99·35	18·00	
4·80	3·25	0·95		100·30	27·00	
4·19	2·30	2·50		102·80	36·00	Point representing centre of proposed cross road.
4·05	4·00	0·19		102·99	45·00	
3·00	4·80		0·75	102·24	54·00	
	4·99		1·99	100·25	63·00	Centre of Brookfield Road.
26·74	26·49	5·39	5·14	100·00		
26·49		5·14		100·25		
·25		·25		·25		

The level is set up at point 1, and staffs are held at a and b.

The reading on staff a is 2·50. It is the first reading taken, and is therefore entered in the backsight column of the level book. It is likewise the commencement of the line, and the distance 0·00 is therefore entered in the distance column, on a line with it.

The reading on staff b is 4·90; it is the last reading which will be taken before the position of the level is changed, and is therefore entered in the foresight column. Point b is 900 links from point a, and 900 links is therefore entered in the distance column.

The reading at b was higher than that at a, indicating a fall, and the latter is therefore subtracted from the former, and the result entered in the column of falls.

Now a height above base or datum of 100ft. below the first point in the section, has been assumed; that is, it has been assumed that point a is 100ft. above a base line which is required for the purpose of plotting the section,

a   b   c   d   e   f   g   h

BS FS BS FS BS FS BS FS BS FS BS FS BS FS BS FS

1 2 3 4 5 6 7

$4.90 - 2.50 =$ Fall 2.40	$4.00 - 2.25 =$ Rise 1.75	$4.20 - 3.25 =$ Rise 0.95	$4.80 - 2.30 =$ Rise 2.50	$4.19 - 4.00 =$ Rise 0.19	$4.30 - 4.05 =$ Fall 0.75	$4.99 - 3.00 =$ Fall 1.99
L.V. 100.00	L.V. 97.60	L.V. 99.35	L.V. 100.30	L.V. 102.80	L.V. 102.99	L.V. 102.24
− F. 2.40	+ R. 1.75	+ R. 0.95	+ R. 2.50	+ R. 0.19	− F. 0.75	− F. 1.99
L.V. 100.00	L.V. 97.60	L.V. 99.35	L.V. 100.30	L.V. 102.80	L.V. 102.99	L.V. 102.24
L.V.	L.V. 97.60	L.V. 99.35	L.V. 100.30	L.V. 102.80	L.V. 102.99	L.V. 102.24	L.V. 100.25

100.00 | 97.60 | 99.35 | 100.50 | 102.80 | 102.99 | 102.24 | 100.25

1000. | 900. | 1800. | 2700. | 3600. | 4500. | 5400. | 6300.

-Section-

JLLUSTRATION No. 17º.

and therefore as b is 2·40 lower than a, the height above base of b must be 100·00 minus 2·40 or 97·60, which is obtained by subtracting the fall 2·40 from the previous reduced level, and the result 97·60 is entered in the column of reduced levels.

The backsight to the staff still held at b, when the level has been set at point 2, reads 4·00; and the foresight to the staff at c is 2·25. The lower reading indicates a rise, and the 2·25 is therefore subtracted from 4·00, and their difference is entered in the column of rises.

Now point b is 97·60ft. above base, and as c is 1·75 higher, we must add this difference to the reduced level at b to obtain the reduced level of c, 99·35, which is entered accordingly.

Point c is 1,800 links from a, and 1,800 is therefore entered in the distance column.

This process of calculating the rises and falls, and adding or subtracting to give the heights above base, is continued until the whole of the levels have been reduced.

It is usual, in addition to booking the backsights, foresights and distances, to calculate the *rises* and *falls*, in the field, so that errors of a serious character may be detected; but usually the *reduced levels* are calculated on return to the office.

When the rises and falls have been ascertained, and before the reduced levels are entered, these columns should be totalled, when the difference between the totals of foresights and backsights and those of the rises and falls will be identical if no mistake has been made in the work of reducing.

This identity of results shows that the calculations have been accurately performed, and the columns should always be totalled and the comparison made, to guarantee accuracy.

As soon as the work in connection with calculating the rises and falls has thus been proved, the former are added

and the latter subtracted from the reduced levels immediately preceding. This reduced level at the commencement will be the assumed height above base.

When the column of reduced levels, or heights above base, has been completed, the difference between the first and last reduced levels will equal the difference between the total backsights and total foresights if the work in connection with that column has been accurately performed.

It is so common to hear students say that their levels are satisfactory because the book proves, and on the other hand that they must have made some mistake in their levels because the book will not prove, that although it may seem to those of some experience an unnecessary reminder, for the benefit of those who are quite unaccustomed to the work, it may be as well to point out (a) that any figures entered in the level book at random will prove if the calculations are accurate; so that if a book does not prove there must be a mistake in the calculations, which should be searched out and corrected; and (b) that the fact of the book proving is no guarantee of the accuracy of the levels at all, but merely a check on the arithmetical work in reducing. Check levelling, which has already been touched on, and which will be further dealt with in this chapter, is the method of proving the accuracy of the field operations.

LEVEL BOOK IN COMPOUND LEVELLING.

(**304**).—I will now give an example of the level book in a case of compound levelling, although the process of reducing is, of course, precisely similar to that in the case of simple levelling.

By carefully following the diagram and section, Illustration No. 180, and comparing them with the accompanying level book, it will be an easy matter to

LEVEL BOOK.—COMPOUND LEVELLING.

Backsight.	Intermediate.	Foresight.	Rise.	Fall.	Reduced Levels in Feet.	Distance in Links.	Remarks.
4·50					100·00	0·00	B.M. N. Corner of step No. 68, Brook St.
	4·70			·20	99·80	0·00	Level of Brook Street (centre of road).
	3·80		·90		100·70	1·60	
	5·01			1·21	99·49	2·30	
	2·60		2·41		101·90	3·50	
	3·20			·60	101·30	4·10	
	5·00			1·80	99·50	4·70	
	4·59		·41		99·91	5·20	
4·80		8·07		3·48	96·43	6·30	
	5·20		·19	·40	96·03	7·50	
	5·01		·20		96·22	8·10	
	3·05		1·96		98·18	8·70	
	3·10			·05	98·13	9·60	
	2·05		1·05		99·18	10·10	
	2·00		·05		99·23	10·45	
	3·20			1·20	98·03	10·95	
4·68		3·30		·10	97·93	11·80	
	4·00			·68	98·61	12·90	Crossing of proposed roads
	4·49			·49	98·12	13·40	
	4·37		·12		98·24	13·90	
	4·03		·34		98·58	14·85	
	4·05			·02	98·56	15·35	
	5·07			1·02	97·54	15·85	
	4·01		1·06		98·60		B.M. set off in wall, 13, Powell Avenue (S. Corner)
		3·00	1·01		99·61	16·95	Level of Powell Avenue (centre of road)
13·98		14·37 / 13·98	10·18	10·57 / 0·18	100·00 / 99·61		
		·39		·39	·39		

trace the steps which have been taken in this case. From the first backsight to the first intermediate, there is a fall; the backsight is therefore subtracted from the intermediate, the result is entered in the column of falls, and also subtracted from the height above base of the first point, which has been assumed to be 100ft., to obtain the height above base of the second point.

There is a rise from the first intermediate to the
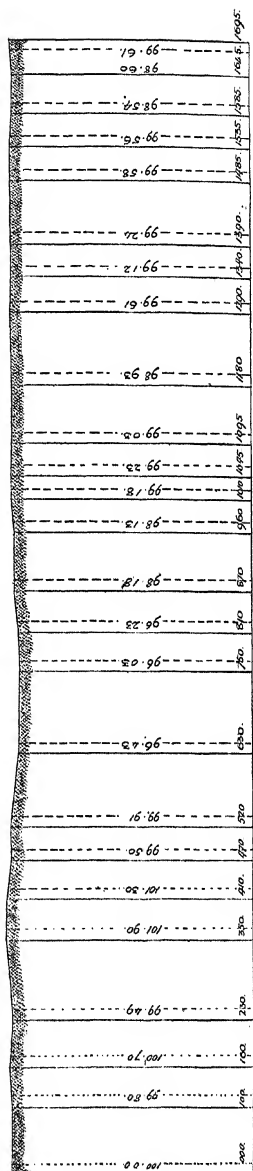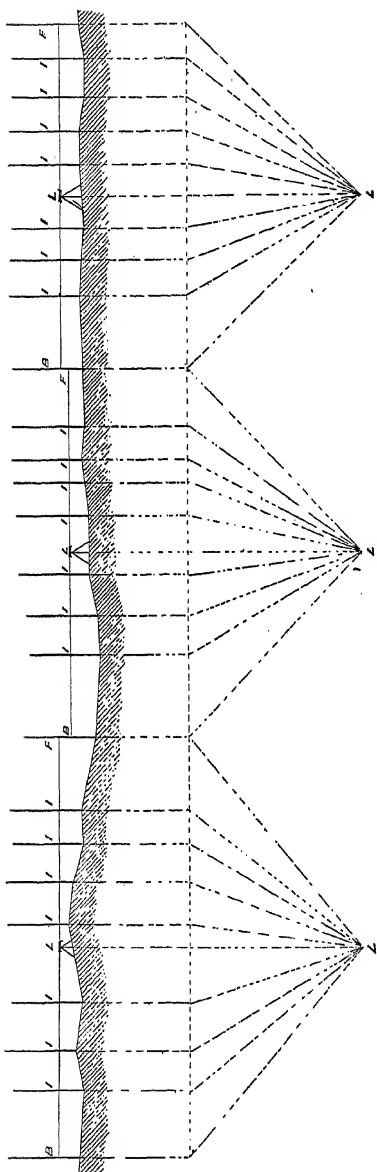
—Section—

ILLUSTRATION No. 180.

second intermediate, and the reading of the second is therefore subtracted from the reading of the first, the difference is entered in the column of rises, and also added to the reduced level of the first intermediate, to get the level of the second intermediate, and this process is repeated to the end of the readings in the first line of collimation.

Finally, the last intermediate is subtracted from the first foresight, the difference entered in the column of falls, and subtracted from the height above base of the previous point, which completes the calculations with regard to the readings taken in the first line of collimation.

The readings in the second line of collimation are next reduced.

There is a fall from the second backsight (the height above base of which has already been fixed in the first line of collimation) to the first intermediate in the second line, the former is therefore subtracted from the latter, the difference is entered in the column of falls and deducted from the height above base of the backsight, to obtain the reduced level of the first intermediate in the second line of collimation.

This process is continued until the whole of the levels have been reduced.

The accuracy of the calculations is proved, as in the last example, by comparing the differences between the totals of backsights and foresights, rises and falls, and first and last reduced levels, and finding them all identical.

THE LEVEL BOOK IN A CASE OF COMPOUND LEVELLING WITH CHECK LEVELS.

(**305**).—I will now give an example of a level book where two lines of section and flying or check levels connecting them have been taken.

LEVEL BOOK.—COMPOUND LEVELLING, WITH CHECK LEVELS.

Backsight.	Intermediate.	Foresight.	Rise.	Fall.	Reduced Levels.	Distance in Chains.	Remarks.
			Line	No. 1.			
4·80					100·00	0·00	Commencement of Line 1.
	3·01		1·79		101·79	1·00	
	5·10			2·09	99·70	2·00	
	10·09			4·99	94·71	3·00	
1·03		14·00		3·91	90·80	4·00	.
	5·70			4·67	86·13	5·00	
	5·89			·19	85·94	6·00	
	4·60		1·29		87·23	7·00	
10·00		2·20	2·40		89·63	8·00	Crossing of section lines.
	11·20			1·20	88·43	9·00	
	5·00		6·20		94·63	10·00	
	5·10			·10	94·53	11·00	
		3·15	1·95		96·48	12·00	End of Line 1.

Check Levels from end of Line No. 1 to commencement of Line No. 2.

Backsight.	Intermediate.	Foresight.	Rise.	Fall.	Reduced Levels.	Distance in Chains.	Remarks.
5·50		8·30		2·80	96.48 93·68		Last point in Line 1. First point in Line 2.
			Line	No. 2.			
8·30					93·68	0·00	Commencement of Line 2.
	11·31		2·91	3·01	90·67	1·00	
	8·40				93·58	2·00	
	8·31		·09		93·67	3·00	
	8·90			·59	93·08	4·00	
12·90		12·35		3·45	89·63	5·00	Crossing of section lines.
	11·00		1·90		91·53	6·00	
	8·00		3·00		94·53	7·00	
	7·10		·90		95·43	8·00	
	5·01		2·09		97·52	9·00	
		2·50	2·51		100·03	10·00	End of Line No. 2.

Check Levels from end of Line No. 2 to beginning of Line No. 1.

Backsight.	Intermediate.	Foresight.	Rise.	Fall.	Reduced Levels.	Distance in Chains.	Remarks.
2·50 9·05		7·60 3·98	5·07	5·10	94·93 100·00		
54·08		54·08	32·10	32·10	100·00		.

The process of reducing the levels is precisely the same as in the last example, but the present will illustrate the way in which the check levels prove the accuracy or inaccuracy of the field operations..

The levels in the foregoing level book have been reduced in precisely the same manner as those in the previous examples, and the accuracy of the calculations has been proved similarly.

A careful perusal of the level book will, it is thought, sufficiently explain what has been done, but the following remarks may be offered.

It will be seen that on completing the levels on line 1, the instrument was moved, and a backsight to the staff still held at the last point in line 1, and a foresight to the staff held at the first point in line 2 taken.   This reading was entered in the book as a foresight, 8·30, with the object of keeping the check levels quite distinct from the others, and the same reading therefore is also entered as a backsight, commencing line 2.

The foresight 12·35 in the second line has been taken at the same point as the foresight 2·20 in the first line, viz., at the point at which the section lines cross each other, and consequently the two sections being connected by the check levels, the reduced level at this point as found in the two lines is identical, as of course it should be.

This in itself is a guarantee of the accuracy of the levelling operations so far, and is very convenient when the time comes for plotting the sections.

The levels in the second line being completed, flying levels were taken back to the commencement of line 1.

The level was not moved between taking the level of the last point in line 2, and the first reading in the flying levels, but the former has been booked as a foresight and the latter as a backsight for the sake of clearness rather than

making one entry of the reading as an intermediate, as might have been done. On reducing the levels, the height above base of point 1, according to the check levels, is 100ft., which equals the height assigned to it at the commencement. This identity of the reduced levels proves the accuracy of the field operations, whilst the differences between the totals of the backsights and foresights, and the totals of the rises and falls being equal, shows that the arithmetical calculations have been correctly made.

PREPARATION OF SECTIONS.

(**306**). *Sections.*—The plotting of the section is a very simple matter, and beyond neat and accurate draughtsmanship little is required.

The careful study of a few well-drawn sections is all that is necessary to give a good idea of the way in which they should be got out.

The examples which can be given here are few, and they must necessarily be small and somewhat diagrammatic in their nature, but a reference to Illustrations Nos. 179 and 180 will give a fair idea.

In the illustrations in question the points at which the levels were taken have been indicated by a hard line, and the height above base of each point has been shown against it with a dotted line. In practical sections the hard line is not necessary, and the dotted line may take its place except at the commencement and end of the section.

The method of plotting is simply as follows :—The base line is laid down on the paper with the T-square or straight-edge, the distances taken from the distance column in the level book are set off on it to a convenient horizontal scale, indefinite perpendiculars are erected at each of the points by aid of a set-square working on the edge of the T-square or straight-edge, the heights above base from the

column of reduced levels are set off to a convenient vertical scale on these perpendiculars at the respective points representing them, and straight lines are drawn from point to point to give the sectional line.

The distances from the distance column are then usually figured on the section just above the base line, and the heights above base or reduced levels are figured against dotted perpendiculars down the centre of the section.

Lines are usually drawn on a plan indicating the positions of the lines on which the levels have been taken, and they are numbered or lettered to correspond with the sections, as section on line No. 1, or section on *A B*, etc., etc.

When the drawings are finished, inked in, printed, and cleaned, a brush charged with sepia is usually carried along just below the sectional line, thus giving a narrow band of colour, as indicated by shading in the sections in Illustrations Nos. 179 and 180. The brush should be fairly large, well charged, and the colour should be thin.

It has already been pointed out that the vertical scale is usually much larger than the horizontal scale. The question of the scale to be used must be governed by the requirements of each particular case. The horizontal scale might be one chain to an inch, and the vertical scale 10ft. to an inch ; or the horizontal scale might be 10 chains to an inch, and the vertical scale 25ft. to an inch.

In the illustrations given in this chapter the vertical scale used in the diagrams is twice that used for the sections.

CHAPTER XXIII.

CONTOURING OR CONTOUR LEVELLING.

Definitions — Practical Object — Utility—Illustrations —
Points of Similarity and Dissimilarity between
Contouring and Ordinary Levelling—Method of
Procedure—Contouring a Town.

DEFINITIONS.

(**307**).—Contouring has already been defined in Chapter
XXII., Art. 289, as " levelling for the purpose of ascer-
" taining a number of points of equal level value, with
" the object of showing on a plan or map contour lines
" representing the variations in level in different parts of
" any tract of land," and contour lines (Art. 290), as
" the lines by which the result of contouring is shown
" practically representing the outlines which would be
" formed by water rising to a number of levels round a
" hill or series of hills, and thus showing the variation
" in altitude between different localities."

PRACTICAL OBJECT.

(**308**).—From these definitions it will be gathered that
the practical object of contouring is to show on a plan
lines which will indicate the general slopes of the land it
represents.

UTILITY.

(**309**).—Obviously a plan or map giving such in-
formation accurately would be very valuable to any one
engaged in connection with undertakings in the district,
such as the formation of roads or construction of sewers,

as suggesting the best course for the proposed roads, etc., to take, in order that satisfactory gradients might be obtained.   Clearly, too, it would be equally valuable to those concerned with the construction of railways, canals, waterworks, etc.

ILLUSTRATIONS.

(**310**).—Suppose we watch, from some point far above the earth, the gradual flooding of a district. We see the water first fill the valleys, cover the lower lying lands and gradually rise until even the top of the highest hill is submerged.

Let the water be rising at the rate of, say, 10ft. per hour, and let us make special observation every five hours.

We observe the outline formed by the water as it breaks round the higher lands, and at each observation note the change in form and gradual reduction in extent of those outlines as the water rises higher and higher.

Suppose it be possible for us at each observation to exactly record on a correct map of the flooded district the outlines formed by the rising water.

These lines would represent contours of 50ft. altitude ; and by consulting such a map we should see at a glance the rise or fall between any number of points.

Again, if we knew the level value of the first or lowest contour in its relationship to some datum—as, for instance, the ordnance datum—we might compare the altitudes of various points with others in different districts, the level values of which according to that standard are known.

Once more, if we could find a hill in the form of a regular cone and ascertain the exact positions of a number of points of equal level value, in a series of rings encircling it and rising tier upon tier above each other, contour lines drawn through the points on each tier would be circles parallel with its base, diminishing in size as the altitude increased.

Thus the contour on *aa* Illustration No. 181 would be
represented on the plan or map by the innermost circle;
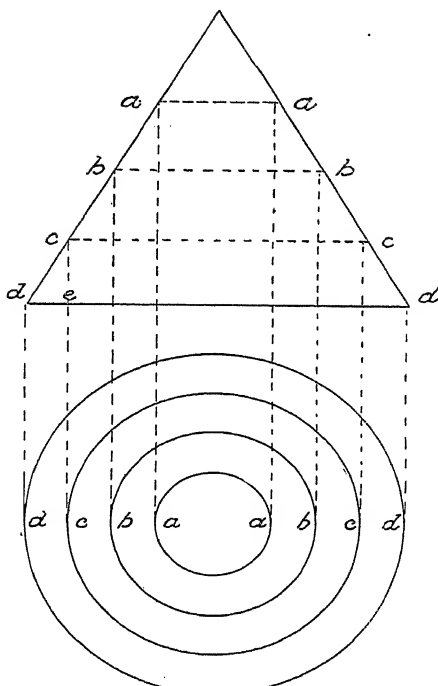
ILLUSTRATION No. 181.

the contour on *bb* by the second circle, that on *cc* by the
third circle, and that on *dd* by the outer circle.

Once more, if the plan were drawn to scale and the alti-
tude between the contours were given, the rise or fall from
point to point could be ascertained at a glance.

Thus, supposing the horizontal distance *cd*, Illustration
No. 181, to be 200ft., and the altitude of the contours 50ft.,
it would be a simple matter to arrive at the gradient of the
land between these points.

POINTS OF SIMILARITY AND DIFFERENCE BETWEEN
ORDINARY LEVELLING AND CONTOURING.

(**311**).—In ordinary levelling we ascertain the difference in level of various points in the earth's surface. In contouring we seek a number of points having the same level value in a series of lines at regular increasing altitudes.

Again, from ordinary levelling we are able to show in *vertical* sections the rise or fall between any number of given points; by contouring, to give in outline on *plan* what may be spoken of as a number of *horizontal* sections, or sections on a horizontal plane.

Contour levelling is for the purpose of giving the general slopes of land in all directions around a centre or number of centres; ordinary levelling, the exact slopes along a particular line or lines only.

There is this similarity of purpose in these two distinct branches of levelling; they are both with the object of giving a correct idea of the inequalities or general slopes of lands; compound levelling—exact and detailed sections; contour levelling—less detailed information, but extending over a wider area.

METHOD OF PROCEDURE IN CONTOURING.

(**312**).—The most simple method of contouring, where the circumstances will admit of its being done, is—

> (1) To lay out a series of lines from boundary to boundary of the tract of country to be contoured, the lines crossing each other in, or radiating from the same point in its highest part, and so ranged that they will as far as possible cut the contour lines at right angles.
>
> (2) To carefully level one of these lines, driving down pegs at each end and at regular altitudes.
>
> (3) Level up from the nearest ordnance bench

mark to the first point, in order that its relative height to ordnance datum may be given and the contour lines on plan be marked with their heights in relation thereto.

(4) Set up the level at a convenient distance from the first point on lowest contour and have the staff held there and carefully note the reading. Now have the staff taken to · the second line and moved up and down on it until the reading is identical with that at the first point, and have a peg driven down to mark the position.

(5) When the distance necessitates the level being removed, a second reading to the staff still held at the last point, with the level in the new position, is taken ; the reading each time in future until the level is again removed being identical with the first reading after the resetting of the instrument. This process is repeated until the first peg in the first contour is returned to, when the second contour is commenced, and so on, until all the contours have been dealt with.

(6) Chain the lines together with sufficient additional lines to enable them to be plotted and proved, and carefully note the positions of all the pegs in the various contours in the field book, so that when the figure is plotted and the positions of the pegs on the lines indicated, the contours may be drawn through them. The positions of the radiating lines may be simply fixed by taking their bearings with the theodolite from the central point in which they meet. (For the theodolite, see Arts. 89 to 104.)

CONTOURING A TOWN.

(**313**).—In contouring a town the same rules are to be followed as near as the circumstances of the case will admit. One of the principal roads may be selected for the line to be levelled, and where a correct map of the district already exists, the positions of the various points in the several contours may be mapped from measurements taken from and to fixed points shown thereon. Otherwise a complete survey of the locality will, of course, be necessary.

Another method is to take a number of "spot" levels at regular intervals, and starting from fixed points, along the roads, selecting those at as nearly as possible right angles with each other, and then to sketch in the contours, guided by the levels obtained.

The altitude interval between the contours must depend on the nature and extent of the country to be contoured and the funds available for the purpose. The most usual interval is from 50ft. to 100ft.

Contouring may be conducted with the ordinary levelling staff, but the contouring staff, which is divided into feet and provided with clamping slides, which may be fixed at any height on it, is of course preferable.

TRAVERSING.

*General Observations—Procedure in the Field—Error in
Reading from the Wrong Vernier—How Guarded
Against—Latitude and Departure—Plotting by—
Calculation of—Traverse Tables—The Traverse Book
—Example of—How made up—To Calculate the
Area from the Traverse Book—Plotting—Magnetic
and True North.*

(**314**). *Definitions.*—Traversing is a method of survey
in which the lengths of lines and their bearings are taken,
and which are subsequently plotted by latitude and
departure.

A traverse may be plotted by protractor, but as it is not
so accurate and there is nothing I know of to recommend
it, I shall not refer further to that method.

A traverse may be (*a*) a closed traverse, or (*b*) an
unclosed traverse. A closed traverse is one in which the
operations are completed at the station from which they
were commenced. An unclosed traverse is one in which
the starting point is not so closed upon.

Traversing may be performed by (*a*) the theodolite; (*b*)
the prismatic compass.

GENERAL OBSERVATIONS.

(**315**).—Traversing is a method of surveying more
particularly applicable to boundary surveys, to the survey
of winding roads, rivers, etc., through dense woods, and
to any case where the land is obstructed so as to prevent
lines being measured across it. It is not confined, how-
ever, to such cases, and may be, and is frequently with
advantage, applied to large surveys.

The traverse survey depends on our ability to obtain :—
(a) the lengths, and (b) the bearings, of a series of lines following the course of the boundaries, roads, etc., to be surveyed.

The lengths of the lines are usually ascertained by chaining, the bearings of the lines are taken with the theodolite or prismatic compass. The former should always be used when great accuracy is required.

The elucidations will be much more simply followed if it is impressed on the mind at the outset—

> (1) It is the bearing of each separate line with regard to the north which is required ;
> (2) All bearings are taken as east of north, except, of course, those which are due north.

In the following explanations I shall suppose the use of the theodolite, as, if traversing with it is perfectly understood, no difficulty will be experienced in performing similar work with the prismatic compass. I shall, however, make reference to traversing with the prismatic compass before the chapter closes.

The theodolite and its use has already been fully dealt with in Chapter VII., Arts. 89 to 104, and it will therefore be assumed that the reader is already thoroughly acquainted with it, or that he will turn to the chapter referred to and familiarise himself therewith.

It is sufficient to say here that all the operations must be performed with the greatest accuracy, and this applies with equal force to the chaining as to the theodolite readings.

From the definitions given at the commencement of this chapter it will be gathered that a closed traverse is one in which the operations are completed on the same point as that from which they were commenced ; and an unclosed traverse is one in which the starting point is not so closed upon.

To be able to close a traverse is obviously a great advantage, as it offers a ready means of proving the accuracy of the work. Indeed, in extensive work, each day's operations should be closed so that its accuracy may be proved before the next day's operations are commenced.

PROCEDURE IN THE FIELD.

(**316**).—The method of procedure in traversing is as follows :—

A number of stations are established round the boundary, etc., to be traversed, each station being in such a position that two others, one on either side, may be sighted therefrom. These stations may be established as the work proceeds.

Illustration No. 182 represents a closed traverse in which the arrows mark the positions of the stations, and the arcs the angles indicating the bearing of the lines east of north.

The theodolite is adjusted over *the extreme western station*, the vernier is set to zero, and the whole instrument turned until the needle reads north. The
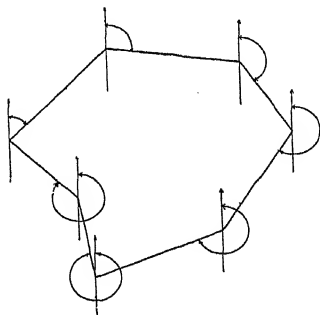
ILLUSTRATION No. 182.

whole instrument is then clamped, the vernier plate released and turned to the right, and the second station

sighted. The angle is now read from the scale and vernier which was set to zero, which, for the sake of distinction, I will call the **A** vernier, as opposed to the other the **B** vernier.

This reading, it will be observed, is the declination of the first line east of north. The line is chained, and the reading and its measurement booked.

The theodolite is now adjusted over the second station, without unclamping the upper plate, that is, with the reading taken at the first station still on the instrument, and station 1 is again sighted.

Now, referring to Illustration No. 183, it will be clear that, if considering the line 1 2 from 1 we find it to
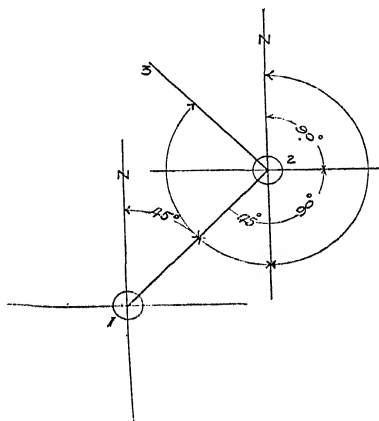
ILLUSTRATION No. 183.

be 45 degrees east of north, considering the same line from station 2, it will be 180 degrees plus 45 degrees, or 225 degrees east of north ; and therefore, as the reading taken from vernier **A** at station 1 is still on the instrument when it is adjusted over station 2, and station 1 is again sighted therefrom, vernier **B**, which reads 180 degrees when vernier **A** reads zero, will correctly give the

declination of line 1 2 east of north as regarded from station 2 ; and that by releasing the vernier plate, turning the telescope to the right, sighting station 3, and taking the reading from vernier **B**, the declination east of north of line 2 3 is obtained.

By this means the necessity of resetting the instrument to zero each time a reading is required is avoided.

Again, with the vernier plate still clamped, that is, with the last reading still upon the instrument, the theodolite is adjusted over station 3, station 2 is sighted a second time, the whole instrument is clamped, the vernier plate is released, the telescope is turned to the right, station 4 is sighted, and the angle is read from the scale and vernier **A**.   This reading is the declination of the line 3 4.

Each line is chained, and the bearing and distance booked as explained in the case of the first line, and this process is repeated until the whole of the lines have been dealt with ; and, in the case of a closed traverse, until the original starting point is returned to.

The readings, it will be noted, are taken alternately from opposite verniers.

Apart from the traverse book an ordinary field book is kept, in which the lines and all off-sets, insets, etc., are entered, as in the case of a chain survey.

TO GUARD AGAINST ERROR IN READING FROM THE WRONG VERNIER.

(**317**).—To guard against reading from the wrong vernier, either one or both of the following precautions are advisable :—

> (1) Have the verniers marked **A** and **B**, and carefully note, in recording each reading, from which vernier it was taken ;

(2) Observe that the readings of the vernier and the magnetic needle agree, or approximately so, which will be the case if the correct vernier is read from.

The traverse book and the method of booking will be better understood after the manner of plotting a traverse has been described.

PLOTTING BY LATITUDE AND DEPARTURE.

(**318**).—Given the lengths of any number of lines and their bearings east of north, it will of course be possible to plot them on paper by the common scale and the protractor, but this is not the most convenient or usual way of plotting a traverse.

In plotting a traverse the latitude and departure of the various lines have first to be obtained. This may be done either by :—

(*a*) Calculation ;
(*b*) Reference to traverse tables, which will be dealt with later on.

The latitude is the distance north or south of the east to west line ; the departure is the distance east or west of the north to south line.

Consequently, given the latitude and departure of any number of lines, and they may be accurately plotted without the aid of the protractor, indeed they may be far more simply and accurately plotted with the common scale.

For example, referring to Illustration No. 184, the N.S. and E.W. lines are drawn on the sheet of paper crossing each other at *A*, which represents the first station in the traverse.

Now, instead of laying down the line *A B* 300 links long and with a bearing 75 degrees east of north with a protractor, the latitude, which is in this case 77·6 north, is laid off on the north to south line, and the departure, which is in this

case 289·8 east, is laid off on the east to west line.   Lines
running east and west and north and south are drawn
through these points, as shown by dotted lines, and by their
intersection indicate the point to which the line *A B* is to
be drawn in order that it may occupy its correct position and
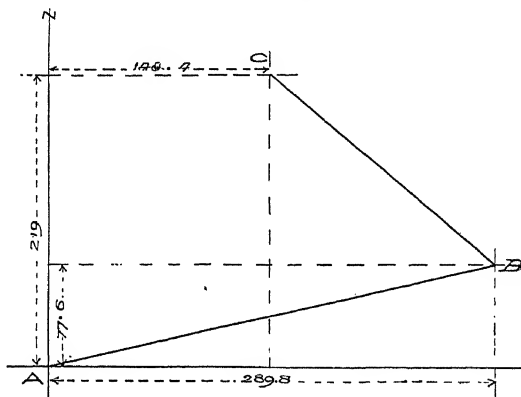
ILLUSTRATION No. 184.

be of its true length. ·Thus the position and length of the
line is more accurately plotted than could be done with
the protractor.

Again the line *B C* is 315 degrees east of north, and its
length is 200 links.   The latitude of this line is 141·4
north, and the departure 141·4 west, the latitude and
departure in this case, however, being in relation to
point *B*.

Now the latitude of line *A B* was 77·6 north, and there-
fore the latitude of line *B C*, which is also north, must be
added to it to get the total latitude from station *A*, and so
we get as the total latitude 77·6 + 141·4 = 219 north.
This is laid off on the north to south line from *A*, as in
the case of the first line.   The departure of line *A B* was
289·8 east, and the departure of the line we are now dealing
with is 141·4 west, and therefore to get the actual (spoken

of as the total) departure from A, the latter must be subtracted from the former to get the total departure from A. Thus we get $289 \cdot 8 - 141 \cdot 4$, or $148 \cdot 4$. This is laid off on the east to west line from A, and the position of the second line $B\ C$ is fixed, as in the case of the first line $A\ B$.

The latitude and departure, and the total latitude and total departure of all the lines of the traverse are found and plotted in the same way.

In the case of a closed traverse, assuming the work has been performed with accuracy in the field, and all the calculations and the plotting are perfect, the last line will be found to terminate at the exact same point as that from which the work was commenced.

It will be observed that, if the work is always commenced at the extreme *western* point, the *total* departure will always be *east*.

There are some advantages from following this course, that is, commencing at the extreme western point, and therefore I shall assume the same in the examples which follow.

The foregoing simple illustrations are given only to convey a clear idea of the method by which a traverse is plotted, and the latitude and departure are assumed to have been calculated or ascertained from traverse tables.

LATITUDE AND DEPARTURE—HOW CALCULATED.

(**319**).—I will now show how latitude and departure may be calculated.

To make this branch of our subject perfectly clear we have only to observe :—

> (1) Having given the hypotenuse of a right-angled triangle and the angle subtended by the perpendicular (the side opposite the given angle), we may find the lengths of the base and perpendicular respectively by trigonometry ;

(2) That what we really do in traversing is to obtain this information concerning a number of right-angled triangles, as indicated by Illustration No. 185 ;

(3) That in calculating the latitude and departure we merely find the base and perpendicular respectively of these right-angled triangles.

Referring to the right-angled triangle, Illustration No. 185, we may find the base and perpendicular by trigonometry, by the following formulæ :—

Base (side adjacent to the given angle) = Hyp. × Cos. A.
Perp. (side opposite the given angle) = Hyp. × Sine A.
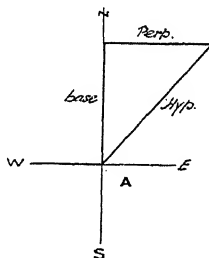
ILLUSTRATION No. 185.

As the base in this case is identical with latitude, the perpendicular is identical with departure, and the length measured in traversing is identical with the hyp., we may write the formulæ thus :—

Lat. = Length × Cos. A.
Depart. = Length × Sine A.

The working out of these formulæ is an extremely simple matter with the aid of logarithms, and as the subject is really part of trigonometry it has been dealt with in Chapter XXVII. on "The Solution of Triangles." (See Article 368.)

TRAVERSE TABLES.

(**320**).—It has already been pointed out that latitude and departure may be either calculated or obtained from traverse tables, and the mention of the latter will be a sufficient intimation that the former is not frequently resorted to for ordinary purposes.

The best traverse tables are Boileau's; they are more extensive than some others published.

There are one or two points to be noticed with regard to the traverse tables :—

> (1) They only give angles up to 90 degrees;
>
> (2) The angles given are those formed by the line chained and the north to south line, viz., the vertical angles, and not the declination of that line east of north as read in the field;
>
> (3) Consequently no distinction is made in the tables as to whether the latitude is north or south, or the departure east or west;
>
> (4) That the distances are simply given as integers, so they may be regarded as links or otherwise.

From these observations it will be gathered that some slight calculations are necessary before the tables can be consulted.

To determine latitude and departure, the circle by which we measure angles must be considered as divided into quadrants, the quadrant in which a line falls determining whether latitude is north or south, and the departure east or west.

To ascertain the angle to be looked out in the tables, viz., the vertical angle, the angle formed by the given line and the north to south line, in whichever quadrant it may fall, must be calculated.

Thus, if the declination east of north is less than 90 degrees, this will be the vertical angle, and being in the

first quadrant, the latitude will be north and the departure east. The latitude and departure, then, in this case may be found direct from the table without the slight calculation necessary in other cases.

Again, if the declination is greater than 90 degrees but less than 180 degrees, or in the second quadrant, to find the vertical angle, viz., the angle which the given line forms with the north to south line, we must subtract the reading from 180 degrees, and the latitude will be south and the departure east.

Again, if the declination is greater than 180 degrees but less than 270, or in the third quadrant, to find the vertical angle we must subtract 180 degrees from the reading, and the latitude will be south and the departure west.

Once more, if the declination east of north is greater than 270 but less than 360, or in the fourth quadrant, to obtain the vertical angle in that quadrant, we must subtract the reading from 360, and the latitude will be north and the departure west.

Illustration No. 186 shows at a glance the latitude and departure for each of the quadrants.
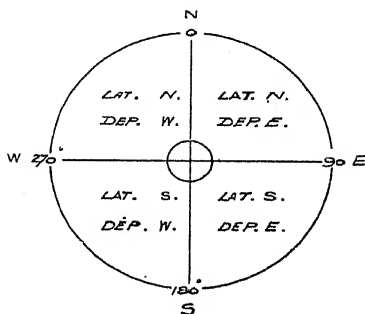
ILLUSTRATION No. 186.

The traverse tables give the latitude and departure by inspection. The vertical angles are given at the top and bottom of each page and the length of the line—the

distance—latitude and departure, are all found adjoining each other in the same horizontal line. No difficulty can be found in consulting the tables.

THE TRAVERSE BOOK.

(**321**).—The traverse book may be in many forms, but the example given on the following page will be found a convenient one. The entries made in it are those relating to the traverse supposed in the following example.

In following this example the entries in the book, Illustration No. 187, and the traverse tables should be referred to, and the various entries, etc., carefully compared.
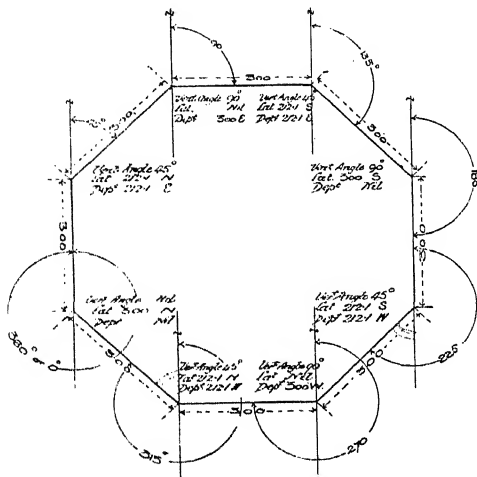
ILLUSTRATION No. 187.

If this is done, no difficulty will be experienced in following the figures in the various columns, nor the process by which they have been arrived at.

The latitude and departure have been taken from "Chambers' Mathematical Tables."

EXAMPLE OF TRAVERSE BOOK.

Line.	Distance.	Vernier.	Bearing.	Vertical Angle.	LATITUDE.		DEPARTURE.		TOTAL LATITUDE.		Total Departure East.	Double Longitude.	DOUBLE AREAS.	
					North.	South.	East.	West.	North.	South.			North.	South.
1	300	A	45	45	212·1		212·1		212·1		212·1	212·1	44986·41	
2	300	B	90	90	··		300		212·1		512·1	724·2		262219·23
3	300	A	135	45		212·1	212·1			300	724·2	1236·3		434520·00
4	300	B	180	90		300		212·1		512·1	724·2	1448·4		262219·23
5	300	A	225	45		212·1		212·1		512·1	512·1	1236·3		
6	300	B	270	90	··			300		300	212·1	724·2		
7	300	A	315	90	212·1	··	··	212·1	··	··	··	212·1	44986·41	
8	300	B	360	45	300		··							

```
                                89972·82        958958·46
                                                 89972·82
                                              2)868985·64
                                                 4·34492
                                                      4
                                                 1·37968
                                                     40
                                                15·18720
```

AREA.—4 acres, 1 rood, 15 poles.

I have taken for illustration a regular octagon, because, in so doing, a figure is secured, the properties of which are well known, and the area of which can be so simply calculated, that a ready proof of the accuracy of the rule for calculating the acreage from double longitudes is afforded. This is only continuing the method which has been as far as possible followed throughout the work, viz., that of taking examples which make the truth of the rules propounded self-evident.

In Illustration No. 187, the intersections of the north to south and the east to west lines represent the positions at which the theodolite was set up. The angles as read by the instrument are shown, the stations are numbered, and the lengths of the lines are given; whilst a note against each station shows the quadrant in which the line falls, the vertical angle, and the latitude and departure for each line.

TO MAKE UP THE TRAVERSE BOOK.

(**322**).—When we complete our day's work in the field, the first four columns of the traverse book only are filled up.

The first thing we do in the way of completing the book is to make the slight calculations referred to in Art. 320, and fill in the column of vertical angles.

Then, taking the distances and the vertical angles from the traverse book, we consult the traverse tables and fill in therefrom the latitude and departure for each line.

Now, so long as the latitude is unchanged, viz., so long as it continues north or south, we add the total latitude of one line to the latitude of that which follows for total latitude, and enter the results in the column of total latitude; but when the latitude changes from north to south or south to north, we subtract the latitude of that line from the last entry in the column of total latitudes for

the next entry in that column, we do likewise with regard
to departure, adding or subtracting as the case may be,
and filling in the results in the column of total departure.

To obtain the double longitudes, we add the total
departure of one line to the total departure of the line
immediately preceding it. Thus in the case of the first
line there is no preceding figure to add, and therefore the
double longitude and the total departure are indentical.
Then we have added 212·1 to 512·1 and entered 724·2 in
the double longitude column. Again, we have added 512·1
to 724·2 and entered the result 1236·3 likewise, and so on
to the end. When we arrive at the last point, that on
which the work commenced and finished, there is no
latitude or departure of course, and there being nothing
to add therefore to the last figure, 212·1, we enter it in the
column of double longitude.

TO CALCULATE THE AREA FROM THE TRAVERSE BOOK.

(**323**).—To obtain the area by calculation from the
traverse book, we simply multiply the double longitude
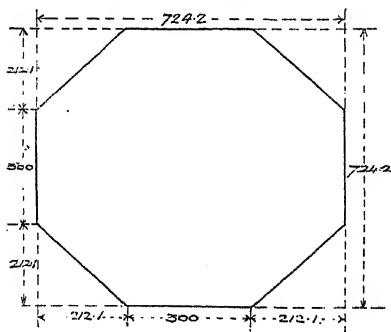
ILLUSTRATION No. 188.

for each line by the latitude for that line, entering the
result in the north or south columns headed double areas,
according as the latitude is north or south. We then

total the two columns and subtract the lesser total from
the greater, which gives the double area (assuming the
measurements to have been taken in links) in square
links ; this may be reduced to acres, roods and poles, as
explained in Chapter XVI., Art. 213, on " The Compu-
tation of Areas."

I pointed out (Art. 321) that a regular octagon had been
taken for example, because in so doing we had a figure the
properties of which were well known, and the area of
which could be so simply calculated that a ready proof of
the accuracy of the rule for calculating the area from
double longitudes would be offered.

Now, referring to Illustration No. 188 and the traverse
book, we see that we might compute this figure as a
square, thus :—

$$
\begin{array}{r}
300 \\
212 \cdot 1 \\
212 \cdot 1 \\
\hline
724 \cdot 2 \\
724 \cdot 2 \\
\hline
14484 \\
28968 \\
14484 \\
50694 \\
\hline
524465 \cdot 64
\end{array}
$$

From which subtract for the four triangles :—

$$
\begin{array}{r}
212 \cdot 1 \\
212 \cdot 1 \\
\hline
2121 \\
4242 \\
2121 \\
4242 \\
\hline
44986 \cdot 41 \times 2 =
\end{array}
$$

$$
\begin{array}{r}
89972 \cdot 82 \\
\hline
4\,34492 \cdot 82 \\
4 \\
\hline
1 \cdot 37971 \cdot 28 \\
40 \\
\hline
15 \cdot 18851 \cdot 20
\end{array}
$$

which gives the area 4a. 1r. 15p., or the same as by calculation from the traverse book.

TO PLOT THE TRAVERSE FROM LATITUDE AND DEPARTURE.

(**324**).—The traverse book having been made up and found to prove, it will be a simple matter to plot the traverse.

The north to south and east to west lines are drawn with the aid of the T-square on a clean sheet of drawing paper, the point at which they bisect each other being the first station in the traverse.

The latitude of the first line is then taken from the total latitude column, and set off to the scale to which the plan is to be plotted, on the N. to S. line, north or south of the east to west line, according as the latitude is north or south.

The departure of the first line, taken from the total departure column, is similarly set off on the east to west line, east of the north to south line.

Through each of these points lines are drawn with the T-square and at right angles to the lines on which they have been plotted, and the intersection of these lines gives the point to which the first line in the traverse has to be drawn, or the second station in the traverse.

The first line of the traverse is then drawn in from station 1 to station 2.

The latitude and departure for the second line is similarly set off on the N. to S. and the E. to W. lines, and the length and bearing of line 2 plotted in a similar manner, and so on with all the lines of the traverse.

The example given is that of a closed traverse, and the perfect plotting of the last line further proves the accuracy of the work.

MAGNETIC AND TRUE NORTH.

(**325**).—It must not be forgotten that the bearings of lines are taken in the field from magnetic north, and that therefore the proper allowance for the declination of the needle must be made in laying down the north to south line on the paper, when the top of the sheet is to represent true north, as it usually should do. In other cases the north point must be shown in its true relative position.

THE DECLINATION OF THE NEEDLE.

(**326**). *Definitions.*—Declination of the needle is its want of accuracy as a correct indicator of true north.

Allowance for declination is the difference between the north as given by the needle and the true meridian.

There are five points which it will be sufficient for our purpose if we note. They are as follows :—

(1) The declination varies in different parts of the world, and the allowance true for one locality is not therefore true for another.

(2) It is ever changing, so that time is a factor to consider in arriving at the allowance to be made.

(3) Declination is at present west. It attained its extreme western error about 80 years ago, and is now approaching accuracy again.

(4) It is not regular in its variation from year to year, nor even at all times throughout each year, but differs with different seasons.

(5) There are some slight local variations.

(6) Roughly speaking it may be said that the alteration which is now taking place is about seven minutes per annum.

From what has been said it will be gathered that it will be necessary in all important cases, where the compass is relied upon, to ascertain the exact declination, and this is done in either of the following ways :—

      (a) By reference to charts and tables which will be found in the Nautical Almanac ;

      (b) From observations to the sun ;

      (c) From observations to the pole star.

The observations must be at fixed hours, according to the day of the year on which they are made.   (For Table see Appendix.)

The theodolite is the instrument to be used in making the observations with the object of finding the true meridian.

In making observations to the sun or moon, the telescope will point true north, whilst the needle will not read to 360, but show the difference or declination.

In correcting by the sun, a dark glass will, of course, be necessary.

In sighting to' the sun the vertical web of the telescope must be made to bisect the globe ; or it may be made to cut the circumference on either side, and an allowance made from tables.

These and other useful tables will be found in the Nautical Almanac.

CHAPTER XXV.

CURVES.

(**327**). *General Observations.*—It is frequently required to lay out curves connecting straight lines, in which case, in order that there may be no break or angle at the points of junction, the latter should be tangents to the former. These curves may be required for railway or other purposes.

With the higher branches of railway engineering we are not concerned, but the following methods of laying out curves will be useful to the land surveyor, and they are therefore given here.

The first thing we shall require in laying out a curve is its radius, and this may be obtained either (*a*) by scaling from a correct plan, or (*b*) calculation in the field.

TO FIND THE RADIUS FROM A PLAN.

(**328**). *Rule.*—Produce the tangents *Aa*, *Bb*, Illustration No. 189, to meet at *C ;* bisect the angle by
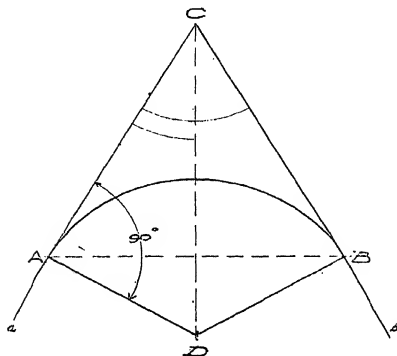
ILLUSTRATION No. 189.

line *CD ;* erect the perpendicular at *A* to cut *CD ;* and *D* is centre, and *AD* radius.

TO FIND THE RADIUS ON THE GROUND.

(**329**).—Range the lines *Aa*, *Bb*, Illustration No. 189, to meet in *C ;* set up the theodolite at *C*, and read the angle *ACB*. The straight lines to be connected by the curve being tangents, they will be at right angles to the radius ; and the angle *CAD* will, therefore, be 90°. Subtract half the angle *ACB* from 90° to obtain the angle *ADC*. Chain the line *AC*. Now, by trigonometry, as the sine of angle *ADC* is to the side *AC*, so is the sine of the angle *ACD* to the radius *AD*.

TO CALCULATE THE OFF-SET.

(**330**).—The first thing to be done is to settle on some *distance* not exceeding one-twentieth the *radius* of the curve. Next divide the square of this distance by twice the radius, to obtain the *off-set.*

TO LAY OUT A CURVE WITH THE CHAIN.

(**331**).—Now (see Illustration No. 190) lay out the " distance" from point 1, at which the curve is to start, in a line with the tangent AA ; set off the off-set at right angles to it at a, its extremity, to find point 2 in the curve.
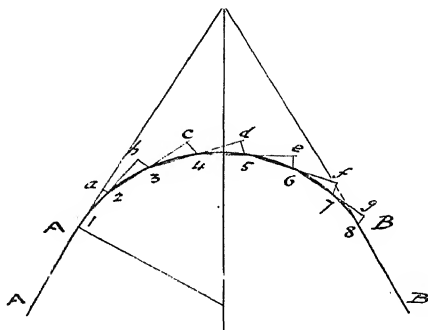
ILLUSTRATION No. 190.

Range a line from point 1 through point 2, and lay off the " distance" on it from point 2 to b, and this time set off *twice* the off-set at right angles to the distance line, to find the third point in the curve.

Drive a line from the second point through the third, and lay off the distance from point 3 to c; again set off the double off-set to find the fourth point, and repeat this process until tangent B is reached. The last must be a *single* off-set, and will exactly cut the tangent if the work has been accurately executed.

Stout pegs should be driven down at points 1, 2, 3, 4, 5, 6, 7 and 8, through which the curve is to pass.

METHOD OF LAYING OUT CURVES WITH THE THEODOLITE.

(**332**).—The manner in which curves are laid out with the aid of the theodolite will be readily understood if the following illustration is carefully followed.

Suppose we had an accurate plan with the tangents and curve desired to be laid out correctly shown thereon. It would be an extremely simple matter to divide the curve into any number of equal parts, and then by drawing lines from each of those divisions to A and E, the points in the tangents from which the curve respectively starts and finishes, to construct a series of triangles within the curve, each with its apex forming a point in the curved line, as shown in Illustration No. 191.
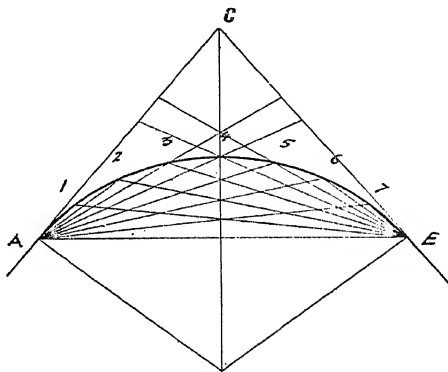
ILLUSTRATION No. 191.

Again, given the chord $A E$, and the angles of each triangle, and they might easily be plotted by the protractor, and the curve laid down by that means, the curved line being drawn through the apex of each triangle.

Now, just as each of these triangles might be plotted on paper with the protractor, so they might be laid down in the field by the aid of two theodolites, one adjusted over each of the points A and E.

The lines of sight representing the respective sides of each triangle would cross each other in the apex of that triangle, and by sending an assistant out to put down a picket at the point of intersection of those

lines (which he could be aided in doing by signals from the surveyors with the instruments) the point could be established, and the same might be done with each of the triangles, until all the points in the curve had been fixed.

The foregoing illustrates the method by which curves are laid out with the theodolite, with this difference, that the angles to be set out are calculated instead of being taken from a plan.

To calculate these angles we need to have

      (*a*) the radius of the curve to be set out, and

      (*b*) the straight distance between the points in the curve as 1—2, 3—4, etc., Illustration No. 191.

The method of calculating the former has already been given ; the latter is merely a selected measurement, generally one chain, but sometimes a longer distance, when obstructions stand in the way of the points in the curve being fixed at such short intervals.

TO CALCULATE THE TANGENTIAL ANGLES.

(**333**).—The angles to be set out with the theodolite to fix the various points in the curve are called tangential angles, they are generally set out from the tangents *A C* and *E C*, Illustration No. 191, but it will be seen that except in case of obstruction they might equally well be laid out from the chord *A E*, the angle *C A E* being the same as the angle *C E C*.

The tangential angle is found by dividing the common distance 1—2, 2—3, etc., by twice the radius. The result is the sine of the arc, and the angle corresponding to the sine is found in the column of natural sines in Chambers's or other similar tables.

It is more simple, however, to use logs to perform the calculation ; and in that case the angle corresponding to the sine is, of course, found in the tables of *log*. sines.

(**334**).—It may be useful to note the following facts :—

    (1) That the angle obtained from $\frac{distance}{rad \times 2}$ is the
    angle subtended by the distance, as angle b in
    triangle $a\,b\,c$, Illustration No. 192.
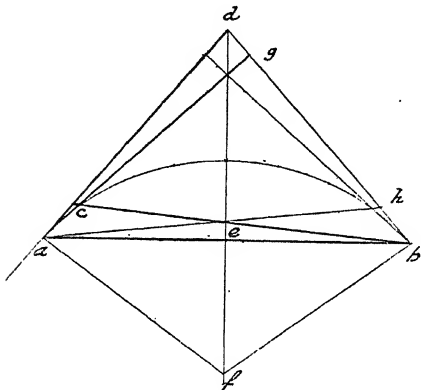
ILLUSTRATION No. 192.

    (2) That this angle is the same as the angle a in
    triangle $d\,a\,g$.

    (3) That the angle a in triangle $c\,a\,b$ may be
    found by subtracting angle a in triangle
    $d\,a\,g$ from angle f in triangle $a\,f\,c$, and
    consequently the triangle may be set out
    from either a or b.

    (4) That the distances being equal, the angles they
    subtend are equal ; and consequently having
    calculated one angle, as the angle b in triangle
    $c\,b\,a$, which is equal to the angle a, in triangle
    $d\,a\,g$, the series of angles to be set out by the
    theodolites from the tangents ad, bd, may be
    found by simply multiplying the angle calcu-
    lated by 2, 3, 4, etc.

(5) That the same angles are set out by each of the theodolites, but not at the same time; thus the first angle a in triangle $d\,a\,g$, set out by the theodolite at a, is the same as the last angle b in triangle $h\,b\,a$ set out by the theodolite at b, and vice versa.

It is best to write out a table of the angles to be set out simultaneously by each theodolite, which will be similar, only the angle which appears first in the table for the theodolite at A will appear last in the table for the theodolite at B.

TO LAY OUT A CURVE WITH THE THEODOLITE.

(**335**). (*a*) *When only One Instrument is employed.*— When only one theodolite is used the angle between the distance line and the tangent only will be required.

The theodolite is adjusted over the point from which the curve is to start, the plates are clamped at zero, and the whole instrument is turned to sight a picket put down at the point at which the tangents would meet if extended, which lines must of course be carefully ranged out. When the picket is sighted the whole instrument is clamped and finely adjusted, so that the cross wires accurately cut the picket.

The vernier plate is then released and set accurately to the required angle. The chain is now extended from the starting point over which the theodolite is adjusted, and in a direct line with the line of sight, the office of the theodolite being to give the direction, and a peg is driven down at the end of the chain.

The upper plate of the theodolite is again released and set to the required angle for the second point, which is twice the angle for the first, and the chain is once more extended from the second point in such a direction that the end of it is brought into the line of sight seen

through the telescope of the theodolite, and a third peg is driven down.

The theodolite is then set for the third angle, the chain extended from point 2 to point 3, the end brought into the line of sight and a fourth peg is driven down, and so on until all the points in the curve have been established.

(**336**). (*b*) *When Two Theodolites are employed.*— Much time is saved by using two theodolites, and the chain is dispensed with.

The instruments are adjusted over the points in each of the tangents at which the curve will join them, and the angles previously calculated as directed are set out simultaneously, either from the chord *A B* or the tangents.

An assistant is sent out with a picket, and the surveyors with the theodolites direct him by signalling with the hand, or otherwise, until he brings the picket to the point at which the direct lines of visions as given by the telescopes cross each other.

A stout peg is driven down at each point as it is thus established.

TABLES OF OFF-SETS FOR RAILWAY CURVES.

(**337**).—It may also be noted here, that the *off-sets* for laying out curves with the chain may be obtained without calculation from tables of off-sets for railway curves.

CHAPTER XXVI.

THE USE OF LOGARITHMS.

————

DEFINITION.

(**338**).—Logs are a collection of ascertained numbers so constructed that by adding *logs* we perform multiplication of their *common numbers*, by *subtracting logs* we perform *division of their common numbers*, by *multiplying logs* we perform the *raising of powers of their common numbers* by *dividing logs* we perform the *extraction of roots of their common numbers*.

THE PURPOSE OF LOGS.

(**339**).—The purpose of logs is to lessen labour and save time in calculations.

The logs of the numbers to be dealt with are found, and when the required operations of multiplying, dividing, raising of powers or extraction of roots, etc., have been conducted by their aid, the common numbers, which the

resulting logs represent, are found in the tables, and thus considerable calculations are performed with great ease and the saving of much time.

ILLUSTRATION.

Thus if we wish to multiply 36894793 into 49763182, we find the log. corresponding to each of these numbers in the tables, and add them together, we then find in the tables the common number corresponding to the resulting log., viz., the sum of the addition, which represents the quotient obtained by multiplying the two numbers referred to into each other, and thus the operation is performed very simply and in far less time than could be done without the aid of logs.

The above illustration is only intended to show the practical use of logs, the method of using them will now be dealt with in detail.

The examples which follow have, by kind permission, been taken in modified form, from "Chambers' Mathematical Tables," with a copy of which the reader should provide himself. They will be required in following the examples in this chapter and again in connection with the solution of triangles.

THE NATURE OF LOGS.

(**340**).—From the definition given it will be gathered that a log. must be a number representing the power to which some fixed base must be raised to produce the number of which it is the log.

In the Tables of Common Logs the base is always 10.

The common log. of any number therefore is the *power* to which 10 must be raised to produce that number, thus :—

$$10^1 = 10 \therefore 1 = \log \text{ of } 10 \text{ called log. } 10$$
$$10^2 = 100 \therefore 2 = \text{,, } 100 \text{ ,, } 100$$
$$10^3 = 1000 \therefore 3 = \text{,, } 1000 \text{ ,, } 1000$$
$$10^4 = 10000 \therefore 4 = \text{,, } 10000 \text{ ,, } 10000$$
$$10^5 = 100000 \therefore 5 = \text{,, } 100000 \text{ ,, } 100000$$

And so on.

Also :—

$$10^0 \text{ or } \tfrac{10}{10} = 1 \qquad \therefore \quad 0 = \text{log. of } 1$$
$$10^{-1} \text{ ,, } \tfrac{10}{100} = \tfrac{1}{10} = \cdot 1 \therefore -1 = \text{ ,, } \cdot 1$$
$$10^{-2} \text{ ,, } \tfrac{10}{1000} \text{ or } \tfrac{1}{100} = \cdot 01 \therefore -2 = \text{ ,, } \cdot 01$$

And so on as we divide successively by 10.

Since

$$10^1 \text{ equals } 10 \text{ and therefore } 1 \text{ equals log. of } 10$$
$$\text{and } 10^0 \text{ ,, } 1 \text{ ,, } 0 \text{ ,, } 1$$

the log. of any number between 1 and 10 must be between 0 and 1, that is to say, some decimal without an integer.

Again, since

$$10^1 \text{ equals } 10 \text{ and therefore } 1 \text{ equals log. of } 10$$
$$\text{and } 10^2 \text{ ,, } 100 \text{ ,, } 2 \text{ ,, } 100$$

the logs of any numbers between 10 and 100 must be something between 1 and 2 or 1 plus a decimal, and likewise, where there are *3* integral figures the log. will be *2* plus a decimal.

The integral part of a log. is called the characteristic ; the decimal part is called the mantissa.

Since $10^0 = 1$ and therefore $0 = \text{log. of } 1$, the log. of any proper fraction, or decimal without an integer, must be less than 0, or in other words negative.

Example.—As an illustration let us consider the log. of ¾.

Divide 1 into 4 parts and take 3 of them, thus :—

$$\text{log. of } 1 = 0$$
$$\text{ ,, } 4 = \cdot 6020600$$

Subtract these logs, which is the same as dividing common numbers, and we obtain .. .. $\bar{1}\cdot 3979400$

$$\text{log. of } 3 = \cdot 4771213$$

And adding these logs, which is equal to multiplying common numbers, we get the log. of ¾ or ·75 .. .. .. $\bar{1}\cdot 8750613$

From the above we see that :—

    (1) The characteristic only is negative and the mantissa positive, which is always the case ;

    (2) That the position of the decimal point does not affect the mantissa at all, but that the characteristic is affected.

RULES FOR FINDING THE CHARACTERISTIC.

(**341**).—The tables only supply the mantissa; the characteristic must be supplied by the calculator.

This is done by the following rules :—

    (1) When the number whose log. is sought contains one or more integral figures, the characteristic is always *one less* than the number of integral figures, and is positive. Thus, the characteristic of 322 would be 2; of 4783 it would be 3; of 5·6481 it would be 0; of 56·6841 it would be 1, and so on.

    (2) If the number is wholly a decimal, its characteristic is the same as the place from the decimal point which its first significant figure occupies, and is *negative*

The negative sign is placed *over* the characteristic, thus $\bar{5}$.

The characteristic of ·1468 is $\bar{1}$; of ·068 it is $\bar{2}$; of ·0008 it is $\bar{4}$, and so on.

To Use the Tables.

TO FIND THE LOG. CORRESPONDING TO A GIVEN NUMBER.

(**342**). *Where the number whose log. is sought contains not more than three figures.*—The mantissa will be found opposite the number on one of the first five pages of Chambers's Tables; the characteristic must be supplied as before directed.

Example.—To find the log. of 852 turn to page 5, and in the second line of the third column we find

No.	Log.
852	9304396.

9304396, then, is the mantissa, and there being three figures in the number, 852, the characteristic will be 2, and the complete log. is 2·9304396.

(**343**). *Where the number contains four figures.*—The mantissa is found on pages 6 to 185, under 0 at the top of the page and opposite the number itself in the extreme left-hand column.

Example.—To find the log. of 374·5 turn to page 60, and there in the extreme left-hand column you find 573, and under 0 at the top of the page, 4518, so that the mantissa is 5734518, and we have to supply the characteristic, which, as there are three integral figures in the number, 374·5, is 2, thus we have the complete log. 2·5734518.

(**344**.) *Where the number contains five figures.*—The mantissa is found on pages 6 to 185, opposite the first four figures on the left-hand side of the page and under the fifth at the top of the page.

Example.—To find the log. of 87647, turn to page 161 and opposite 876·4 we find 942, and under 7 on the same line 7371, and the mantissa is therefore 9427371, and there being five figures in the number, the characteristic will be 4.

(**345**). *Where the log. sought is that corresponding to a number containing six or seven figures.*—Find the mantissa for the first *five* figures as before directed, then take the difference between this log. and the next higher, multiply this difference by the figures above five considered as decimals and add the product to the log. of the first five figures for the complete log.

Example.—Thus, to find the log. of 387548.

The log. of 38754	=	·5883165
The log. of the next higher figure, viz., 38755	=	·5883277
Difference	=	112
The figure above five is 8		·8
And 112 × ·8 (8 considered as a decimal)	=	89·6
	or say	90
The log. of the first five figures, as above, is	=	5883165
	plus	90
which gives us the correct mantissa	=	5883255

and the characteristic 5 (there being 6 figures in the
number whose log. is sought) must be supplied to give us
the complete log. 5·5883255.

The difference may also be found by inspection from the
marginal right-hand column; thus, on page 63 under 112
and opposite 8 we find 90, or the same proportional part
as we calculated in the last example.

At the commencement of the tables, on pages 1 to 13,
the differences are not all given in the marginal columns,
but if the nearest difference is taken it will be sufficiently
accurate for most purposes.   If great accuracy is required
correct by the rule before given.

TO FIND THE NUMBER CORRESPONDING TO A GIVEN LOG.

(**346**).   *General Observations*.—I have dealt with the
method of finding the *logarithm corresponding to a given
number*, and will now give the method of finding the
number corresponding to a given logarithm.

Method of Practice.—It is obvious that the two opera-
tions are the reverse of each other, and a good method
of practice to acquire readiness and accuracy in using the
tables is first to find the logs corresponding to given
numbers, and then to find the numbers from the tables
corresponding to the ascertained logs, without reference
to previous work, and finally to compare results to ascertain
whether the operations have been accurately performed.

When the exact log. is found in the tables.—In some
cases, of course, the exact logs will be found in the tables,
and then no calculation will be necessary.

Assume that we wish to find the *number* corresponding
to the log. 4·6410575.

First of all there is the characteristic 4.   In seeking
logs the characteristic is supplied by the calculator and is
not found in the tables, so we shall look in the tables for

the number corresponding to the *mantissa* only, viz. : we shall look in the tables of logs for 6410575, and when we have found the log. *and the corresponding number*, which is what we are seeking, we shall put the decimal point one place farther to the right than the number of the characteristic, because that is the reverse operation to making the characteristic one less than the integral figures, as we do when seeking the *log*. corresponding to a given number.

Then, as the reverse operation of looking for the *number* and taking out the corresponding mantissa, we look for the *mantissa* and take out the corresponding number.

On page 73 of Chambers's Tables we find in the first column of logs 641, and in the line above this and in the column numbered 8 at the top of the page we find $\overline{0575}$. The line drawn over the figures in the tables signifies that the number which should be prefixed to them from the first column of logs is 641 in the line below, not 640, which relates to all the other numbers except those over which the line is drawn, and the number corresponding to the log 6410575 is that in the first column (*the column of numbers*) in a line with *0575*, not that in a line with *641*, and the number at the top of the column in which 0575 is found, viz., 8.

The number corresponding to 6410575 is, therefore, 43758·, the decimal point coming after the *5th* figure because the characteristic in the given log. is *4* or one *less*.

So much for the case where the exact log. may be found in the tables.

When the exact log. is not found in the tables.— When the exact log. is not found in the tables, the following is the method of finding the number corresponding to a given log.

Suppose it is required to find the *number* corresponding to the log. 3·9212074.

Take out the next *lower* log. and 5 figures of the
number belonging to it; then find the difference between
this logarithm and the given one, and under the corre-
sponding tabular difference find the proportional part
which is equal to this difference, and opposite it is found
the sixth figure of the number; but if the difference is not
exactly found among the proportional parts, take the next
lower part, and the figure opposite it is the sixth figure
of the number; subtract this part from the given differ-
ence, annex to the remainder a cipher, consider it as a new
proportional part, and find the corresponding figure as
before, and it will be the seventh figure of the number.

EXAMPLE.

	NUMBER SOUGHT.	GIVEN LOG.
Given log.		3·9212074
Next lower log.		3·9212025
Number corresponding to next lower log.	8340·7	
Difference between given and next lower log.		49
Now look to the *marginal* column of *tabular differences* for 49, but the exact figure is not found, so the next lower, 47, is taken and the corresponding number is 9. This 9 is annexed to the number already found, thus ..	·09	47
The difference is 2, a 0 is annexed and it becomes 20, and now we look in the tabular differences for 20, but 21 is the nearest found and 4 is the corresponding number. The 4 is annexed to the number already found as the 7th figure	·004	20
Which gives us the correct num- ber corresponding to the log. 3·9212074 as	8340·794	

If the foregoing process is carefully followed it will be
seen that the operation is the reverse of that employed in

finding a log. corresponding to a given number, and the comparison will greatly help in impressing the mind with the rules for finding logs corresponding to given numbers and numbers corresponding to given logs.

CALCULATIONS WITH LOGS.

I shall now deal with :—

    (1) Addition of logs ;

    (2) Subtraction of logs ;

    (3) Multiplication of logs ;

    (4) Division of logs ;

    (5) To perform multiplication of common numbers by the aid of logs ;

    (6) To perform division of common numbers by the aid of logs ;

    (7) To perform involution by logarithms ;

    (8) To perform evolution by logarithms ;

    (9) To find the logarithmic sine or other trigono-metrical quantities of an arc ;

    (10) To find the degrees, minutes and seconds corresponding to any given logarithmic sine or other quantity.

ADDITION OF LOGS.

It may be as well to note :—

    (1) The *mantissa* of logs is always *positive* ; the *characteristic* may be *positive* or *negative*.

    (2) The rules of algebraic addition apply.

(**347**).—*To add two negative characteristics.*

Rule.—Take their sum, and make it negative. $\overline{2}$ and $\overline{3}$ give $\overline{5}$.

To add a positive and a negative characteristic.

Rule.—Take their difference, giving the sign of the greater. 6 and $\bar{2} = 4$; 5 and $\bar{2} = 3$; $\bar{5}$ and $2 = \bar{3}$; $\bar{2}$ and $1 = \bar{1}$.

$$
\begin{array}{rl}
 & 5{\cdot}3468541 \quad \text{added} \\
\text{to} & \bar{3}{\cdot}2685427 \\
\hline
 & 2{\cdot}6153968 \\
\end{array}
$$

NOTE.—There is plus 1 carried from the mantissa, therefore 7 and $\bar{2}$ give 5.

$$
\begin{array}{l}
6{\cdot}3874654 \\
\bar{2}{\cdot}9245636 \\
\hline
5{\cdot}3120290 \\
\end{array}
$$

(**348**).—*To subtract a negative characteristic.*

Rule.—Change its sign to plus and add *according to the above rules for addition.* $\bar{3}$ from $2 = 5$; $\bar{5}$ from $\bar{2} = 3$; $\bar{3}$ from $\bar{5} = \bar{2}$.

$$
\begin{array}{rl}
\text{From} & 2{\cdot}6847658 \\
\text{subtract} & \bar{3}{\cdot}2468543 \\
\hline
 & 5{\cdot}4379115 \\
\end{array}
$$

NOTE.—Here there is 1 carried from the mantissa which must be subtracted, thus $\bar{2}$ becomes $\bar{3}$ and $\bar{5}$ and 5 give 2.

$$
\begin{array}{rl}
\text{From} & \bar{2}{\cdot}3468537 \\
\text{subtract} & \bar{5}{\cdot}7654626 \\
\hline
 & 2{\cdot}5813911 \\
\end{array}
$$

(**349**).—*To multiply a logarithm with a negative characteristic.*

Rule.—Multiply the fractional part by the common rules, then multiply the negative characteristic, which will give a negative result, and add the carriage to it by the rule above.

$\bar{2} \times 5 = \overline{10}$ (minus and plus gives minus) and assuming 2 of carriage to be added, the result would be $\bar{8}$.

$$
\begin{array}{r}
\bar{2}{\cdot}3685464 \times 2 \\
2 \\
\hline
\bar{4}{\cdot}7370928 \\
\end{array}
$$

NOTE.—Here $\bar{3} \times 6 = \overline{18}$ with carriage 4, gives $\overline{14}$.

$$
\begin{array}{r}
\bar{3}{\cdot}7856473 \times 6 \\
6 \\
\hline
\overline{14}{\cdot}7138838 \\
\end{array}
$$

(**350**).—*To divide a log. having a negative characteristic.*

Rule.—Where the characteristic is divisible by the divisor, write down the quotient with a negative sign, but where such is not the case, add such a *negative* number to it as will make it exactly divisible, and *prefix* an equal *positive* integer to the fractional part of the log., and then divide the increased negative characteristic and the other part of the log. separately.

Thus divide : $\overline{14} \cdot 3268472$ by 9. Now 14 cannot be exactly divided by 9, and we therefore add $\overline{4}$, when it becomes $\overline{18}$ (which is exactly divisible by 9), and to $\cdot 3268472$ we add plus 4, when it becomes 4.3268472, thus we have not changed the value of the log., which is now $\overline{18} + 4 \cdot 3268472$, and we divide as follows :—

$$9 \;)\; \overline{18} + 4 \cdot 3268472$$
$$\overline{\; 2 \cdot 4807608}$$

(**351**).—*To perform multiplication of common numbers by the aid of logarithms.*

Rule.—Find the logs representing the given numbers to be multiplied by each other. Add these logs, which will give the log. of the product, and find the number corresponding thereto for the answer.

(**352**).—*To perform division of common numbers by the aid of logarithms.*

Rule.—Find the logs representing the divisor and dividend, and from the one subtract the other for the log. of the quotient. Then find the common number which this resulting log. represents, for the answer.

(**353**).—*To perform involution by the aid of logarithms.*

Rule.—Multiply the log. of the given number by the exponent of the power to which it is to be raised (to square, multiply by 2 ; to cube, multiply by 3, and so on) and the product will be the log. of the required power, and the number corresponding thereto must be found from the tables.

(**354**).—*To perform evolution by the aid of logarithms.*

Rule.—Divide the log. of the given number by the exponent of the root which is required to be extracted (2 for square root, 3 for cube root, etc.), and the quotient will be the *log.* of the required root, and the number corresponding thereto must be found in the tables.

TO FIND THE LOG. SINE OF AN ARC

(that is, to find the log. sine representing the degrees, minutes and seconds in an arc).

(**355**). *When the degrees are under 45.—Rule.*—They are found in the tables in column marked sine, the degrees are given at the *top* of the page and the minutes in the left-hand column. The log. sine is found opposite the given minutes.

When the degrees exceed 45.—Rule.—The degrees are given at the *foot* of the page in the column marked sine, and the minutes are given in the *right*-hand column. The log. sine is found in a line with the given minutes in the arc.

When the arc contains degrees, minutes and seconds.— The tables do not give *seconds* ; they have to be calculated as follows.—*Rule.*—Find the degrees and minutes as before advised, and for the seconds multiply the tabular difference in the difference column next the log. sine for the degrees and minutes found, by the number of seconds, divide the product by 60 and add the quotient as a proportional part to the log. sine for the degrees and minutes already found.

The same rules apply to the other trigonometrical quantities.

TO FIND THE DEGREES, MINUTES AND SECONDS CORRESPONDING TO ANY LOG. SINE.

(**356**). *Rule.—When the exact log. is found in the tables.*—The degrees will be found at either the top or

the foot of the column in which the log. sine is contained, and the minutes will be found in a line with the log. sine.

Rule.—*When the exact log. is not found in the tables.*— Find the next *lower* log., subtract same from given log., multiply the difference by 60 and divide the product by the tabular difference, and the result will be the number of seconds required.

The rules for other trigonometrical quantities are the same.

As, in the case of logarithms, finding the numbers corresponding to given logs is simply the reverse operation of finding the logs corresponding to given numbers, so finding the degrees, minutes and seconds corresponding to a given log. sine, etc., is simply the reverse operation to finding the log. sine, etc., corresponding to the degrees, minutes and seconds of a given angle.

CHAPTER XXVII.

SOLUTION OF TRIANGLES.

General Remarks—Reduction of Problems to workable form—Examples—Method of Study—Brief Explanation—Sine—Tangent—Functions of Angles above 90°, how found from the Tables—The Ambiguous Case—To determine the Correct Solution when more than one—Log. Sines and Log. Tangents—Cases 1, 2, 3, 4, 5 and 6, with Worked Examples.

GENERAL REMARKS.

(**357**).—It is not my intention to attempt to write a chapter on trigonometry. For this I have three reasons :—

(1) There are already some excellent text books on that subject ;

(2) Any one who wishes to pursue it would be well advised to obtain personal tuition ;

(3) A knowledge of trigonometry in the strictest sense is not necessary for our purpose.

It is quite necessary, however, that we should be able to solve triangles, which, so far as it goes, is trigonometry of course, and this I shall deal with.

Most books on land surveying which have included the subject at all, have either contained a heavy chapter on trigonometry far more difficult for students to follow than the majority of complete works on the subject ; or they have given trigonometrical formulæ without indicating how they may be applied.

My attempt will be to take a middle course ; to supply sufficient formulæ for our purpose, and so far to give ample illustration of their application, and worked examples.

No doubt, one reason why students generally find so much difficulty in following works on this and like subjects, is that they are usually filled with algebraic formulæ which, however beautiful and concise to the master mathematician, are very annoying to those who do not understand them.

The examples, too, generally given, although good, have been of the school type—not applied particularly to the surveyor's work, not such in fact as the candidate for the profession is likely to encounter in examinations in land surveying.

In a subject like this it is only after having answered a great many questions that one can handle them safely, hence the importance of intending examinees following a number of examples set as surveying problems rather than mere exercises.

REDUCTION OF PROBLEMS TO WORKABLE FORM.

(**358**).—To understand a problem, to be able to reduce it to a mere example in which certain parts are given and other parts required, is often more difficult than the merely mechanical work of calculating, when that has been done.

I always recommend my pupils to make a sketch or diagram (if time permits to scale) and to figure thereon the dimensions given. In this way the most complicated problems may generally be reduced to a triangle or number of triangles, certain parts of which are given, and the remaining parts of which (or some of them) are required. Even in quite simple cases, this making of diagrams often prevents slips which would otherwise occur.

EXAMPLES.

I will give one or two illustrations. Suppose the question is as follows :—

Example 1.—It is required to know the distance between two buildings, *DE*, which cannot be measured owing to obstructions.

A line, *AB*, was measured exactly 200ft. long some distance from the buildings, and false pickets put down at its extremities. The theodolite was then set up over each of the false pickets, and the following angles read: from station *A*, and between picket *B* and the building nearest thereto, 30° 5′ 10″; between the two buildings, 15° 20′ 30″; and from station *B* between first picket and building nearest thereto, 45° 6′ 2″; between the buildings, 12° 30′ 15″. What is the distance between the buildings?

Now, making a sketch, as advised, we have Illustration No. 193, from which exactly what is given and what sought are at once apparent.
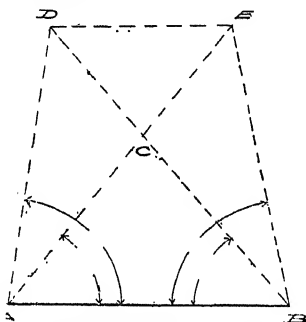
ILLUSTRATION No. 193.

Then by subtracting the two angles read in triangle *ABC* from 180° we get angle *ACB*, and as angle *DCE* is equal, we have that also. Then again, by subtracting angle *ACB* from 180° we obtain angle *ACD*, and as angle *BCE* is equal, we have that also. Again, by subtracting from 180 the sum of angles *DAC* and *DCA* we obtain angle *ADC*, and similarly we may find angle *CEB* by subtracting the sum of angles *EBC* and *ECB* from 180 degrees.

Then sine ∠*ADB* is to side *AB* as sine ∠*DBA* is to side *DA*.

Then sine $\angle DCA$ is to side AD (found) as sine $\angle DAE$ is to side DC.

The same may be done on the other side of the figure to find line CE, when we should have lines DC and EC and the included angle.

Then $\frac{1}{2}$ ($\angle EDC + \angle DEC$) = $90 - \frac{1}{2} \angle DCE$ and side (DC + side EC) is to (side DC − side EC) as tan. $\frac{1}{2}$ ($\angle EDC + \angle DEC$) is to tan. $\frac{1}{2}$ ($\angle EDC - \angle DEC$).

Thus we shall have calculated half the sum and half the difference of angles EDC and DEC, and by adding these we get the greater and by subtracting them we get the less.

Then, as sine angle CED is to side DC, so is sine angle DCE to side DE, which represents the distance to be calculated.

Again, suppose a problem as follows :—

Example 2.—It is required to lengthen a base line AB for a trigonometrical survey. Show how that may be done without measuring the line on the ground.

Now, again, making a sketch we obtain Illustration No. 194, in which we set out the steps to be followed.
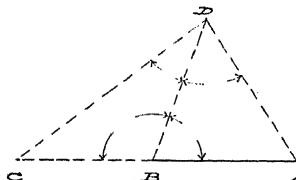
ILLUSTRATION No. 194.

Set up the transit theodolite at B and sight to A. Turn the telescope right over on its axis and sight to C, which will give the straight line in which C is the point to which the line is to be extended. Set up the theodolite at B and read the angles CBD and DBA, which together equal $180°$. Set up the theodolite at point D, some distance outside the line AB, and read angles CDB and

BDA. Subtract the sum of angles *DBA* and *BDA* from 180° for angle *DAB*, and the sum of angles *DBC* and *CDB* from 180 to find angle *DCB*.

Then sine angle *BDA* is to side *AB*, as sine angle *DAB* is to side *DB* ; and again sine angle *DCB* is to side *BD* (just calculated) as sine angle *CDB* is to side *BC*.

Thus we shall have calculated the distance *CB*, which, added to distance *AB*, gives the full length of the extended base line.

The same operation should be repeated on the other side of the base line, and it may likewise be extended from the other end in a similar manner.

METHOD OF STUDY.

(**359**). *Course recommended.*—In all subjects involving figures and calculations, it is absolutely necessary to master every part of the subject as it proceeds, before passing on to that which follows. Knowing it is not sufficient, we must be able to apply our knowledge readily. This subject is no exception to the rule. In studying from a book, thoroughly master each article before you proceed to the following one, and work out all the examples given, turning to the tables and taking out the logarithms, logarithmic sines, tangents, etc., for yourself as they occur. If this is done with close attention, no insurmountable difficulty will be met with.

FUNCTIONS.

(**360**). *Brief Explanation.* — With the object of simplifying matters I shall give :—

    (1) Rules rather than formulæ.

    (2) As few rules as possible, although there may be others equally good for the purpose.

    (3) Worked examples in each case.

We will first notice that in every triangle there exists a

ratio between the various parts, and therefore if any three parts are given (excepting only the three angles) the remaining parts may be calculated.

But as we have to deal with angles, which we are accustomed to express in degrees (°), minutes (′) and seconds (″), in conjunction with the lengths of the sides expressed in ordinary numbers, and from them to calculate other angles and sides, it will be necessary that we have the values of the angles expressed in like terms, viz., in common numbers.

Some idea of how the values of angles are expressed in common numbers may be gathered from the following considerations.

We know if we construct a right-angled triangle with base and perpendicular equal, the angles opposite the right angle will also be equal, or 45 degrees each ; also that the hypotenuse will bear a given proportion to the sides, thus the square on the base plus the square on the perpendicular will equal the square on the hypotenuse, and that the longer side will be opposite the greater angle.

Here then we see that the proportion of the sides implies given angles, and the angles imply a relationship of sides.

Just in the same way, then, that the proportion of sides in the right-angled triangle alluded to implies given angles, so in every conceivable right-angled triangle the proportion of sides indicates the angles.

Now, referring back to Illustration No. 185, if we divide the *perpendicular* by the *hypotenuse* we obtain a result which is called the *sine of angle A*, and thus we have a result which, whilst being expressed in common numbers, nevertheless indicates a certain angle. Similarly, by dividing the *base* by the *hypotenuse* we obtain the cosine, and by dividing the *perpendicular* by the *base*, the tangent. Also, by dividing the *hypotenuse* by the *perpendicular* we obtain

the cosecant, by dividing the *hypotenuse* by the *base* we obtain the secant, and by dividing the *base* by the *perpendicular* we obtain the cotangent.

(**361**). *Sine, Cosine, Tangent, etc.*—Calculations have been made for every degree, minute and second of the arc, up to 90 degrees, and embodied in tables, which also give the degrees, minutes and seconds corresponding, and so we can find on inspection the number representing the sine, cosine, tangent, etc., of any angle, or the angle represented by any sine, cosine, tangent, etc.

(**362**).—In solving triangles we operate with the functions, and if the case is one in which we are seeking an angle and the result of our calculations is therefore a sine, tangent, etc., we finally turn to the tables and take out the angle corresponding.

The tables I have alluded to would be tables of natural sines, etc.; but there are tables of logarithmic sines which give the logarithms of the numbers represented, with ten added to prevent negative characteristics.

We shall not need therefore to calculate the sines and tangents, as they are taken from the tables, and as we shall usually be working in logarithms, because of the reduction of labour they afford, it will be the tables of logarithmic sines, etc., we shall consult.

It will be observed that we shall not in the majority of cases be dealing with right-angled triangles; but then there is in every triangle a proportion between the sine of one angle and the side subtending it, and the sine of any other angle and the side subtending that angle, and therefore where the particulars given permit that application, the calculation becomes a mere case of simple proportion. But where this is not so, a certain relationship always exists by which the parts sought may be calculated from the parts given. This will be made clear in the following rules and examples.

HOW THE FUNCTIONS OF ANGLES OVER NINETY DEGREES ARE FOUND FROM THE TABLES.

(**363**).—The sine, cosine, tangent, etc., of any angle over 90 degrees is obtained by taking out the sine, cosine, tangent, etc., of its supplement, viz., the angle found by subtracting the given angle from 180 degrees. Thus, when the given angle is 130 degrees, we have 180 minus 130 equal 50 degrees, and we look out the sine or tangent for 50 degrees.

THE AMBIGUOUS CASE.

(**364**).—When we have a triangle to solve, such as that shown in Illustration No. 195, the given parts being two of the sides and the angle opposite one of them, as angle A, and sides a and c, and the angle given
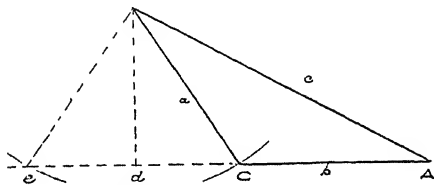
ILLUSTRATION No. 195.

is acute, side a is longer than the perpendicular $B\,d$, but shorter than the side c, there may be two solutions, for side a would plot equally well at C or e, with B as centre, and the angle might be either ACB or its supplement $A\,e\,B$, which is equal to $B\,C\,e$.

TO DETERMINE THE CORRECT SOLUTION.

(**365**).—If by the calculations the side a is found to be longer than the side c, then the acute angle $B\,e\,A$ and not the obtuse angle BCA will be the correct one.

Should the calculations show that the side a is equal to the sine of angle A multiplied by the side b, then it

must be a right-angled triangle, and there can only be one solution. If neither is the case, then there are two possible solutions.

TABLES OF LOG. SINES, TANGENTS, ETC.

(**366**).—As in the tables ten has been added to the logarithms of the numbers representing the sines, tangents, etc. (see Art. 362), it follows that ten must be deducted in making calculations with them, as will be shown in the following examples.

RULES AND EXAMPLES.

I will now give the few rules which it is necessary to know, and some simple examples in illustration of their application.

Before studying this chapter it is absolutely necessary that the previous one, on the use of logarithm tables, should be mastered.

(**367**). *Case 1.*—To find the base or perpendicular of a right-angled triangle when one side (not the hypotenuse) and an adjacent angle are given.

Rule.—Multiply the tangent of the given angle by the given adjacent side (not the hypotenuse) for the other side.

Example.—I desired to know the width of a river at a certain point, and set up the theodolite exactly 20ft. from the water's edge and in the line the length of which I wished to ascertain. I ranged out another line on the bank, at right angles to that to be ascertained, exactly 100ft. long, and setting up the theodolite over its extremity found the angle between lines drawn therefrom to the point previously occupied by it and an object at the water's edge on the opposite bank in the line to be ascertained, to be exactly 45 degrees.   What was the width of the river ?

Now we know at once from the angles that we have here a right-angled triangle whose perpendicular and base are equal, and therefore the width of the river is 100ft.

less 20ft., the distance of the theodolite from the water's edge in its first position. But this simple figure is taken for example purposely, because, the case being self-evident, we are convinced of the truth of the rule. The calculation would be precisely the same if the most awkward figures had been selected, and by carefully following the workings any example coming under this case can be dealt with.

Proceeding then as though the case required solution, and following the advice of Art. 358, we make a sketch, as per Illustration No. 196.
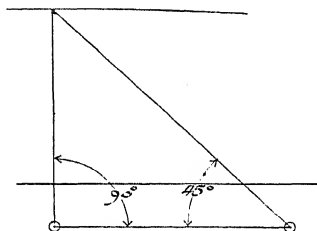
ILLUSTRATION No. 196.

Then the log. of	200	2·3010300
Log. tan. 45 degrees		10·0000000
		12·3010300
Deduct 10 added in tables		10

2·3010300 the number corresponding
to which is 200.

Thus we find by calculation the distance between the position first occupied by the theodolite and the opposite bank of the river to be 200ft., as we know must be the case, and we have therefore proof of the truth of the rule.

(**368**). *Case 2.*—To find the base and perpendicular of a right-angled triangle when the hypotenuse and one of the angles opposite the right angle are given.

Rule.—(*a*) To find the base : Multiply the hypotenuse by the cosine of the given angle.

(*b*) To find the perpendicular : Multiply the hypotenuse by the sine of the given angle.

Example 1.—Latitude and Departure.—What is the latitude and departure respectively of the line which is 300ft. long, and has a bearing of 35 degrees?

Here (see Art. 320) the latitude is north and the departure east.

Making a sketch, Illustration No. 197, we see the
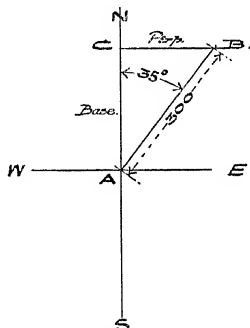
ILLUSTRATION No. 197.

latitude equals base, and the departure the perpendicular of the right-angled triangle *ABC*.

The latitude equals :—

Log. 300	= 2·4771213
Log. cos. 35 degrees =	9·9133645
	12·4904858
Deduct 10 added in tables	10·
	2·4904858
Next lower log.	2·4904782 number corresponding 245·74

The latitude is therefore 245·74 feet east

The departure equals :—

Log. 300	= 2·4771213
Log. sin. 35 degrees =	9·7585913
	12·2357126
Deduct 10 added in tables	10·
	2·2357126
Next lower log.	2·2357052 number corresponding 172·1

The departure is therefore 172·1 feet north.

NOTE.—From this example we gather how traverse tables may be compiled.

Example 2.—Correction of Inclines.—Where chaining is done on hilly ground an allowance has to be made to reduce hypotensual to horizontal measure (see Arts. 77 and 190).

Given the length on the slope and the angle of acclivity or declivity and the horizontal line might, of course, be ascertained as in the last example.

Since, however, the ratio between the diagonal and the base is found direct in the table of natural secants no such calculation is necessary.
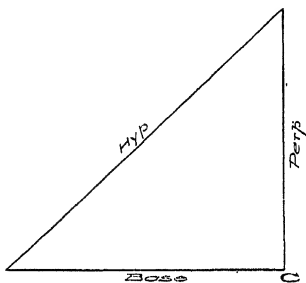
ILLUSTRATION No. 198

Referring to Illustration No. 198, the natural secant of angle $A = \frac{hyp.}{base}$ or the ratio existing between hyp. and base having regard to the angle A.

In chaining on the sloping surface the length of hyp. is obtained, and we require to know what to deduct therefrom to find the base.

Suppose the following case:—A line 10 chains long was measured on the surface of land which sloped 5 degrees. What deduction should be made from this chainage to obtain the horizontal measurement?

Looking in the table of natural secants, and under 5 degrees, we find 1·0038198; the ratio, therefore, is as 1 to 1·0038198, and ·0038198 of a link must be deducted from every link, or ·38198 links from every chain, or $3\frac{4}{5}$ links only from the 10 chains.

But the deduction to be made rapidly increases as the angle of slope increases, thus, from the tables, we find for 10 degrees the natural secant is 1·0154266, so that ·0154266 of a link must be deducted from every link, or 1·54266 links from every chain, or 15·4266 links from the 10 chains. Whilst, therefore, in this case the angle of slope is double, the deduction is, roughly speaking, four times as great in the one case as it is in the other. (See Appendix for " Table for Correction of Inclines.")

SOLUTION OF TRIANGLES OTHER THAN RIGHT-ANGLED TRIANGLES.

(**369**). *Case 3.*—To find the remaining sides and angle of a triangle when one side and the two adjacent angles are given.

Rule.—Subtract the sum of the given angles from 180 degrees to find the remaining angle. Then, as the sine of that angle is to the given side, so is the sine of either of the other angles to its opposite side.

Example.—I desired to continue two chain lines obstructed by a river. They crossed each other at some
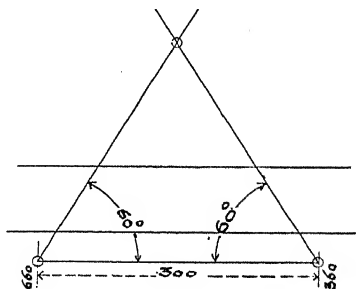
ILLUSTRATION No. 199.

distance on the other side of the water, and each started from a chain line on this side at 360 and 660 links respectively. I set up the theodolite at each of these

points, and measured the angles between the lines, each of which I found to be exactly 60 degrees. What length must be added to each of the chain lines when chained from the crossing point on the other side of the river?

Now, making a sketch as advised in Art. 358, we have Illustration No. 199.

Here again, we know from the fact that the angles are equal and 60 degrees, we have an equilateral triangle, and for information there is no need of calculation, but as an example it is self-evident and proves the truth of the rule. Working, then, in logs, we have :—

$$180° - (60° + 60°) = 60°$$

Then, sine 60° is to 300 as sine 60° is to the side subtending that angle, and working this out in logs, we have:—

Log. sin. 60	=	9·9375306
Log. 300	=	2·4771213
		12·4146519
Log. sin. 60	=	9·9375306
		2·4771213 the number corresponding to which is 300.

Thus by calculation the line is found to equal 300 links, which we know must be the case, having regard to the angles. The other side would, of course, be calculated in precisely the same way.

(**370**). *Case 4.*—To find the remaining side and angles of a triangle when two sides and one of the angles opposite one of the sides are given.

Rule.—The side given is to the opposite angle as the other given side is to its opposite angle. Multiply the side adjacent to the given angle by the sine of the given angle, and divide the product by the other given side to obtain the sine of the opposite angle.

Example.—I made a chain survey of a triangular field two sides only of which could be measured accurately owing to difficulties, but chaining past many obstructions I made the third line 1202 links. On using the theodolite

I found it impossible to take the angles between the lines measured, and was successful in reading one only of the other two angles, which I found to be exactly 60 degrees. The two lines chained exactly 1200 links each. Was my measurement past the obstructions correct?

Making a sketch we have Illustration No. 200.
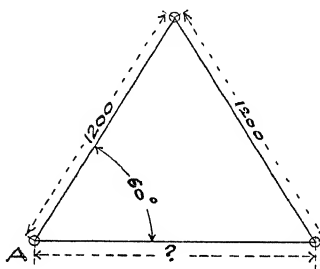
ILLUSTRATION No. 200.

Then by the rule, side *BC*, 1200 links, is to sine angle *A*, 60 degrees, as side *AC*, equal 1200 links, is to the sine angle *B*.

Working this by logs. precisely in the same manner as we should if it was a case requiring solution, we get :—

Log. sin. 60 degrees	=	9·9375306
Log. 1200	=	3·0791812
		13·0167118
Log. 1200	=	3·0791812
		9·9375306

which is the log. sin. of an angle of 60 degrees, as will be seen from the tables.

We have now two of the angles of the triangle and may easily find the third by subtracting their sum from 180 degrees, and then :—

Log. 1200	=	3·9791812
Log. sin. 60 degrees	=	9·9375306
		13·9167118
Log. sin. 60	=	9·9375306
		3·9791812

the number corresponding to which is 1200.

This shows that the true length of the third line is, as we know must be the case, 1200 links, and, therefore, the chaining past the obstructions was in error to the extent of two links.

(371). *Case 5.*—When two sides and the included angle are given, to find the other side and angles.

The rules applicable to the solution in this case being somewhat more complicated than those previously dealt with, the student should give them special attention.

Questions set in examinations usually involve the application of a combination of the rules, and it is absolutely necessary, therefore, that all of them should be thoroughly mastered.

Rule.—The sum of the sides is to their difference, as the tangent of half the sum of the angles sought is to the tangent of half their difference. Half their difference added to half their sum equals the larger, and half their difference subtracted from half their sum equals the smaller of the angles sought.

The sum of the angles sought is found by subtracting the given angle from 180 degrees.

Or giving the rule in another form :—Multiply the difference of the given sides by the tangent of half the sum of the angles sought, and divide the product by the sum of the given sides for the tangent of half the difference of the given angles. Look out the angle represented in the tables and add this angle to half the sum of the angles sought for the larger angle, and subtract this angle from half the sum of the angles sought for the smaller angle.

Example.—Two straight roads, which meet in a town, incline at an angle of 90 degrees. What is the length of a straight road which runs from the one to the other at a distance therein from the point of junction in the town of respectively 300 and 400 yards ?

Making a sketch we have Illustration No. 201.
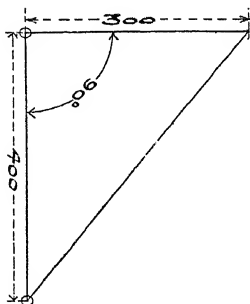
ILLUSTRATION No. 201.

$$400 + 300 = 700.$$
$$400 - 300 = 100.$$

Sum of angles in triangle	180 degrees.
Given angle	90 ,,
Sum of angles sought	90

Then by the rule, 700 is to 100 as tan. 45 degrees (half sum of angles sought) is to the tan. of half the difference of the angles sought. Working this in logs we have :—

Log. 100	=	2·0000000
Log. tan. 45 degrees	=	10·0000000
		12·0000000
Log. 700	=	2·8450980
		9·1549020
Nearest log. tan.		9·1541739 which represents an angle of 8° 7′ 0″

Difference   7281

Multiply by   60

Divide by tab. difference 9030)436860(48″   which added to angle
found direct from
tables   ..   ..   48″

Gives half difference of angles sought 8° 7′ 48″

Half sum of angles sought	= 45°
Half difference of ditto	= 8° 7′ 48″
Larger angle sought	53° 7′ 48″

Then deducting 53° 7′ 48″ from 90° we obtain 36° 52′ 12″ which is the third angle.

Then to find the side sought, by Case 3—As sin. angle 36° 52′ 12″ is to 300, so is sin. 90° to the side sought, and working this in logs we have :

```
Log. 300                =   2·4771213
Log. sin. 90            = 10·0000000

                          12·4771213
Log. sin. 36° 52′ = 9·7781186
Tab. diff.  =  1685
Seconds  =     12
            ____
         60 )20220(    337
                      ____   9·7781523

                             2·6989690  the number corresponding to
                             _____       which log. is 500.
```

Thus, by calculation, we find the side sought to be 500 yards, which we know must be the correct result, having regard to the fact that it is a right-angled triangle with its base and perpendicular in the proportion of three to four.

(**372**). *Case 6.*—When the three sides only are given to find the angles.

Rule.—From half the sum of the three sides subtract each side separately, multiply the three remainders continually together and divide the product by the half sum. Extract the square root of the quotient, and divide the result separately by the three remainders to obtain the tangents of half of each of the angles respectively.

This rule may seem somewhat complicated, but its application will be found extremely simple with the aid of logs., as will be seen from the following example.

Example.—I desired to know the angle formed by two lines, but found it impossible to use the theodolite owing to obstructions. I managed, however, with difficulty, to measure the lines for part of their distances, and likewise obtained a tie line from 300ft. in line 1 to 400ft. in

line 2, and this I found to measure 500ft. What is the angle?

Now following the rule—
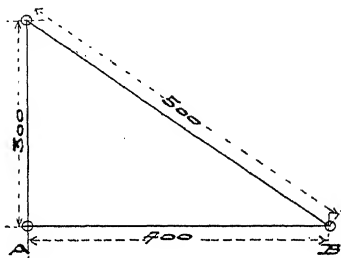
ILLUSTRATION No. 202.

$$
\text{Three sides} \begin{cases} 300 \\ 400 \\ 500 \end{cases}
$$

2)1200 sum of three sides.

600	600	600 half sum three sides
300	400	500
300	200	100

Log. 100 =   2·0000000
   200 =   2·3010300
   300 =   2·4771213

         6·7781513
Log. 600 =   2·7781513

Extract square root 2)4.0000000

         2·0000000
   Add 10·0000000

         12·0000000
Log. 300   = 2·0000000

Result = 10·0000000, which is the log. tan. 45 degrees, and represents the tan. of half the larger angle sought.

Then to obtain the other two angles, following the rule—

		12·0000000 as found above.	
Log. of	200 =	2·3010300	

		9·6989700	
Nearest log. tan.	= 9·6986847, which represents an angle of		26° 33' 0''

Difference	=	2853	
Multiply by	=	60	

Divide by tab. diff. 3159)171180(54 seconds, which added to angle found direct from tables gives angle	0 0 54
	26° 33' 54''

The results, being half the actual angles, must be multiplied by 2 for the correct angles. Thus we have—

45° 0' 0'' multiplied by 2 =	90° 0' 0''
26° 33 54 multiplied by 2 =	53 7 48
	143 7 48
which deducted from 180 degrees gives as the value of the third angle	36 52 12
	180° 0' 0''

This result, compared with those in the example given in the preceding case, in which the figures used were the same, will further convince of the truth of the rules, as the same answer has been arrived at in the two cases, though by a different method.

APPENDIX.

APPENDIX.

MENSURATION RULES—LINES AND AREAS.

RIGHT-ANGLED TRIANGLES.

GIVEN.	SOUGHT.	HOW FOUND.
The two legs	The hypotenuse..	Square the legs, add the products, and extract the square root of the sum.
The hypotenuse and one leg.	The other leg ..	(a) Multiply the sum of the hypotenuse and leg by their difference, and extract the square root of the product to find the other leg. OR (b) From the square of the hypotenuse subtract the square of the given leg, and extract the square root of the other leg.
Base and perpendicular.	Area	Multiply the base by the perpendicular and take half the product for the area.
Area and base ..	Perpendicular ..	Divide twice the area by the base to find the perpendicular.
Area and perpendicular.	Base	Divide twice the area by the perpendicular to find the base.

EQUILATERAL TRIANGLES.

GIVEN.	SOUGHT.	HOW FOUND.
Length of side ..	Area	First find the perpendicular as follows: 1 : ·866 :: Side : Perp. Then compute the area by multiplying base by perpendicular and taking half product.
Area	Length of side ..	By simple proportion the area given is to ·43301 as 1² is to the square of the side sought.

It may be well to note the fact that a hexagon is divisible into six equilateral triangles, the area of one of which may be computed as above explained, and multiplied by 6 for the area of the hexagon.

TRIANGLES.

GIVEN.	SOUGHT.	HOW FOUND.
Area and perpendicular	Base	Divide twice area by perpendicular.
Area and base ..	Perpendicular ..	Divide twice area by base.
Base and perpendicular	Area	Multiply base by perpendicular and take half product.
Three sides	Area	From half the sum of the three sides subtract each side separately, multiply the half sum and the three remainders continually together, and extract the square root of the product for area.

TRIANGLES (*continued*).

GIVEN.	SOUGHT.	HOW FOUND.
Three sides	Perpendicular and area.	To find the perpendicular.—The same is to the sum of the other two sides as the difference of those sides is to the difference between the segments into which the base would be divided by a perpendicular let fall from the apex. Then half this difference added to half the base will equal the longer of the segments, and half the difference subtracted from half the base will equal the shorter segment. The area may then be computed from base and perpendicular.
Two sides of a triangle and one corresponding part of a similar triangle.	The other corresponding side of the similar figure.	By simple proportion the two parts of the first figure given are to each other as the similar part of the second figure is to its corresponding side.

It is worth notice—

(a) That triangles whose sides are in the proportion of three, four and five are right-angled triangles, and therefore the area may be computed from the base and perpendicular.

(b) That where triangles are equilateral—the three sides equal—a perpendicular let fall from any angle will bisect the opposite side. Therefore,

> although the three sides only are given, the
> area may be computed from base and per-
> pendicular, the latter being first calculated.

It may also be noted that if the diameter of a circle is
drawn and any chord at right angles thereto, and the
extremities of the diameter and one end of the chord are
united, three similar triangles are formed, given any two
parts of one of which and a corresponding part of another,
and the second corresponding part may be found by the
simple rule given above. Thus, given the chord and the
versed sine of an arc of a circle and the diameter and the
chord of half the arc may be easily calculated.

It is only necessary to make a diagram to see how the
following rules have been arrived at.

Diameter equals half the chord of the whole arc squared,
divided by the versed sine, plus the versed sine.

The chord of half the arc equals the square root of half the
chord of the whole arc squared, plus the versed sine squared.

The diameter equals the chord of half the arc squared
divided by the versed sine.

The versed sine equals the chord of half the arc squared
divided by the diameter.

The chord of half the arc equals the square root of the
diameter multiplied by the versed sine.

SQUARE.

GIVEN.			SOUGHT.		HOW FOUND.
Side	..	..	Area	..	Square the side for the area.
Area	..	..	Side	..	Extract square root of area

It may be noted that the area of a square circum-
scribing a circle is twice the area of the inscribed
square.

RECTANGULAR FIGURES.

GIVEN.	SOUGHT.	How FOUND.
Sides	Area ..	Multiply the sides together for the area.
Area and one side ..	Other side ..	Divide the area by the given side for the other side.
Area and the proportion which the sides bear to each other.	Lengths of sides ..	Divide the given area by the product of the terms of proportion, extract the square root of the quotient and multiply the result separately by the terms of proportion for the respective sides.

TRAPEZOID.

GIVEN.	SOUGHT.	How FOUND.
Lengths of sides and perpendicular distance between them.	Area ..	Multiply half the sum of the sides by the perpendicular distance between them.

TRAPEZIUM.

GIVEN.	SOUGHT.	How FOUND.
Diagonal and the perpendicular distances to each of the opposite angles.	Area ..	Multiply the diagonal by the sum of the perpendiculars and take half the product for the area.
Four sides, when the opposite angles are supplements.	Area ..	From half the sum of the four sides subtract each side separately, multiply the four remainders continually together and extract the square root for the area.

RHOMBUS AND RHOMBOID.

GIVEN.	SOUGHT.	How FOUND.
The length of sides and perpendicular distance between them.	The Area ..	Multiply the side by the perpendicular distance between the sides for the area.

NARROW STRIPS BOUNDED BY CURVED LINES.

Simpson's Rule.—Divide the straight boundary up into an even number of equal parts, at each of which divisional points take off-sets to the curved boundary; or, where the boundaries are both curvilineal, the off-sets may be taken from a chain line running through the centre of the strip, similarly divided.

Then to the sum of the first and last off-sets add four times all the even off-sets, and twice all the odd off-sets (not including the first and last); multiply this sum by the common distance between the off-sets from fence to fence, and take one-third of the product for the area.

SIMILAR FIGURES.

GIVEN.	SOUGHT.	How FOUND.
A side and the area of a figure and a corresponding side of a similar figure.	The area of the similar figure.	The area given is to the square of the side of the figure comprising it, as the square of the given corresponding side is to the area of the similar figure.
The areas of two similar figures and the side of one of them.	The corresponding side of the similar figure.	The areas of the figures are to each other as the square of their corresponding sides.

CIRCLES.

GIVEN.	SOUGHT.	How FOUND.
Diameter	Circumference ..	Multiply the diameter by 3 1416.
Circumference ..	Diameter ..	Divide the circumference by 3·1416.
Chord of half arc and whole arc.	Length of arc ..	From eight times the chord of half the arc subtract the chord of the whole arc, and take one-third the quotient for the length of arc.
Degrees in the arc and the radius of the circle of which it is part.	Length of arc ..	180° is to the degrees in the arc, as the radius multiplied by 3·1416 is to its length,
Radius	Area of circle ..	Multiply square of the radius by 3·1416.
Circumference ..	Area of circle ..	Divide the square of the circumference successively by 3·1416 and by 4.
Degrees in sector and radius of circle.	Area of sector ..	360° is to the degrees in the sector as area of circle is to area of sector.
Radius and the length of the arc of the sector.	Area of circle ..	Multiply radius by half length of arc.
Degrees in sector and radius of circle, or radius of circle and length of arc of sector.	Area of segment ..	Find the area of the sector and that of the triangle formed by the radii and the chord of the segment, and subtract the latter from the former to obtain the area of the segment.
The radius of the circle and the width of the ring.	The area of a circular ring.	Find the areas of the outer and inner circles by rules already given, and subtract the area of the smaller from that of the greater for the area of the ring.

ELLIPSE.

GIVEN.	SOUGHT.	HOW FOUND.
The axes	Area	Add the axes, divide the sum by 2, and reserve the result. Square each of the axes, add the products, divide by 2, extract the square root of the sum, and reserve the sum. Add the reserved sums and multiply the result by 3·1416 for the circumference.
The axes	Area	Multiply the product of the semi-axes by 3·1416.

TABLE OF POLYGONS.

NAME OF POLYGON.	No. of Sides.	Inter. Angle.	Central Angle.	The Side of the Polygon being 1		
				Radius of inscribed circle or perpendiculars.	Radius of circumscribed circle.	Area.
Triangle ..	3	60°.0′	120°.0′	·2887	·5773	0·43301
Square ..	4	90.0	90.0	·5000	·7071	1·00000
Pentagon ..	5	108.0	72.0	·6882	· 506	1·72047
Hexagon ..	6	120.0	60.0	·8660	1·0000	2·59807
Heptagon ..	7	128.34$\frac{2}{7}$	51.25$\frac{5}{7}$	1·0383	1·1524	3·63391
Octagon ..	8	135.0	45.0	1·2071	1·3066	4·82842
Nonagon ..	9	140.0	40.0	1·3737	1·4619	6·18182
Decagon ..	10	144.0	36.0	1·5388	1·6180	7·69420
Undecagon ..	11	147.16$\frac{4}{11}$	32.43$\frac{7}{11}$	1·7028	1·7747	9·36564
Dodecagon ..	12	150.0	30.0	1·8660	1·9319	11·19615

CONDITIONS OF POLYGONS OFFERING MEANS FOR PROVING
THE ACCURACY OF TRIGONOMETRICAL SURVEYS.

Divide any polygon into a number of triangles by drawing lines from a centre point within the figure to the several angles, and—

(1) The sum of the angles of each triangle will be 180°;

(2) The sum of the angles meeting in the centre point will be equal to 360°, or four right angles; ·

(3) The sum of the interior angles of the polygon will equal twice as many right angles as the figure has sides, minus four right angles;

(4) Regarding the angles of the several triangles into which the polygon divides, from the centre point, the sum of the sines of the angles on the right will equal the sum of the sines of those on the left;

(5) If a point is selected outside a closed polygon and lines drawn from it to its several angles, a number of triangles will be formed, some falling wholly outside the polygon, the others including its area. Call the former *negative* and the latter *positive*. Likewise, regarding the angles of the several triangles from the point chosen outside the polygon, call them *right* and *left* for each triangle. Then the sines of the " right positive " angles plus the sines of the " left negative " angles will equal the sines of the " right negative " angles, plus the sines of the " left positive " angles.

TRIGONOMETRICAL FORMULÆ.

Referring to Illustration No. 203.
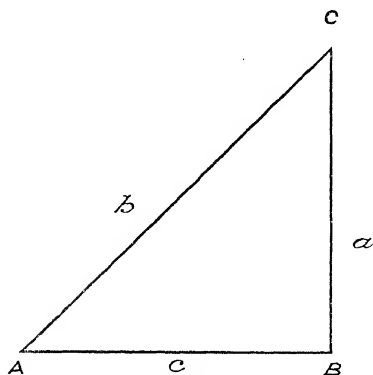
ILLUSTRATION No. 203.

Sine A is he	$\dfrac{\text{perp.}}{\text{hyp.}} = \dfrac{a}{b}$	Therefore	$\begin{cases}\text{Sine} \times \text{hyp.} = \text{perp.} \\ \text{Sine} \div \text{perp.} = \text{hyp.}\end{cases}$
Cos. A	$\dfrac{\text{base}}{\text{hyp.}} \;\; \dfrac{c}{b}$	,,	$\begin{cases}\text{Cos.} \times \text{hyp.} = \text{base.} \\ \text{Cos.} \div \text{base} = \text{hyp.}\end{cases}$
Tan. A	$\dfrac{\text{perp.}}{\text{base}} = \dfrac{a}{c}$	,,	$\begin{cases}\text{Tan.} \times \text{base} = \text{perp.} \\ \text{Tan.} \div \text{perp.} = \text{base.}\end{cases}$
Cot. A	$\dfrac{\text{base}}{\text{perp.}} = \dfrac{c}{a}$	,,	$\begin{cases}\text{Cot.} \times \text{perp.} = \text{base.} \\ \text{Cot.} \div \text{base} = \text{perp.}\end{cases}$
Secant A	$\dfrac{\text{hyp.}}{c} = \dfrac{b}{c}$	,,	$\begin{cases}\text{Secant} \times \text{base} \quad \text{hyp.} \\ \text{Secant} \div \text{hyp.} \quad \text{base.}\end{cases}$
Cosecant	$\dfrac{\text{hyp.}}{\text{perp.}} = \dfrac{b}{a}$	,,	$\begin{cases}\text{Cosecant} \times \text{perp.} = \text{hyp.} \\ \text{Cosecant} \div \text{hyp.} = \text{perp.}\end{cases}$

Also—

$$\sqrt{(1 - \sin.^2)} = \cos.$$
$$\text{Sin.} \div \text{tan.} = \cos.$$
$$\text{Sin.} \times \text{cot.} = \cos.$$
$$\text{Sin.} \div \cos. = \tan.$$
$$\text{Cos.} \div \sin. = \cot.$$
$$\text{Cos.} \div \cot. = \sin.$$
$$\text{Tan.} \div \sin. = \sec.$$
$$\text{Tan.} \div \sec. = \sin.$$
$$\text{Tan.} \times \cot. = \text{rad.}$$

$$\sqrt{(1 - \cos.^2)} = \sin.$$
$$1 \div \cot. = \tan.$$
$$1 \div \sin. = \text{cosec.}$$
$$1 \div \cos. = \sec.$$
$$1 \div \text{cosec.} = \sin.$$
$$1 \div \sec. = \cos.$$
$$1 \div \tan. = \cot.$$

Complement of an angle = its difference from   90°
Supplement   ,,   ,, =   ,,   ,,   180°

The sine of any angle is to the side opposite as the sine of any other angle is to the side opposite that angle ; and *vice versa*,—

The length of any side is to the sine of the opposite angle as the length of any other side is to the sine of its opposite angle.

From which is obtained the simple proportion—

Sine angle A : a :: sine angle C : c

or

a × sine angle C.

sine A

and

a : sine angle A :: b : sine angle B

or

$$\frac{b \times \text{sine angle A}}{a} = \text{sine angle B}$$

When one angle only and the sides forming it are given—

The given angle subtracted from 180° equals the sum of the other two angles, and

The sum of the two sides is to their difference, as the tan. of half the sum of the two angles not given, is to the tan. of half their difference, or

$$\frac{\text{Diff. of two sides} \times \text{tan. of half sum of two angles}}{\text{Sum of two angles}} = \begin{cases} \text{Tan. of half diff.} \\ \text{of two angles.} \end{cases}$$

Then, by adding half the difference of the two angles to half their sum the greater of the two angles is obtained, and by subtracting half the difference of the two angles from half their sum the smaller of the two angles is obtained.

PROPORTION WHICH PLAN BEARS TO LAND REPRESENTED IN VARIOUS SCALES.

CHAIN SCALES.

1 chain to an inch	...	=	$\frac{1}{792}$
2 chains ,,	...	=	$\frac{1}{1584}$
3 ,, ,,	...	=	$\frac{1}{2376}$
4 ,, ,,	...	=	$\frac{1}{3168}$
5 ., ,,	...	=	$\frac{1}{3960}$
6 ,, ,,	...	=	$\frac{1}{4752}$

FOOT SCALES.

100 feet to 1 inch	...	=	$\frac{1}{1200}$
200 ,, ,,	...	=	$\frac{1}{2400}$
10 ,, ,,	...	=	$\frac{1}{120}$
20 ,, ,,	...	=	$\frac{1}{240}$
30 ,, ,,	...	=	$\frac{1}{360}$
40 ,, ,,	...	=	$\frac{1}{480}$

OTHER SCALES.

6 inches to 1 mile	...	=	$\frac{1}{10560}$
4 ,, ,,	...	=	$\frac{1}{15840}$
1 inch ,,	...	=	$\frac{1}{63360}$
25·344 (usually spoken of as 25″ Ordnance Scale)		=	$\frac{1}{2500}$
10·56 feet to 1 mile	...	=	$\frac{1}{500}$
10 ,, ,,	...	=	$\frac{1}{528}$

TABLE SHOWING THE ANGLE OF SLOPE REPRESENTED BY VARIOUS GRADIENTS.

GRADIENT.	ANGLE OF SLOPE.			GRADIENT.	ANGLE OF SLOPE.		
	Degrees.	Minutes.	Seconds.		Degrees.	Minutes.	Seconds.
1 in 100	0	34	20	1 in 18	3	10	50
,, 90	0	38	10	,, 17	3	22	00
,, 80	0	43	0	,, 16	3	34	30
,, 70	0	49	10	,, 15	3	48	50
,, 60	0	57	20	,, 14	4	05	10
,, 50	1	08	40	,, 13	4	24	00
,, 45	1	16	20	,, 12	4	45	40
,, 40	1	26	00	,, 11	5	11	40
,, 35	1	38	10	,, 10	5	42	40
,, 30	1	54	30	,, 9	6	20	20
,, 25	2	17	30	,, 8	7	07	30
,, 20	2	51	20	,, 7	8	07	50
,, 19	3	00	50	,, 6	9	27	40

CURVATURE.

Rules for approximately calculating the allowance to be made for curvature and refraction.

Miles.—Take two-thirds the square of the distance in miles for the correction in feet.

Yards.—Multiply the square of the distance in yards by 257 and cut off 5 decimal places for the correction in inches.

Chains.—Divide the square of the distance in chains by 800 for the correction in inches.

Refraction.—The allowance for refraction may be taken at $\frac{1}{6}$ or $\frac{1}{7}$ that for curvature and must be deducted therefrom.

TABLE OF ALLOWANCE FOR CURVATURE AND REFRACTION WHEN THE DISTANCE IS GIVEN IN CHAINS.

Distance.	Allowance for			Distance.	Allowance for		
	Curvature.	Refraction.	Curvature and Refraction.		Curvature.	Refraction.	Curvature and Refraction.
3	·0009	·0001	·0008	17	·0801	·0043	·0258
3½	·0013	·0002	·0011	17½	·0319	·0046	·0273
4	·0017	·0002	·0015	18	·0338	·0048	·0290
4½	·0021	·0003	·0018	18½	·0357	·0051	·0306
5	·0026	·0004	·0022	19	·0376	·0054	·0322
5½	·0031	·0004	·0027	19½	·0396	·0056	·0340
6	·0037	·0005	·0032	20	·0417	·0060	·0357
6½	·0044	·0006	·0038	20½	·0438	·0063	·0375
7	·0051	·0007	·0044	21	·0459	·0066	.0393
7½	·0058	·0008	·0050	21½	·0481	·0069	·0412
8	·0067	·0010	·0057	22	·0504	·0072	·0432
8½	·0075	·0011	·0064	22½	·0527	·0075	·0452
9	·0084	·0012	·0072	23	·0551	·0079	·0472
9½	·0094	·0013	·0081	23½	·0575	·0082	·0493
10	·0104	·0015	·0089	24	·0600	·0086	·0514
10½	·0115	·0016	·0099	24½	·0625	·0089	·0536
11	·0126	·0018	·0108	25	·0651	·0093	·0558
11½	·0138	·0020	·0118	25½	·0677	·0097	·0580
12	·0150	·0021	·0129	26	·0704	·0100	·0604
12½	·0163	·0023	·0140	26½	·0731	·0104	·0627
13	·0176	·0025	·0151	27	·0759	·0108	·0651
13½	·0190	·0027	·0163	27½	·0788	·0113	·0675
14	·0204	·0029	·0175	28	·0817	·0117	·0700
14½	·0219	·0031	·0188	28½	·0846	·0121	·0725
15	·0234	·0033	·0201	29	·0876	·0125	·0751
15½	·0250	·0036	·0214	29½	·0906	·0129	·0777
16	·0267	·0038	·0229	30	·0937	·0134	·0803
16½	·0284	·0041	·0243				

TABLE OF ALLOWANCE FOR CURVATURE AND REFRACTION WHEN THE DISTANCE IS GIVEN IN MILES.

Distance.	Allowance for			Distance.	Allowance for		
	Curvature.	Refraction.	Curvature and Refraction.		Curvature.	Refraction.	Curvature and Refraction.
¼	·042	·006	·036	12½	104·167	14·881	89·286
½	·167	·024	·143	13	112·667	·16·095	96·572
¾	·375	·054	·321	13½	121·500	17·357	104·143
1	·667	·095	·572	14	130·667	18·666	112·001
1½	1·501	·215	1·286	14½	140·167	20·024	120·143
2	2·668	·381	2·287	15	150·000	21·428	128·572
2½	4·169	·596	3·573	15½	160·167	22·881	137·286
3	6·003	·856	5·147	16	170·667	24·381	146·286
3½	8·171	1·167	7·004	16½	181·500	25·928	155·572
4	10·672	1·525	9·147	17	192·667	27·524	165·143
4½	13·547	1·980	11·517	17½	204·167	29·167	175·000
5	16·675	2·382	14·293	18	216·000	30·857	185·143
5½	20·177	2·882	17·295	18½	228·167	32·581	195·586
6	24·012	3·430	20·582	19	240·667	34·381	206·286
6½	28·181	4·026	24·155	19½	253·500	36·214	217·286
7	32·683	4·669	28·014	20	266·667	38·075	228·592
7½	37·519	5·360	32·159	21	294·000	42·000	252·000
8	42·688	6·098	36·590	22	322·667	46·095	276·572
8½	48·191	6·884	41·307	23	352·667	50·381	302·286
9	54·027	7·718	46·309	24	384·000	54·857	329·143
9½	60·197	8·600	51·597	25	416·667	58·095	358·572
10	66·700	9·529	57·171	26	450·667	64·381	386·286
10½	71·520	10·217	61·303	27	486·000	69·428	416·572
11	80·687	11·527	69·160	28	522·667	74·667	448·000
11½	88·167	12·594	75·573	29	560·667	80·095	480·572
12	96·000	13·714	82·286	30	600·000	85·714	514·286

ANGULAR MEASURE.

60	Seconds ('')	..	..	..	..	..	1 Minute (')
60	Minutes	..	..	..	..	..	1 Degree (°)
30	Degrees	..	..	..	..	..	1 Sign
60	,,	..	..	..	..	..	1 Sextant
90	,,	..	..	..	..	..	1 Quadrant or Right Angle
360	,,	..	..	..	..	..	1 Circle or Circumference

TABLE FOR REDUCING HYPOTENSUAL TO HORIZONTAL
MEASURE.

Angles.		Reduction in Links.	Angles.		Reduction in Links.	Angles.		Reduction in Links.
3°	0′	0·137	12°	30′	2·370	21°	30′	6·958
3	30	0·187	13	0	2·553	22	0	7·282
4	0	0·244	13	30	2·763	22	30	7·612
4	30	0·308	14	0	2·970	23	0	7·950
5	0	0·381	14	30	3·185	23	30	8·294
5	30	0·460	15	0	3·407	24	0	8·645
6	0	0·548	15	30	3·637	24	30	9·004
6	30	0·643	16	0	3·874	25	0	9·369
7	0	0·745	16	30	4·118	25	30	9·741
7	30	0.856	17	0	4·370	26	0	10·121
8	0	0·973	17	30	4·628	26	30	10·507
8	30	1·098	18	0	4·894	27	0	10·899
9	0	1·231	18	30	5·168	27	30	11·299
9	30	1·371	19	0	5·448	28	0	11·705
10	0	1·519	19	30	5·736	28	30	12·118
10	30	1·675	20	0	6·031	29	0	12·538
11	0	1·837	20	30	6·333	29	30	12·964
11	30	2·008	21	0	6·642	30	0	13·397
12	0	2·185						

STATUTE MEASURE.

The following three statutes form the basis of our
Imperial standard measure :—

    34 Henry VIII.

    5 George IV., cap. 74.

    5 and 6 William IV., cap. 63.

The first of these statutes fixed the acre at 10 square
chains.

The Act of 5 George IV., cap. 74, enacted that the yard
(that at present adopted) should be called the "Imperial
Standard Yard "; that a foot be $\frac{1}{3}$ of a yard; an inch $\frac{1}{12}$
part of a foot; a pole $5\frac{1}{2}$ yards; a furlong 220 yards;
a mile 1,760 yards.

Superficial measure was to be computed by the
standard yard, and a rood to be 1,210 square yards;
an acre 4,840 square yards.

The Statute 5 and 6 William IV., cap. 63, abolished all
customary weights and measures.

LINEAR MEASURES.

Inches	Links	Feet	Yards	Poles	Chains	Furlongs	Miles
7·92 =	1						
12	1·5151 =	1					
36	4·5454	3 =	1				
198	25	16·5	5·5 =	1			
792	100	66	22	4 =	1		
7920	1000	660	220	40	10 =	1	
63360	8000	5280	1760	320	80	8 =	1

SQUARE MEASURES.

Inches	Links	Feet	Yards	Poles	Chains	Roods	Acres	Miles
62·7264 =	1							
144	2·2956 =	1						
1296	20·6611	9 =	1					
39204	625	272·25	30·25 =	1				
627264	10000	4356	484	16 =	1			
1568160	25000	10890	1210	40	2·5 =	1		
6272640	100000	43560	4840	160	10	4 =	1	
4014489600	64000000	27878400	3097600	102400	6400	2560	640 =	1

NAUTICAL MEASURE.

6 Feet	make	1 Fathom
120 Fathoms	,,	1 Cable's Length
1,000 Fathoms, or 2027·3 yards..	,,	1 Nautical Mile or Knot
3 Nautical miles	,,	1 League
20 Leagues, or 60 nautical miles	,,	1 Degree

GUNTER'S CHAIN MEASURE—LENGTH.

7·92 Inches .. .. make	1 Link
12 ,, or 1·515 links .. .. ,,	1 Foot
36 ,, or 4·545 links .. .. ,,	1 Yard
198 ,, or 25 links .. .. ,,	1 Pole or Perch
792 ,, or 100 links, or 66 feet, or 22 yards, or ,, 4 poles	1 Chain
7,920 ,, or 1,000 links, or 10 chains .. .. ,,	1 Furlong
63,360 ,, or 8,000 links, or 80 chains	1 Mile

LENGTH OF A MILE IN VARIOUS COUNTRIES IN ENGLISH YARDS.

Russia	1,100	yards.
Germany	5,866	,,
Sweden and Denmark	7,233	,,
Hungary	8,680	,,
France .. .. league	3,666	,,
The English Statue Mile	1,760	,,
The English Nautical Mile or Knot	2,227·3	,,
The Irish Mile..	2,240	,,
The Scots Mile	1,984	,,
The Italian Mile	1,467	,,
The Polish Mile	4,400	,,
The Spanish Mile	5,028	,,
The Geographical Mile	2,026·6	..

LENGTH OF A FOOT IN VARIOUS COUNTRIES IN ENGLISH INCHES.

England .. .. .. 12·00	Prussia .. .. .. 12·36
Spain .. .. .. 11·03	Austria .. .. .. 12·45
Holland .. .. .. 11·14	Portugal.. .. .. 12·96
Sweden .. .. .. 11·69	Russia .. .. .. 13·75
America .. .. .. 12·00	Norway .. .. .. 24·00
Denmark.. .. .. 12·35	France .. .. metre 39·37

TABLE OF LAND MEASURES IN USE IN DIFFERENT COUNTRIES OF EUROPE.

In English Square Yards.

Amsterdam, a *morgen* is ..	.. ..	9,722 Square Yards.
Berlin, a *great morgen* is..	.. ..	6,786 ,, ,,
Dantzic, a *morgen* is ..	.. ..	6,650 ,, ,,
France, a *hectare* is ..	.. ..	11,960¼ ,, ,,
Geneva, an *arpent* is ..	.. ..	6,179 ,, ,,
Hamburg, a *morgen* is ..	.. ..	11,545 ,, ,,
Portugal, a *geiza* is ..	.. ..	6,970 ,, ,,
Prussia, a *morgen* is ..	.. ..	3,053 ,, ,,
Rhineland, a *morgen* is ..	.. ..	10,185 ,, ,,
Rome, a *pezza* is	.. ..	3,158 ,, ,,
Russia, a *Dersetina* is ..	.. ..	13,066 ,, ,,
Spain, a *Fanegada* is ..	.. ..	5,500 ,, ,,
Sweden, a *Tunneland* is ..	.. ..	5,900 ,, ,,
Switzerland, a *faux* is ..	.. ..	7,854 ,, ,,
Tuscany, a *quadretto* is ..	.. ..	4,074 ,, ,,
Vienna, an *ioch* is.. ..	.. ..	6,889 ,, ,,

TABLE FOR CONVERTING DECIMAL PARTS OF AN ACRE INTO ROODS AND PERCHES.

Perches.	0 Rood.	1 Rood.	2 Roods.	3 Roods.	Perches.	0 Rood	1 Rood.	2 Roods.	3 Roods.
0	·000	·250	·500	·750	21	·131	·381	·631	·881
1	·006	·256	·506	·756	22	·137	·387	·637	·887
2	·012	·262	·512	·762	23	·144	·394	·644	·894
3	·019	·269	·519	·769	24	·150	·400	·650	·900
4	·025	·275	·525	·775	25	·156	·406	·656	·906
5	·031	·281	·531	·781	26	·162	·412	·662	·912
6	·037	·287	·537	·787	27	·169	·419	·669	·919
7	·044	·294	·544	·794	28	·175	·425	·675	·925
8	·050	·300	·550	·800	29	·181	·431	·681	·931
9	·056	·306	·556	·806	30	·187	·437	·687	·937
10	·062	·312	·562	·812	31	·194	·444	·694	·944
11	·069	·319	·569	·819	32	·200	·450	·700	·950
12	·075	·325	·575	·825	33	·206	·456	·706	·956
13	·081	·331	·581	·831	34	·212	·462	·712	·962
14	·087	·337	·587	·837	35	·219	·469	·719	·969
15	·094	·344	·594	·844	36	·225	·475	·725	·975
16	·100	·350	·600	·850	37	·231	·481	·731	·981
17	·106	·356	·606	·856	38	·237	·487	·737	·987
18	·112	·362	·612	·862	39	·244	·494	·744	·994
19	·119	·369	·619	·869	40	·250	·500	·750	1·000
20	·125	·375	·625	·875					

SIZES OF DRAWING PAPERS.

Antiquarian	..	53 × 31 inches.	Royal ..	..	24 × 19 inches.
Double Elephant		40 × 26¾ ,,	Medium	..	22 × 17½ ,,
Atlas ..	..	34 × 26 ,,	Demy ..	..	20 × 15½ ,,
Columbier	..	34½ × 23½ ,,	Foolscap	..	17 × 13½ ,,
Imperial	..	30 × 22 ,,	24 Sheets	..	1 Quire
Elephant	..	28 × 23 ,,	20 Quires	..	1 Ream
Sup. Royal	..	27 × 19 ,,			

CHAIN TESTING.

The standards in London for testing chains and other measures are situated in the Guildhall, and on the north side of Trafalgar Square.

A Gunter's chain ought to be 1½ or 2 inches over 66 feet to allow for sagging, which cannot be entirely avoided.

1 in 1,000 is the allowable error in good chaining.

THE MAGNETIC COMPASS.

The magnetic North Pole, or the Pole to which a suspended needle points, is a considerable distance from the North Pole of the earth—its exact distance being 75° N. longitude, 96° 45′ W. The magnetic South Pole is latitude 75½° S., longitude 154° E.

North, South, East and West are terms used to express the relative positions of places to one another. They are the four CARDINAL POINTS of the compass—an instrument used to determine the respective bearings of places. There are altogether thirty-two points of the compass—twenty-eight of the number being intermediate between the four cardinal points. The point lying midway between north and east is called *North-east;* that midway between north and west is *North-west.* In like manner, the points lying midway between the east and west and the south points are called *South-east* and *South-west.*

These points are sometimes called the four Collateral Points. The *Cardinal Points* are generally abbreviated thus: N. S. E. W. The *Collateral Points* thus: N.E., N.W., S.E., S.W.

DECLINATION OF THE NEEDLE.

TABLE OF DATES AND HOURS WHEN THE POLE STAR IS DUE NORTH.		TABLE OF DATES WHEN THE SUN IS DUE SOUTH AT NOON.
January	6 p.m.	
February	4 a.m.	
March	2 a.m.	
April	12 midnight.	April 16th.
May	10 p.m.	June 15th.
August	4 a.m.	September 1st.
September	2 a.m.	December 25th.
October	12 midnight.	
November	10 p.m.	
December	8 p.m.	

POINTS OF THE COMPASS AND THEIR CORRESPONDING
ANGLES WITH THE MERIDIAN.

POINTS.	ANGLE.	NORTH.		SOUTH.	
	° ′ ″				
¼	2 48 45				
½	5 37 30				
¾	8 26 15				
1	11 15 0	N. by E.	N. by W.	S. by E.	S. by W.
1¼	14 3 45				
1½	16 52 30				
1¾	19 41 15				
2	22 30 0	N.N.E.	N.N.W.	S.S.E.	S.S.W.
2¼	25 18 45				
2½	28 7 30				
2¾	30 56 15				
3	33 45 0	N.E. by N.	N.W. by N.	S.E. by S.	S.W. by S.
3¼	36 33 45				
3½	39 22 30				
3¾	42 11 15				
4	45 0 0	N.E.	N.W.	S.E.	S.W.
4¼	47 48 45				
4½	50 37 30				
4¾	53 26 15				
5	56 15 0	N.E. by E.	N.W. by W.	S.E. by E.	S.W. by W.
5¼	59 3 45				
5½	61 52 30				
5¾	64 41 15				
6	67 30 0	E.N.E.	W.N.W.	E.S.E.	W.S.W.
6¼	70 18 45				
6½	73 7 30				
6¾	75 56 15				
7	78 45 0	E. by N.	W. by N.	E. by S.	W. by S.
7¼	81 33 45				
7½	84 22 30				
7¾	87 11 15				
8	90 0 0	E.	W.	E.	W.

ORDNANCE SURVEY.

Ordnance datum as applied to the plans of Great Britain is mean sea level at Liverpool.

Mean sea level is ˙650 feet above mean sea level at Liverpool.

The datum as applied to islands at a distance from the mainlaid is the local sea level.

The ordnance datum for Ireland is spring tide low water mark, Poolbeg Lighthouse, Dublin Bay.

Trinity high-water-mark is 12˙48 feet above the ordnance datum.

Ordnance sheets may be obtained to several scales.

The map to a scale of one inch to a mile is the most useful as a general road map. The new outline edition shows contours at 100 feet intervals and levels along the roads.

The 25 inch map shows buildings and a considerable amount of detail. The levels are given along the streets to one decimal place.

The most usual town map is drawn to a scale of 10˙56 feet to a mile. It shows a very considerable amount of detail of buildings, and may be had in outline, in colours, or cross-ruled.

Indexes to plans of all towns are published, scale 6 inches to a mile. These show roads and sufficient detail for identification purposes.

The contours on the one inch and six inch maps are generally at 100 feet intervals after the first two contours, which are respectively 50 feet and 100 feet above ordnance datum.

The areas of the various enclosures are given on the revised sheets. In the case of the old editions the areas are given in a book of reference.

Areas are computed to the centre of hedges or other boundaries.

Questions of disputed ownership of hedges, etc., cannot be settled, nor the acreage of estates correctly computed from the ordnance maps.

A most useful pamphlet entitled " Ordnance Survey Maps of the United Kingdom : a Description of their Scales, Characteristics, etc.," has been written by Col. D. A. Johnson, R.E., Director-General of the Ordnance Surveys, and printed by Messrs. Darling and Son, 1-3, Gt. St. Thomas Apostle, E.C., price sixpence, which every one anxious to acquaint himself with the ordnance survey and maps, and the abbreviations, symbols, characteristics, etc. used on them, etc., should secure. It may be obtained from all mapsellers.

APPELLATIONS.

Moors.—Large uncultivated tracts of land.

Heath.—Open ground overgrown with shrub or heath.

Downs.—Open pastures.

Fells.—Large open lands.

Wolds.—High open grounds.

Fens.—Low wet ground.

Marshes.—Low swampy grounds.

Mosses.—Turfy, boggy moors.

Forests.—Wild uncultivated tracts of ground abounding with trees.

Ings.—Large open meadows.

Holmes.—Hilly, fenny or level grounds adjoining brooks.

Open Fields.—Uninclosed lands.

SQUARE MEASURE.

	Contents—Inches in Feet.						Decimals of a Foot.		No. of Yards and Feet in Perches.		
NO.	CM	XM	M	C	X	U	D.	INCHES.	P.	YDS.	FEET.
9	6250·5	3	2	·1	·1	·2	·9	129·6	1	30¼	272¼
8	5556·	..	..	..	..	..	·8	115·2	2	60½	544½
7	4861·5	..	..	..	..	..	·7	100·8	3	90¾	816¾
6	4167·	..	..	..	..	..	·6	86·4	4	121	1089
5	3472·5	..	..	..	..	..	·5	72·	5	151¼	1361¼
4	2778·	..	..	..	..	..	·4	57·6	6	181½	1633½
3	2083·5	..	..	..	..	..	·3	43·2	7	211¾	1905¾
2	1389·	..	..	..	..	..	·2	28·8	8	242	2178
1	694·5	2	1	·1	·2	·3	·1	14·4	9	272¼	2450¼

	Contents of Yards, in Acres and Decimals of an Acre.				Value of Decimals of an Acre.				
NO.	CM	XM	M	C	D.	R. P.	D.	P.	YDS.
9	185·9504	2	1	·1	·9	3 24	·09	14	12·1
8	165·2891	..	..	..	·8	3 8	·08	12	24·2
7	144·6280	..	..	..	·7	2 32	·07	11	5·7
6	123·9668	..	..	..	·6	2 16	·06	9	18·1
5	103·3056	..	..	..	·5	2	·05	8	0·
4	82·6446	1	·1	·2	·4	1 24	·04	6	12·1
3	61·9833	..	..	..	·3	1 8	·03	4	24·2
2	41·3222	..	..	..	·2	32	·02	3	6·
1	20·6611	..	..	..	·1	16	·01	1	18·1

30¼ yards = 1 perch.

AVERAGE WIDTH OF FENCES ON VARIOUS FORMATIONS.

Formation.	Width in	
	Links.	Feet.
Chalk	12	8
Colite	12	8
Coralline Oolite ..	14	$9\frac{1}{4}$
Red Sandstone ..	15	10
Oxford Clay	16	$10\frac{3}{8}$
Kimmeridge Clay ..	$16\frac{1}{2}$	11
Lias	18	12

Ditches from 3 to 6 feet in width.

TABLE OF FACTORS.

$105 = 7 \times 5 \times 3$	$210 = 10 \times 7 \times 3$	$336 = 12 \times 7 \times 4$	$512 = 8 \times 8 \times 8$
$112 = 8 \times 7 \times 2$	$216 = 12 \times 9 \times 2$	$343 = 7 \times 7 \times 7$	$528 = 12 \times 11 \times 4$
$125 = 5 \times 5 \times 5$	$220 = 11 \times 10 \times 2$	$350 = 10 \times 7 \times 5$	$540 = 10 \times 9 \times 6$
$126 = 9 \times 7 \times 2$	$224 = 8 \times 7 \times 4$	$352 = 11 \times 8 \times 4$	$550 = 11 \times 10 \times 5$
$128 = 8 \times 8 \times 2$	$225 = 9 \times 5 \times 5$	$360 = 12 \times 10 \times 3$	$560 = 10 \times 8 \times 7$
$132 = 11 \times 12$	$231 = 11 \times 7 \times 3$	$363 = 11 \times 11 \times 3$	$567 = 9 \times 9 \times 7$
$135 = 9 \times 5 \times 3$	$240 = 8 \times 3 \times 10$	$378 = 9 \times 7 \times 6$	$576 = 12 \times 12 \times 4$
$140 = 10 \times 7 \times 2$	$242 = 11 \times 11 \times 2$	$384 = 8 \times 8 \times 6$	$588 = 12 \times 7 \times 7$
$144 = 12 \times 12$	$243 = 9 \times 9 \times 3$	$392 = 8 \times 7 \times 7$	$594 = 11 \times 9 \times 6$
$144 = 9 \times 16$	$245 = 7 \times 7 \times 5$	$396 = 11 \times 9 \times 4$	$605 = 11 \times 11 \times 5$
$144 = 8 \times 18$	$250 = 10 \times 5 \times 5$	$400 = 10 \times 10 \times 4$	$616 = 11 \times 8 \times 7$
$147 = 7 \times 7 \times 3$	$256 = 8 \times 8 \times 4$	$405 = 9 \times 9 \times 5$	$630 = 10 \times 9 \times 7$
$150 = 10 \times 5 \times 3$	$264 = 11 \times 6 \times 4$	$420 = 10 \times 7 \times 6$	$640 = 10 \times 8 \times 8$
$154 = 11 \times 7 \times 2$	$270 = 10 \times 9 \times 3$	$432 = 12 \times 9 \times 4$	$648 = 9 \times 9 \times 8$
$160 = 10 \times 8 \times 2$	$275 = 11 \times 5 \times 5$	$440 = 11 \times 10 \times 4$	$660 = 11 \times 10 \times 6$
$165 = 11 \times 5 \times 3$	$280 = 10 \times 7 \times 4$	$448 = 8 \times 8 \times 7$	$672 = 12 \times 8 \times 7$
$168 = 8 \times 7 \times 3$	$288 = 12 \times 12 \times 2$	$450 = 10 \times 9 \times 5$	$700 = 10 \times 10 \times 7$
$175 = 7 \times 5 \times 5$	$294 = 7 \times 7 \times 6$	$462 = 11 \times 7 \times 6$	$704 = 11 \times 8 \times 8$
$176 = 11 \times 8 \times 2$	$297 = 11 \times 9 \times 3$	$480 = 12 \times 10 \times 4$	$729 = 9 \times 9 \times 9$
$180 = 10 \times 9 \times 2$	$300 = 10 \times 10 \times 3$	$484 = 11 \times 11 \times 4$	$750 = 10 \times 15 \times 5$
$189 = 9 \times 7 \times 3$	$308 = 11 \times 7 \times 4$	$486 = 9 \times 9 \times 6$	$756 = 12 \times 9 \times 7$
$192 = 12 \times 8 \times 2$	$315 = 9 \times 7 \times 5$	$490 = 10 \times 7 \times 7$	$768 = 12 \times 8 \times 8$
$196 = 7 \times 7 \times 4$	$320 = 10 \times 8 \times 4$	$495 = 11 \times 9 \times 5$	$840 = 12 \times 10 \times 7$
$198 = 11 \times 9 \times 2$	$324 = 9 \times 9 \times 4$	$500 = 10 \times 10 \times 5$	$900 = 10 \times 10 \times 9$
$200 = 10 \times 10 \times 2$	$330 = 11 \times 10 \times 3$	$504 = 9 \times 8 \times 7$	

SQUARES AND SQUARE ROOTS—NUMBERS 1 TO 240.

No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.
1	1·0000	49	7·0000	97	9·8488	145	12·0415	193	13·8924
2	1·4142	50	7·0710	98	9·8994	146	12·0830	194	13·9283
3	1·7320	51	7·1414	99	9·9498	147	12·1243	195	13·9642
4	2·0000	52	7·2111	100	10·0000	148	12·1655	196	14·0000
5	2·2360	53	7·2801	101	10·0498	149	12·2065	197	14·0356
6	2·4494	54	7·3484	102	10·0995	150	12·2474	198	14·0712
7	2·6457	55	7·4161	103	10·1488	151	12·2882	199	14·1067
8	2·8284	56	7·4833	104	10·1980	152	12·3288	200	14·1421
9	3·0000	57	7·5498	105	10·2469	153	12·3693	201	14·1774
10	3·1622	58	7·6157	106	10·2956	154	12·4096	202	14·2126
11	3·3166	59	7·6811	107	10·3440	155	12·4499	203	14·2478
12	3·4641	60	7·7459	108	10·3923	156	12·4899	204	14·2828
13	3·6055	61	7·8102	109	10·4403	157	12·5299	205	14·3178
14	3·7416	62	7·8740	110	10·4880	158	12·5698	206	14·3527
15	3·8729	63	7·9372	111	10·5356	159	12·6095	207	14·3874
16	4·0000	64	8·0000	112	10·5830	160	12·6491	208	14·4222
17	4·1231	65	8·0622	113	10·6301	161	12·6886	209	14·4568
18	4·2426	66	8·1240	114	10·6770	162	12·7279	210	14·4913
19	4·3588	67	8·1853	115	10·7238	163	12·7671	211	14·5258
20	4·4721	68	8·2462	116	10·7703	164	12·8062	212	14·5602
21	4·5825	69	8·3066	117	10·8166	165	12·8452	213	14·5945
22	4·6904	70	8·3666	118	10·8627	166	12·8840	214	14·6287
23	4·7958	71	8·4261	119	10·9087	167	12·9228	215	14·6628
24	4·8989	72	8·4852	120	10·9544	168	12·9614	216	14·6969
25	5·0000	73	8·5440	121	11·0000	169	13·0000	217	14·7309
26	5·0990	74	8·6023	122	11·0453	170	13·0384	218	14·7648
27	5·1961	75	8·6602	123	11·0905	171	13·0766	219	14·7986
28	5·2915	76	8·7177	124	11·1355	172	13·1148	220	14·8323
29	5·3851	77	8·7749	125	11·1803	173	13·1529	221	14·8660
30	5·4772	78	8·8317	126	11·2249	174	13·1909	222	14·8996
31	5·5677	79	8·8881	127	11·2694	175	13·2287	223	14·9331
32	5·6568	80	8·9442	128	11·3137	176	13·2664	224	14·9666
33	5·7445	81	9·0000	129	11·3578	177	13·3041	225	15·0000
34	5·8309	82	9·0553	130	11·4017	178	13·3416	226	15·0332
35	5·9160	83	9·1104	131	11·4455	179	13·3790	227	15·0665
36	6·0000	84	9·1651	132	11·4891	180	13·4164	228	15·0996
37	6·0827	85	9·2195	133	11·5325	181	13·4536	229	15·1327
38	6·1644	86	9·2736	134	11·5758	182	13·4907	230	15·1657
39	6·2449	87	9·3273	135	11·6189	183	13·5277	231	15·1986
40	6·3245	88	9·3808	136	11·6619	184	13·5646	232	15·2315
41	6·4031	89	9·4339	137	11·7046	185	13·6014	233	15·2643
42	6·4807	90	9·4868	138	11·7473	186	13·6381	234	15·2970
43	6·5574	91	9·5393	139	11·7898	187	13·6747	235	15·3297
44	6·6332	92	9·5916	140	11·8321	188	13·7113	236	15·3622
45	6·7082	93	9·6436	141	11·8743	189	13·7477	237	15·3948
46	6·7823	94	9·6953	142	11·9163	190	13·7840	238	15·4272
47	6·8556	95	9·7467	143	11·9582	191	13·8202	239	15·4596
48	6·9282	96	9·7979	144	12·0000	192	13·8564	240	15·4919

CUBES AND CUBE ROOTS—NUMBERS 1 TO 240.

No.	Cube Roots.	No.	Cube Roots.	No.	Cube Roots.	No.	Cube Roots.	No.	Cube Roots.
1	1·0000	49	3·6593	97	4·5947	145	5·2535	193	5·7789
2	1·2599	50	3·6840	98	4·6104	146	5·2656	194	5·7889
3	1·4422	51	3·7084	99	4·6260	147	5·2776	195	5·7988
4	1·5874	52	3·7325	100	4·6415	148	5·2895	196	5·8087
5	1·7099	53	3·7562	101	4·6570	149	5·3014	197	5·8186
6	1·8171	54	3·7797	102	4·6723	150	5·3132	198	5·8284
7	1·9129	55	3·8029	103	4·6875	151	5·3250	199	5·8382
8	2·0000	56	3·8258	104	4·7026	152	5·3368	200	5·8480
9	2·0800	57	3·8485	105	4·7176	153	5·3484	201	5·8577
10	2·1544	58	3·8708	106	4·7326	154	5·3601	202	5·8674
11	2·2289	59	3·8929	107	4·7474	155	5·3716	203	5·8771
12	2·2894	60	3·9148	108	4·7622	156	5·3832	204	5·8867
13	2·3513	61	3·9364	109	4·7768	157	5·3946	205	5·8963
14	2·4101	62	3·9578	110	4·7914	158	5·4061	206	5·9059
15	2·4662	63	3·9790	111	4·8058	159	5·4175	207	5·9154
16	2·5198	64	4·0000	112	4·8202	160	5·4288	208	5·9249
17	2·5712	65	4·0207	113	4·8345	161	5·4401	209	5·9344
18	2·6207	66	4·0412	114	4·8488	162	5·4513	210	5·9439
19	2·6684	67	4·0615	115	4·8629	163	5·4625	211	5·9533
20	2·7144	68	4·0816	116	4·8769	164	5·4737	212	5·9627
21	2·7589	69	4·1015	117	4·8909	165	5·4848	213	5·9720
22	2·8020	70	4·1212	118	4·9048	166	5·4958	214	5 9814
23	2·8438	71	4·1408	119	4·9186	167	5·5068	215	5·9907
24	2·8844	72	4·1601	120	4·9324	168	5·5178	216	6·0000
25	2·9240	73	4·1793	121	4·9460	169	5·5287	217	6·0092
26	2·9624	74	4·1983	122	4·9596	170	5·5396	218	6·0184
27	3·0000	75	4·2171	123	4·9731	171	5·5504	219	6·0276
28	3·0365	76	4·2358	124	4·9866	172	5·5612	220	6 0368
29	3·0723	77	4·2543	125	5·0000	173	5·5720	221	6·0459
30	3·1072	78	4·2726	126	5·0132	174	5·5827	222	6·0550
31	3·1413	79	4·2908	127	5·0265	175	5·5934	223	6·0641
32	3·1748	80	4·3088	128	5·0396	176	5·6040	224	6·0731
33	3·2075	81	4·3267	129	5·0527	177	5·6146	225	6·0822
34	3·2396	82	4·3444	130	5·0657	178	5·6252	226	6·0911
35	3·2710	83	4·3620	131	5·0787	179	5·6357	227	6·1001
36	3·3019	84	4·3795	132	5·0916	180	5·6462	228	6·1091
37	3·3322	85	4·3968	133	5·1044	181	5·6566	229	6·1180
38	3·3619	86	4·4140	134	5·1172	182	5·6670	230	6·1269
39	3·3912	87	4·4310	135	5·1299	183	5·6774	231	6·1357
40	3·4199	88	4·4479	136	5·1425	184	5·6877	232	6·1446
41	3·4482	89	4·4647	137	5·1551	185	5·6980	233	6·1534
42	3·4760	90	4·4814	138	5·1676	186	5·7082	234	6·1622
43	3·5033	91	4·4979	139	5·1801	187	5·7184	235	6·1710
44	3·5303	92	4·5143	140	5·1924	188	5·7286	236	6·1797
45	3·5568	93	4·5306	141	5·2048	189	5·7387	237	6·1884
46	3·5830	94	4·5468	142	5·2171	190	5·7488	238	6·1971
47	3·6088	95	4·5629	143	5·2293	191	5·7589	239	6·2058
48	3·6342	96	4·5788	144	5·2414	192	5·7689	240	6·2144

CONVENTIONAL SIGNS.
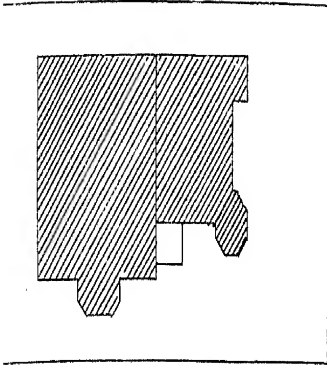
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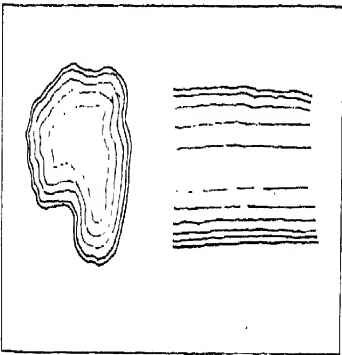
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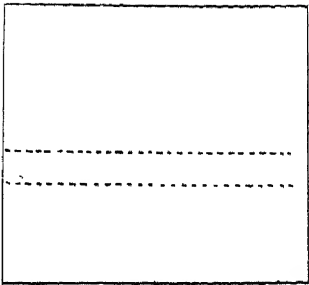
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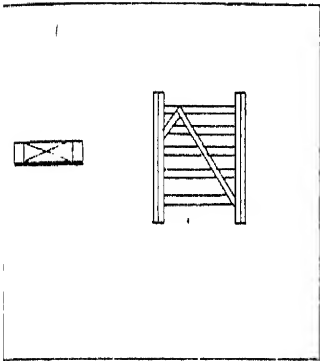
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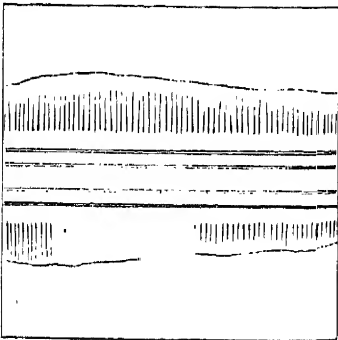
RAILWAY EMBANKMENT.
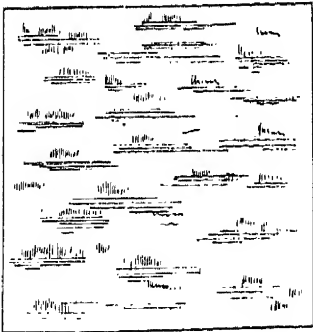
MARSH.
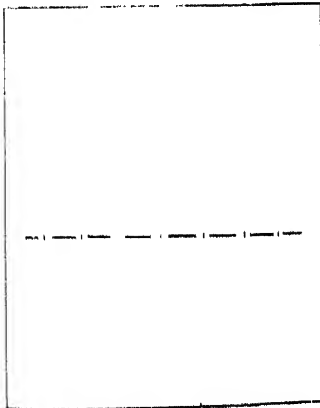
HEDGE AND DITCH.

BOUNDARY.
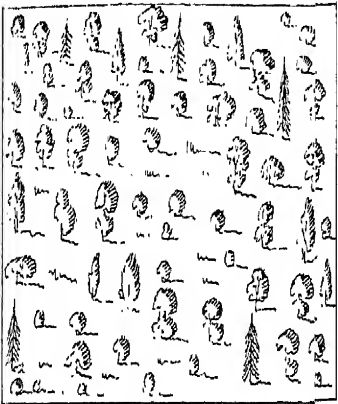
WOOD.
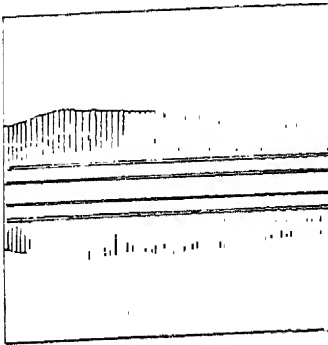
RAILWAY IN CUTTING.

EXAMPLES OF PAPERS SET

SURVEYORS' INSTITUTION EXAMINATIONS.

SURVEYING AND LEVELLING.

Time allowed—Three hours.

NOTE.—*All Candidates are required to attempt questions Nos.* 1, 2, *and* 3.

> *Candidates other than Building Candidates will receive full marks for any* 10 *questions correctly answered.*
>
> *Building Candidates will receive full marks for any* 8 *questions correctly answered.*
>
> *Candidates omitting to leave figures by which results are arrived at, will risk a loss of marks in case of a wrong answer being given through accident.*
>
> *Questions* 1, 2, 3, 6, 7 *and* 9 *carry higher marks than the remainder.*

———

1. On the plan given, draw in pencil the lines it would be necessary to run, to enable you to make a complete survey with the chain only.*

2. Compute the areas of the enclosures in the corner of the plan above mentioned, giving the results in acres, roods and perches; one of these enclosures must be

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* For the plan, enclosures and field notes referred to in questions 1, 2 and 3, see Syllabus issued by the Surveyors' Institution.

computed by means of the ordinary plotting scale, and the other in any way the Candidate may elect. (Enclosure No. 1, if well done and a correct answer arrived at by the ordinary plotting scale, will carry full marks).

3. From the field notes given, lay down the survey lines and plot a plan to a scale of two chains to an inch.

4. Required to set out a circular space for a Reservoir to contain 1 acre, 1 rood, and 20 perches ; give the radius in links.

5. Divide the triangle A B C into three equal portions by lines parallel to the side A B.   A B = 2,500 links ;
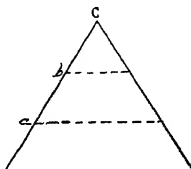
A C = 2,100 ; and B C = 1,800.   Give the area of A B C, and the distances A a, a b, and b c.

6. The points A and B are only both visible from one point C.   Lines C D = 1,260 links and C E = 1,040 links were run, and the following angles were taken, viz. :—
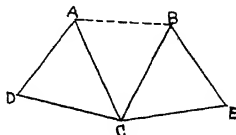
A D C = 67° 30', A C D = 45° 0', A C B = 70° 20', B C E = 39° 10', and B E C = 81° 50'.   Find the length A B in links.

7. Plot the above figure to a scale of one chain to an inch, and give the distance A B as it measures upon your plan.

8. A traverse round a wood is as follows :—

A to B =   290 links, bearing 255° 5'   A .   . B
B to C = 1000 ,,   ,,   194° 10'
C to D = 680 ,,   ,,   77° 12'   D .   . C

give the calculated distance D to A.

9. Protract and plot the above to a scale of one chain to an inch.

10. Convert 17 acres, 1 rood, and 20 perches statute measure into square yards.

11. How would you determine the latitude of any position (on land), and what instrument would you require ?

12. Illustrate and describe in what way you would produce a survey line obstructed by a large tree or building.

13. If a plan is plotted to a scale of 3 chains to an inch, what proportion does the area of the plan bear to the ground ?

AFTERNOON PAPER.

Time allowed—Two hours and a half.

Note.—All candidates are required to attempt questions No. 1 and 2. Candidates other than Building Candidates will receive full marks for any 9 questions correctly answered. Building Candidates will receive full marks for any 7 questions correctly answered. Candidates omitting to leave figures by which results are arrived at will risk a loss of marks in case of a wrong answer being given through accident. Questions 1, 2, 8, 9 and 11 carry higher marks than the others.

1. Make up the following level book :

Back Sight.	Inter-mediate.	Fore Sight.	Rise.	Fall.	Reduced Levels.	Dis-tance.	Remarks.
					Feet.	Chains.	
6·60					45·80	0	
	4·00					1·00	'
	5·70					2·00	
·80		12·20				3·00	
	6·90					4·00	
	11·20					5·00	
·24		13·12				6·00	
	4·80					7·00	
	8·30					8·00	
1·10		13·75				9·00	
	6·70					10·00	
	5·70					11·00	
	8·10					12·00	
2·90		15·05				13·00	{ 1st side of pond
	7·10					14·00	water level.
	10·60					14·80	
	11·70					15·00	
	10·80					16·00	{ 2nd side of pond
	7·10					16·40	water level.
13·75		6·85				17·00	
	11·10					18·00	
	8·60					19·00	
	2·30					20·00	
		·85				21·00	

2. Plot the following section to a horizontal scale of 2 chains to an inch, and to a vertical scale of 20 feet to an inch :—

Height above Base. Feet.	Distances. Chains.
30·00	0·00
31·30	1·00
33·40	2·00
27·85	3·50
27·01	5·00
30·47	6·20
29.43	7·00
26·30	9·00
23·20	10·00
25·20	11·20
29·30	12·00
31·25	14·00
33·31	15·00
34·16	15·60
32·27	17·50
30.25	18·00
29·09	18·50

3. In setting out the centre line for a new road or a railway, illustrate and describe in what way you would proceed to connect two pieces of straight by a curve of, say, 10 chains radius.

4. Before commencing to take a series of levels briefly describe how you would ascertain if your level was in adjustment.

5. The point A being inaccessible and at a considerable altitude above the surrounding country, illustrate and describe in what way you would ascertain its height above the point B (the nearest convenient point of observation), using a theodolite for the purpose.

6. Give the rate of inclination between the given points of level taken upon a line chained along the invert of a water course.

Distance Chains.	Height. Feet.
0·00	41·00
2·40	42·16
3·00	42·50
6·30	44·20
8·00	45·70

7. What is the rate per chain (in feet and decimals) of a gradient rising 1 in 250?

8. Give the levels of points B, C, and D on a continuous section, the level of point A being 25 feet, and the horizontal distances and angles as follows :—

A to B, 12 chains; angle of elevation, 3° 20′
B to C, 9   ,,   ,,   depression, 4° 25′
C to D, 15   ,,   ,,   elevation, 2° 15′

9. The telescope of a theodolite set 4·25 feet above the point A having a level value of 25 feet, is directed towards the bottom of a staff at B, and shows an angle of elevation of 10° 4′ ; it is then directed to 10 feet on the staff, when it shows an angle of elevation of 10° 35′. Required the horizontal distance A to B in feet, and also the level of point B.

10. Illustrate by diagram the difference between "true" and "apparent" level, and give a rule for determining same.

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11. Construct a triangle A B C, having its sides A B = 3 inches, B C = 2½ inches, and A C = 1½ inches. Suppose the points A, B, and C to be trigonometrical stations of a survey, and that from a point D of a traverse A bears 120°, B 150°, and C 165°, find the point D by construction.

12. Explain and illustrate by diagram how you would obtain the distance to an inaccessible point, using only chain and poles.

INDEX

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